CM 2110 Calculus and Statistical Distributions

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Course Syllabus

Pre-requisites

CM 1110

Remark:

This course module contains two main sections: (1) mathematics and (2) statistics. This syllabus is designed for the statistics section. Lectures for mathematics section and statistics section are conducted by two lecturers as two separate sub modules (1.5 hour lectures/Week). End Semester Examination is conducted as a single examination.

Learning Outcomes

On successful completion of this module, students will be able to plan more carefully the design of experiment in advance which provide evidence for or against theories of cause and effect and make inferences about population characteristics based on sample information and thereby solve data analysis problems in different application domains. (R(https://cran.r-project.org/) and RStudio are also freely available to install on your own computer). Get the Open Source Edition of RStudio Desktop. RStudio allows you to run R in a more user-friendly environment.

Outline Syllabus

- Functions of Several Variables
- Linear Algebra
- Coordinate Systems & Vectors
- Differential Equations
- **Statistical Distributions**
- **Estimation**

- **Hypothesis Testing**
- **Design of Experiments**

Method of Assessment

- Mid-semester examination
- End-semester examination

Recommended Texts

- Casella, G., & Berger, R. L. (2002). Statistical inference (Vol. 2, pp. 337-472). Pacific Grove, CA: Duxbury.
- Mood, A.M., Graybill, F.A. and Boes, D.C. (2007): Introduction to the Theory of Statistics, 3rd Edn. (Reprint). Tata McGraw-Hill Pub. Co. Ltd.
- Montgomery, D. C. (2017). Design and analysis of experiments. John wiley & sons.

Lecturer

Dr. Priyanga D. Talagala

Schedule

Lectures:

- Friday [9.15 am - 10.45 am]

Tutorial:

- Friday [11.00 am - 12.30 pm]

Consultation time:

- Friday [8.15 am to 9.00 am]

Chapter 1

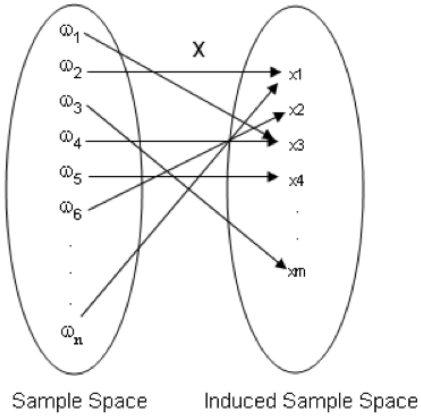
Statistical Distributions

1.1 Random Variable

- Some sample spaces contain quantitative (numerical) outcomes, others contain qualitative outcomes.
- Often it is convenient to work with sample spaces containing numerical outcomes.
- A function that maps the original sample space into the real numbers is called a 'random variable'.
- This is more useful when the original sample space contains qualitative outcomes.

Definition 1: Random Variable

A **random variable** is a function from a sample space S into the real numbers (*i.e.* $X : S \to \Re$)



- In other words, to each one of the outcomes of an experiment or a sample point ω_i, of the sample spaces, there is a unique real number x_i, known as the value of the random variable X.
- The range of the random variable is called the *induced sample space*.
- *A note on notation:* Random variables will always denoted with uppercase letters and the realized values of the random variable (or its range) will be denoted by the corresponding lowercase letters. Thus, the random variable X can take the value x.

1.1.1 Types of Random Variables

- A random variable is of two types
  - Discrete Random Variable
  - Continuous Random Variable

1.1.1.1 Discrete Random Variable

- If the induced sample space is discrete, then the random variable is called a **discrete random variable**.
- A random variable (X) is said to be discrete if it takes only a finite; or an infinite but countable number of values.
- Examples: The following are discrete random variables:
  - Number of children per family
  - Attendance of CM 2110 lectures
  - GPA credit value that you can obtain for CM 2110
  - The number of machine breakdowns during a given day in a company

Example 01 Consider the experiment of tossing a coin

Example 02

Consider the experiment of rolling of a single dice

Example 03

Consider the experiment of tossing two coins and count the number of heads turn up head

1.1.1.2 Continuous Random Variable

- If the induced sample space is continuous, then the random variable is called a **continuous random variable.**
- A continuous random variable is a random variable that can take on any value in a given interval.
- Random variables which consist of measurements are usually continuous.
- For example
  - height of a student in this class
  - the current measured in a thin copper wire in milliamperes
  - Life time of a mobile phone battery
  - SGPA of a level 2 student

Example 04

Lifetime of a bulb

1.2 Probability Mass Function

Definition 2: Discrete density function of a discrete random variable

If X is a discrete random variable with distinct values $x_1, x_2, \dots, x_n, \dots$, then the function, denoted by $f_X(.)$ and defined by

$$f_X(x) = \begin{cases} P(X = x) & \text{if } x = x_j, j = 1, 2, \dots, n, \dots \\ 0 & \text{if } x \neq x_j \end{cases} \tag{1.1}$$

is defined to be the discrete density function of X.

- The values of a discrete random variable are often called *mass points.*
- $f_X(x)$ denotes the *mass* associated with the *mass point* x_j.
- ***Probability mass function*** *discrete frequency function* and *probability function* are other terms used in place of *discrete density function*
- Probability function gives the measure of probability for different values of X.

1.2.1 Properties of a Probability Mass Function

- Let X be a discrete random variable with probability mass function $f_X(x)$. Then,

1. For any $x \in \Re$, $0 \leq f_X(x) \leq 1$.
2. Let E be an event and $I = \{X(\omega) : \omega \in E\}$. Then $P(E) = P(X \in I) = \sum_{x \in I} f_X(x)$.
3. Let $R = \{X(\omega) : \omega \in \Omega\}$. Then $\sum_{x \in \Re} f_X(x) = 1$.

1.2.2 Representations of Probability Mass Functions

1.2.2.1 Using a table

1.2.2.2 Using a function

1.2.2.3 Using a graph

1.3 Probability density function

1.4 Cumulative distribution function

1.5 Descriptive properties of distributions

1.6 Models for discrete distributions

1.7 Models for continuous distributions

Chapter 2

Estimations

2.1 Point Estimation

2.1.1 Methods of finding point estimators

2.1.2 Methods of evaluating point estimators

2.2 Interval Estimation

2.2.1 Interpretation of confidence intervals

2.2.2 Methods of finding interval estimators

2.2.3 Methods of evaluating interval estimators

Chapter 3

Hypothesis Testing

3.1 Null and alternative hypotheses

3.2 Errors in testing hypotheses-type I and type II error

3.3 Significance level, size, power of a test

3.4 Formulation of hypotheses

3.5 Methods of testing hypotheses

Chapter 4

Design of Experiments

4.1 ntroduction to experimental design

4.2 Basic principles of experimental design

4.3 Completely randomized design