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Assignment 3

Problem 1

What is the maximum number of edges in a graph with V vertices and no parallel edges? What is the minimum number of edges in a graph with V vertices, none of which are isolated?''

Maximum : v(v - 1)/2

Proof: Because in an undirected graph with v vertices, assuming we cannot have self-loop as edges, each one can be connected with other (v - 1) vertices, then we have v(v-1) maximum approaches to connect all the vertices. Moreover, since the graph is undirected, there can be only one edge between two vertices, so the maximum number of edges will be v(v - 1)/2.

Minimum: If we have odd v, then the minimum will be (v+1)/2

If we have even v, the the minimum will be v/2 * also assuming that self-loop are not allowed

Proof: Since we allowed several small components in the graph, we can achieve that all the vertices in the graph are grouped into small part, and each part include 2 vertices. So in this case, long-chain vertices can be avoided. But to meet the requirement of that "none of which are isolated", when v is odd, we have to link one left vertices to the 2-vertices small component, so the minimum will be (v+1)/2, and when v is even, the minimum will be v/2.

Problem 2

What is the maximum number of edges in a digraph with V vertices and no parallel edges? What is the minimum number of edges in a digraph with V vertices, none of which are isolated?''

Maximum: Similar to the proof of problem 1, since in this case we have a directed graph which enables two directions between two vertices, then there will be two edges between them. So the maximum edges will be v(v - 1).

Minimum: If we have odd v, then the minimum will be (v+1)/2

If we have even v, the the minimum will be v/2 * also assuming that self-loop are not allowed

Proof: Like the proof in undirected graph, since one direction between 2 vertices in directed graph is like the edge between vertices in undirected one, then the minimum will be the same as the result in problem 1.

Problem 3

Give the transitive closure of the digraph with ten vertices and these edges:

3->7 1->4 7->8 0->5 5->2 3->8 2->9 0->6 4->9 2->6 6->4

Transitive Closure

	0	1	2	3	4	5	6	7	8	9
0	T		T		T	T	T			T
1		T			T					T
2			T		T		T			T
3				T				T	T	
4					T					T
5			T		T	T	T			T
6					T		T			T
7								T	T	
8									T	
9										T

Problem 4

True or false: The reverse postorder of a graph's reverse is the same as the postorder of the graph.

False.

Suppose we have a graph which include two directions between two vertices. When we do reverse the graph, the directions between these two edges will remain the same, so the reverse postorder of it will not produce the same result as the postorder of the original graph.