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Prolog 1

1 N-queens problem

A chessboard consists of 64 squares, which are divided into 8 rows and 8 columns. In chess, a queen can move horizontally, vertically or diagonally. In each direction, she can travel as far as she wants. The 8-queens problem is a chess problem where 8 queens must be positioned on a chessboard in such a way that no queen can capture another queen in **one** move. In this exercise, we generalize the 8-queen problem to the N-queen problem, where N queens must be positioned on a chessboard consisting of N rows and N columns.

Board representation

The configuration of the queens on a chessboard can be represented as a list, where the index represents the column number and the value represents the row number. For example, the list `[2,4,1,3]` denotes a 4-queens configuration where the queens are in row 2 in column 1, row 4 in column 2, row 1 in column 3, and row 3 in column 4.

The proposed representation guarantees that the queens cannot attack each other in one of the four possible directions (i.e., horizontal, vertical, and diagonal in two directions). Which direction is that? How can the same restriction easily be enforced for one of the three other directions?

Assignment

Implement a predicate `queens(N,L)` that, for a given number of queens N, generates a list L representing a valid configuration of the N queens on a chessboard comprising N rows and N columns. The predicate should generate **all** solutions.

Naive solution

A naive solution to the N-queens problem is to first generate all possible configurations for the queens, and then verify which configurations satisfy the imposed restrictions. The validity of a candidate solution can be verified by checking whether each of the queens cannot attack one of the other queens by moving horizontally, vertically or diagonally. Given that a queen B can attack a queen

A if and only if queen A can attack queen B, it suffices to check whether each queen cannot attack any of the queens to her right.

Example

The following is **one** of the 92 solutions for the 8-queens problem, which is also shown visually in Figure 1.

```
?- queens(8,L).
L = [6, 1, 5, 2, 8, 3, 7, 4] ;
```

Table 1 presents the number of solutions for different values of N.

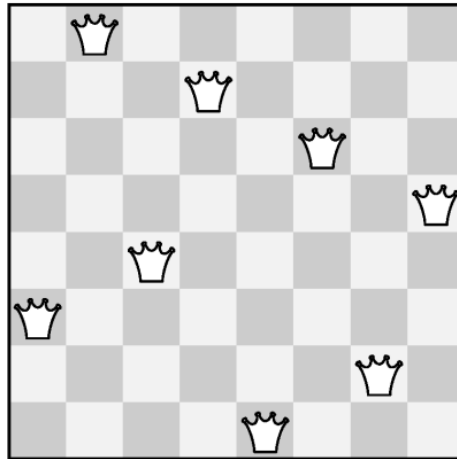


Figure 1: **One** of the 92 solutions for the 8-queens problem.

Number of queens	1	2	3	4	5	6	7	8	9	10
Number of solutions	1	0	0	2	10	4	40	92	352	724

Tabel 1: The number of solutions for a given number of queens.