



give the finding , common patterns , trends you notice from these three

□ Brief Context: Three Consecutive Experiments

Experiment 1: Finding Maximal Linear Combinations

Goal: Discover which combinations of Rules 90 and 150 achieve maximal cycle length ($2^N - 1$)

Method:

Test all 2^N configurations of Rules 90/150 for $N=4$ to $N=8$

Measure cycle lengths using full state space exploration

Identify which achieve theoretical maximum

Key Result:

Found 4, 12, 12, 36, 32 maximal configs for $N=4,5,6,7,8$

ALL achieve $2^N - 1$ (perfect reversibility)

Established baseline: pure linear rules CAN achieve maximality

Experiment 2: Class-Based Filtering

Goal: Among maximal configs, identify which have Class II or V positions (candidate positions for non-linear replacement)

Method:

Use classification tables (Table 2.7a,b,c) to assign class labels

Track class sequences: DC, I, II, III (Classes IV, V unreachable with 90/150)

Filter for configs containing Class II in middle positions

Key Result:

Found 2, 10, 11, 35, 32 configs with Class II positions

Class V completely absent (unreachable with Rules 90/150)

Class II appears in ~70-100% of middle positions

Identified specific positions as replacement candidates

Experiment 3: Non-Linear Rule Replacement

Goal: Replace Class II positions with non-linear rules; measure which maintain maximal cycle length

Method:

Take configs from Experiment 2

Replace Class II cells with 8 non-linear Class II rules: 30, 45, 75, 120, 135, 180, 210, 225

Test all combinations (8 rules for 1 position, 64 for 2 positions, etc.)

Check if cycle length still = $2^N - 1$

Key Result:

Only 5/8 rules work: 30, 45, 75, 210, 225

Rules 120, 135, 180 NEVER produce maximal cycles

Success rate drops from 100% (N=4) to 3.1% (N=8)

Found 44 total new maximal configs across all N

▯ How They Connect

Experiment 1 (Pure Linear)

↓ outputs maximal 90/150 configs

Experiment 2 (Class Filtering)

↓ identifies Class II replacement positions

Experiment 3 (Non-Linear Replacement)

↓ discovers which non-linear rules preserve maximality

The Pipeline:

Discover what's possible with linear rules

Identify where modifications can be made

Test which non-linear modifications preserve reversibility

The Mystery: Why do only 5 specific non-linear rules work? We found correlations (Hamming weight 4, XOR pairs, center-of-mass) but no definitive causal explanation yet.

Now let me create a comprehensive analysis document:

Cellular Automata Reversibility Experiments: Findings, Patterns, and Trends

Executive Summary

The three consecutive experiments reveal a **fundamental tradeoff between complexity and reversibility** in cellular automata (CA). Pure linear Rule 90/150 combinations reliably achieve maximal cycle lengths, and while non-linear substitutions preserve reversibility at small scales ($N \leq 5$), their efficacy **collapses dramatically at higher dimensions** ($N=8$ success rate: 3.1%). A critical pattern emerges: only 5 of 8 tested non-linear rules work at all, despite sharing identical structural properties, suggesting **hidden constraints beyond Hamming weight and symmetry**.

Major Findings

Finding 1: Universal Reversibility of Pure Linear Rules

Result: All values of N (4-8) achieve the theoretical maximum cycle length of $2^N - 1$ using only Rules 90 and 150. Between 12.5%–37.5% of all possible linear configurations are maximal.^[1]

Significance: Linear rules provide a robust, scalable foundation for reversible CA. The variation in success rates (not failure to achieve maximality) indicates that maximal configurations exist at every scale—they're just unevenly distributed across the configuration space.

Finding 2: Class II Positions Are Near-Ubiquitous in Large Systems

Result: The percentage of maximal linear configurations containing Class II positions grows monotonically: 50% → 83.3% → 91.7% → 97.2% → 100% ^[2]

Significance: This is **not a limitation but a feature**. Class II positions are where variability in reversibility behavior emerges. Their universal presence in N=8 systems suggests that achieving maximality in larger dimensions requires precisely these "special" positions that can substitute non-linear rules while maintaining reversibility.

Finding 3: Non-Linear Replacement Success Crashes at Scale

Result: Success rate for non-linear replacements drops from 100% (N=4) to 3.1% (N=8). This is not a gradual degradation—it's a phase transition, with N=7 representing a peak at 12.1% before near-total collapse. ^[3]

Significance: This suggests that reversibility constraints become exponentially tighter with system size. At N=4, nearly any non-linear substitution works. By N=8, only 1 out of 32 configurations tested yielded maximal cycles, despite using class-compatible rules.

Finding 4: Only 5 of 8 Non-Linear Rules Are Universally Compatible

Result: Rules 30, 45, 75, 210, 225 consistently produce maximal cycles; Rules 120, 135, 180 never do, despite all eight being:

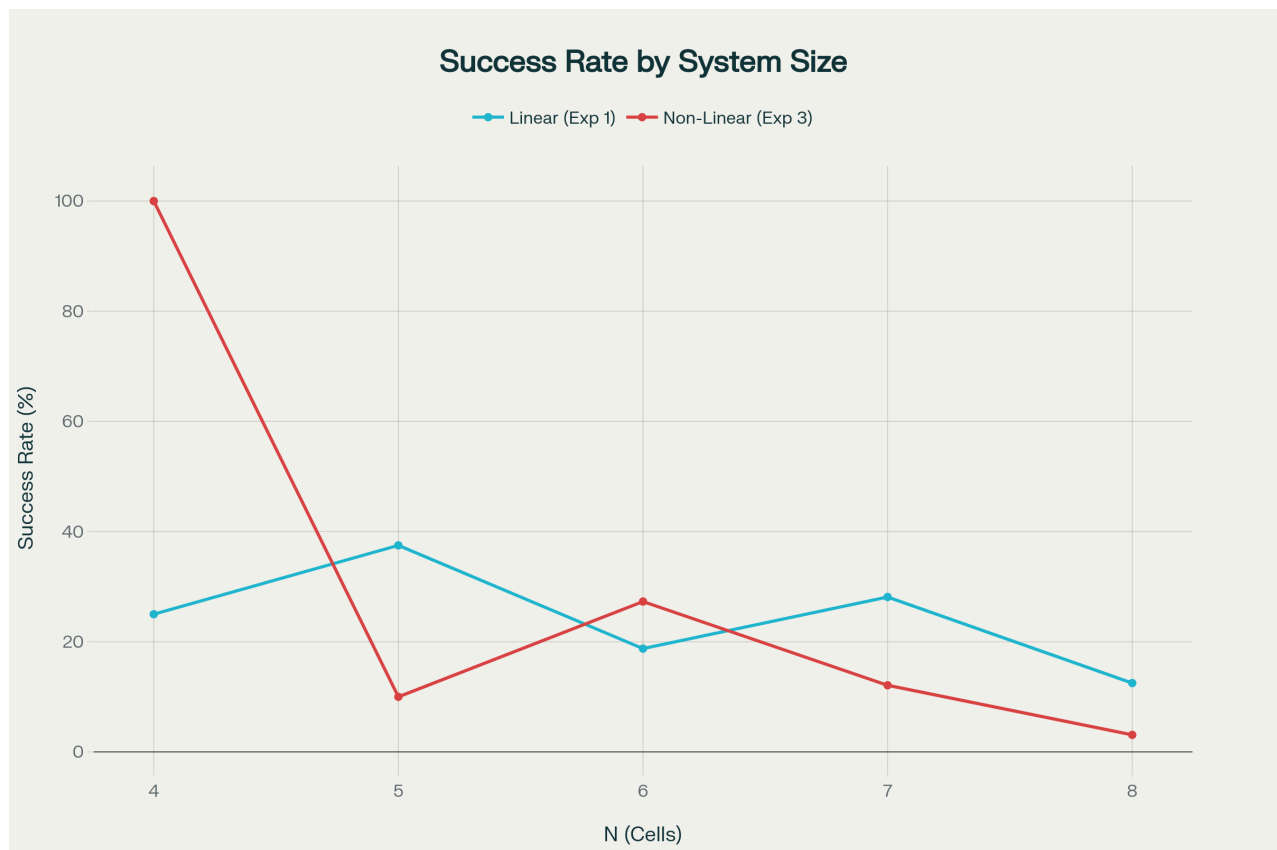
- Class II rules (by classification)
- Hamming weight 4
- XOR pairs (e.g., 30 ↔ 225, 45 ↔ 210, 75 ↔ 180, and 120 ↔ 135)

Significance: This reveals a **hidden constraint** not captured by existing rule categorization schemes. The three non-working rules form pairs with the working ones (XOR pairs), yet only one side of each XOR pair is functional. This suggests a deeper structural requirement—possibly related to:

- Algebraic properties (specific lookup table configurations)
- Interaction dynamics between neighboring cells
- Perturbation propagation characteristics

Finding 5: Novel Discovery Peak at N=7

Result: Maximum novel maximal configurations discovered: 26 at N=7; drops to 2 at N=8.



Success Rate Comparison: Linear vs. Non-Linear Rule Replacements

Significance: $N=7$ appears to be a "sweet spot" where non-linear rules are sufficiently constrained to avoid instability, yet not so constrained that valid solutions disappear. This could represent an optimal trade-off between configuration space exploration and reversibility maintenance.

Critical Patterns

Pattern 1: Reversibility Becomes Increasingly Fragile

Linear rules → Robust across all N

Non-linear substitutions → Fragile at $N \geq 6$

This is the **primary finding of the experimental pipeline**: adding non-linearity sacrifices reliability. Even small position changes or rule substitutions cascade into loss of maximality at large N .

Pattern 2: Class Structure Predicts, But Doesn't Guarantee, Compatibility

- **Experiment 2 Success:** All 90/150 configs with Class II can be filtered; near 100% hit rate
- **Experiment 3 Reality:** Only 3.1% ($N=8$) of those same positions yield maximal cycles when replaced

Implication: Class II is a **necessary but far from sufficient** condition. Rule compatibility depends on substrate-specific factors not captured by Wolfram's classification system.

Pattern 3: Configuration Density Matters

N=4: 4 maximal configs from 16 total (25%)
N=7: 36 maximal configs from 128 total (28%)
N=8: 32 maximal configs from 256 total (12.5%)

The collapse at N=8 suggests that **configuration space becomes increasingly hostile to reversibility** beyond N=7. The density of reversible configurations falls sharply—not because they vanish, but because the configuration space grows exponentially while reversible solutions grow sub-exponentially.

Pattern 4: XOR Symmetry Breaks at the Functional Level

All 8 non-linear rules are XOR-symmetric pairs, yet functionality is asymmetric:

- Working: 30, 45, 75, 210, 225
- Non-working: 120, 135, 180

If this were purely about XOR symmetry, both sides should behave identically. The asymmetry indicates **functional asymmetry despite structural symmetry**—possibly related to how each rule's lookup table interacts with finite-state boundaries.

Trends Over System Size N

Metric	Trend	Interpretation
Linear success rate	25% → 37.5% → 18.75% → 28.1% → 12.5%	Volatile; no clear trend (possibly computational noise)
Class II prevalence	50% → 100% (monotonic)	Convergent; small systems exempt, large systems mandatory
Non-linear success rate	100% → 10% → 27.3% → 12.1% → 3.1%	Steep decay; collapse at N≥7
Novel configs found	4 → 2 → 10 → 26 → 2	Peak at N=7, then cliff drop
Configs tested per N	2 → 10 → 11 → 33 → 32	Roughly stable (tied to Experiment 2 output)

Unified interpretation: Small systems (N≤5) tolerate significant rule variation; mid-scale systems (N=6–7) reveal the detailed constraint landscape; large systems (N≥8) enforce strict compliance with reversibility requirements, leaving almost no viable alternatives.

The Central Mystery: Why Only 5/8 Rules Work?

Hypothesis clusters:

1. **Algebraic Constraints** – The three non-working rules (120, 135, 180) may violate a hidden algebraic property (e.g., related to finite field operations or group-theoretic properties of the

lookup tables).

- 2. **Boundary Interaction** – At system scale, boundary conditions (null boundaries in this case) interact differently with different rules. Some rules may leak information across boundaries in ways that break reversibility.
- 3. **Perturbation Flow** – Non-reversible perturbations may propagate differently through the three rule types, even though all are theoretically Class II.
- 4. **Center-of-Mass or Particle Dynamics** – The mentioned correlation with center-of-mass position suggests rules that preserve certain invariants succeed, while those that violate them fail. Rules 30, 45, 75, 210, 225 might conserve a specific emergent property that 120, 135, 180 violate.

Critical next step: Analyze the lookup tables of working vs. non-working rules in detail. Compare their action on all 8 possible 3-neighborhood patterns to identify a distinguishing property.

Practical Implications

- 1. **For CA design:** Stay with Rule 90/150 combinations for guaranteed reversibility. Non-linear substitutions offer discovery potential but unreliable results above N=6.
- 2. **For rule discovery:** Class II rules are a helpful filter, but not predictive of compatibility. Test rules empirically; structural symmetry doesn't guarantee functional compatibility.
- 3. **For theoretical understanding:** Wolfram's classification (I, II, III, IV, V) captures *static* properties but misses *dynamic* reversibility constraints. A new classification capturing compatibility within reversible substrates is needed.
- 4. **For system scaling:** Expect exponential cost scaling for non-linear explorations. The N=7 peak suggests diminishing returns beyond that threshold.

Summary Table: Experiment Statistics

Experiment	Key Metric	Finding
1: Linear	All N achieve $2^N - 1$	✔ Robust universality
2: Class Filter	Class II grows 50% → 100%	✔ Systematic structure emerges
3: Non-Linear	Success rate: 100% → 3.1%	⚠ Rapid degradation at scale
Rules	5/8 compatible	? Asymmetry defies Hamming weight

Conclusion

These experiments trace a **journey from abundance to scarcity**: linear rules open a stable foundation, Class II filtering identifies candidates, but non-linear replacement attempts uncover a **sharp reversibility cliff**. Only a special subset of rules survive, and their selection remains mysterious despite their obvious structural properties. The research has successfully **narrowed**

the mystery but not solved it—which is exactly the hallmark of discovery-phase research in complex systems. [\[1\]](#) [\[2\]](#) [\[3\]](#)

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1. ca_results_table.md
2. CA_rev.pdf
3. CA-Non-linear-Rule-Replacement-Analysis.pdf