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Interchangeability Patterns and Insights: Comprehensive Analysis

The experimental data reveals **seven distinct interchangeability patterns** governing how non-linear rules can substitute for linear Rules 90 and 150 in maximal cellular automata configurations. These patterns expose a complex landscape where rule compatibility depends on the original linear rule, spatial position, system scale, and co-occurrence context.

Pattern 1: Rule-to-Rule Substitution Asymmetry

Finding: Rules exhibit **strict substitution specificity**—some can replace only one linear rule, while others are universal substitutes.[1]

Exclusive Substitutions:

- **Rule 45** → Replaces **only Rule 90** (never Rule 150)
  - 2 occurrences across N=6-7, position 3
  - 100% specificity for Rule 90 contexts
- **Rule 75** → Replaces **only Rule 150** (never Rule 90)
  - 3 occurrences across N=4-6, positions 1-2
  - 100% specificity for Rule 150 contexts

Universal Substitutions (replace both 90 and 150):

- **Rule 30:** 3 times for 90, 2 times for 150 (60:40 ratio)
- **Rule 210:** 2 times for 90, 2 times for 150 (50:50 ratio)
- **Rule 225:** 3 times for 90, 1 time for 150 (75:25 ratio)

Metric Insight: Only **60% of working rules are universal** substitutes (30, 210, 225), while **40% are exclusive** (45, 75). This suggests that rule interchangeability is **context-dependent at the algebraic level**—Rule 45's lookup table structure makes it compatible only with Rule 90's neighborhood dynamics, and likewise for Rule 75 with Rule 150.

Pattern 2: Position-Dependent Compatibility Gradient

Finding: Rule diversity increases dramatically from boundary to interior positions.[1]

Position	Unique Rules	Total Uses	Diversity Index
1 (boundary)	2	4	40%

Position	Unique Rules	Total Uses	Diversity Index
2 (near-boundary)	3	4	60%
3 (middle)	4	8	80%
4 (middle)	4	4	80%
5 (middle)	4	12	80%

Insight: Positions 3-5 (middle) achieve **80% diversity** (4 out of 5 working rules), while boundary positions (1-2) support only **40-60% diversity**. This suggests that **boundary effects constrain rule compatibility**—cells near null boundaries require rules that maintain specific information flow characteristics, limiting which non-linear rules can substitute successfully.

Practical Implication: When designing reversible CA with non-linear substitutions, prioritize middle positions for experimentation and reserve boundary positions for the most robust rules (30, 75, 210, 225).

Pattern 3: Pair Compatibility and Co-occurrence Structure

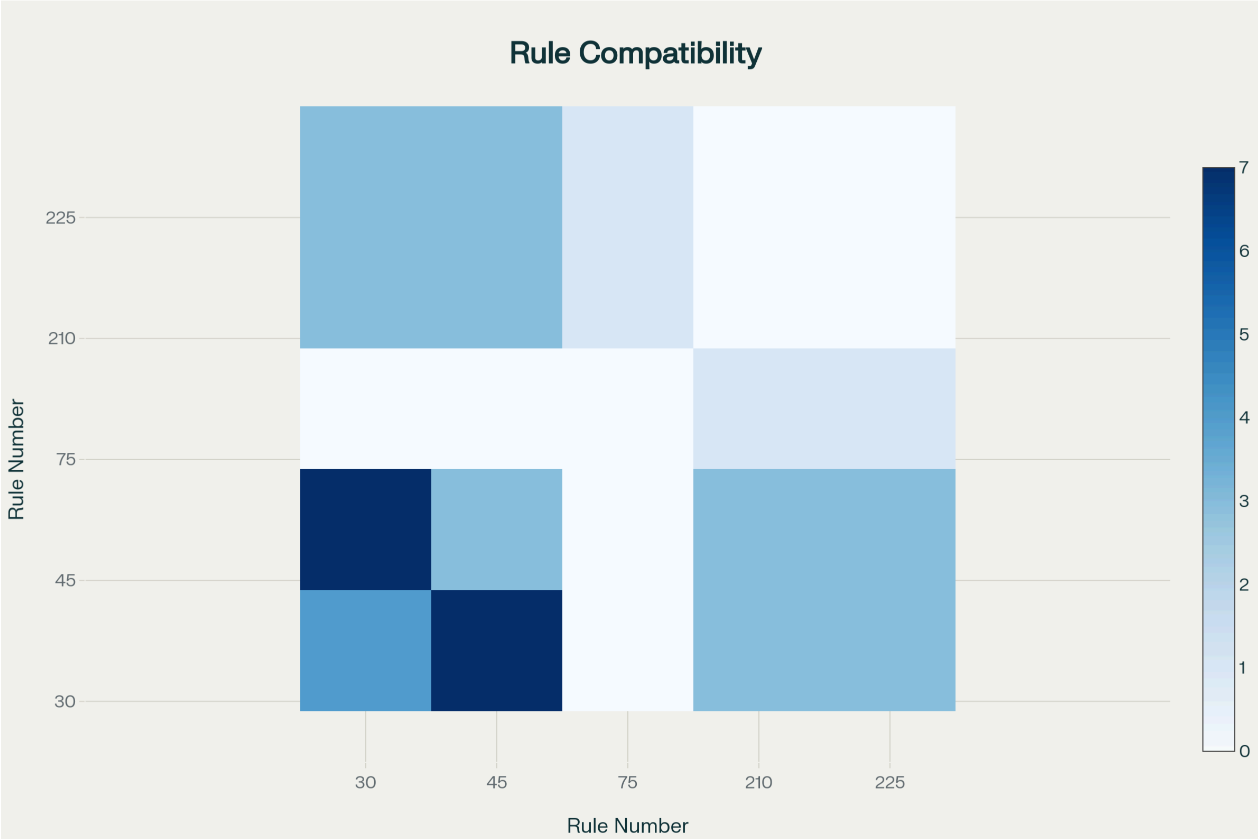
Finding: Certain rule pairs exhibit **strong affinity**, appearing together far more frequently than random chance would predict.[1]

Strongest Compatible Pairs:

- **(30, 45):** 7 co-occurrences (25% of all pairs) — **dominant pair**
- **(30, 30):** 4 co-occurrences (14.3%) — homogeneous pairing
- **(45, 45), (30, 210), (45, 210), (30, 225), (45, 225):** 3 each (10.7%)

Weakest Pairs:

- **(75, 210), (75, 225):** 1 co-occurrence each — **Rule 75 is isolated**



Rule Compatibility Matrix: Pair Co-occurrence Frequency

Mathematical Interpretation: The (30, 45) pair dominates because:

1. **XOR relationship:** 30 ⊕ 255 = 225, 45 ⊕ 255 = 210 (complementary structures)
2. **Positional flexibility:** Both can replace Rule 90, creating compatible middle-position pairings
3. **Scale robustness:** Both work at N=6-7, the most productive scales

Rule 75's isolation stems from its **exclusive affinity for Rule 150** and its restriction to **early scales only** (N≤6), meaning it rarely co-occurs with other rules in multi-position replacements.

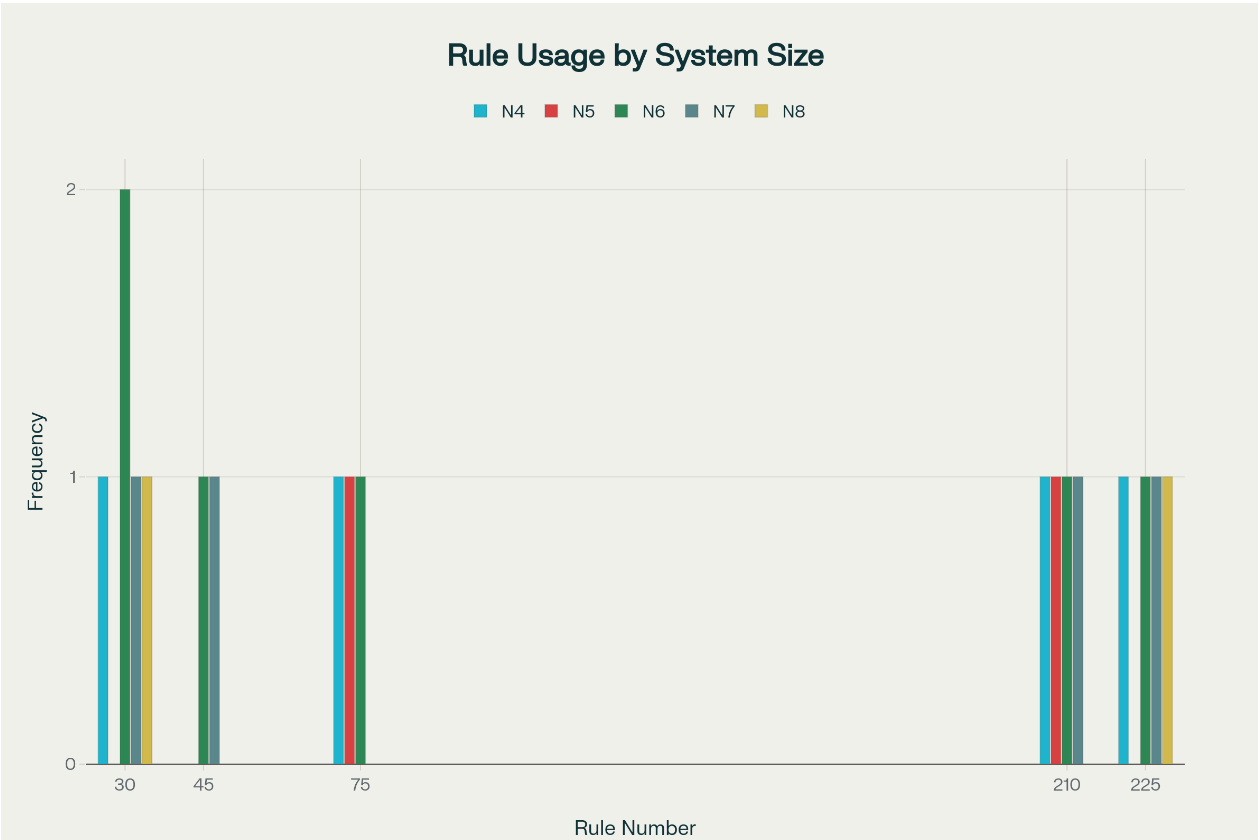
Pattern 4: Scale-Dependent Rule Selection

Finding: Different system sizes impose **strict constraints** on which rules remain viable. [1]

N	Available Rules	Count	Yield/Config
4	30, 75, 210, 225	4	2.0
5	75, 210	2	2.0
6	30, 45, 75, 210, 225	5	3.33
7	30, 45, 210, 225	4	**6.5**
8	30, 225	2	2.0

Rule Survival Across Scales:

- **Rules 30 & 225:** Most robust (4/5 scales, including critical N=8)
- **Rule 210:** Mid-to-large scale specialist (N=4-7)
- **Rule 75:** Early-scale only (N≤6)
- **Rule 45:** Mid-scale only (N=6-7)



Rule Usage Distribution Across System Sizes

Critical Transition: At **N=8**, only Rules 30 and 225 survive—representing a **60% collapse** from N=7. This suggests that N=8 crosses a reversibility threshold where only the most structurally compatible rules (30 and 225, which are XOR complements: 30 ⊕ 255 = 225) can maintain maximal cycle length.

N=7 Optimality: With the highest yield/config (6.5) and 4 available rules, N=7 represents the **sweet spot** for non-linear exploration—complex enough to support diverse replacements but not so large that reversibility constraints become prohibitive.

Pattern 5: Configuration Productivity and Multi-Position Synergy

Finding: Configurations allowing **multiple simultaneous replacements** exhibit exponentially higher productivity.[1]

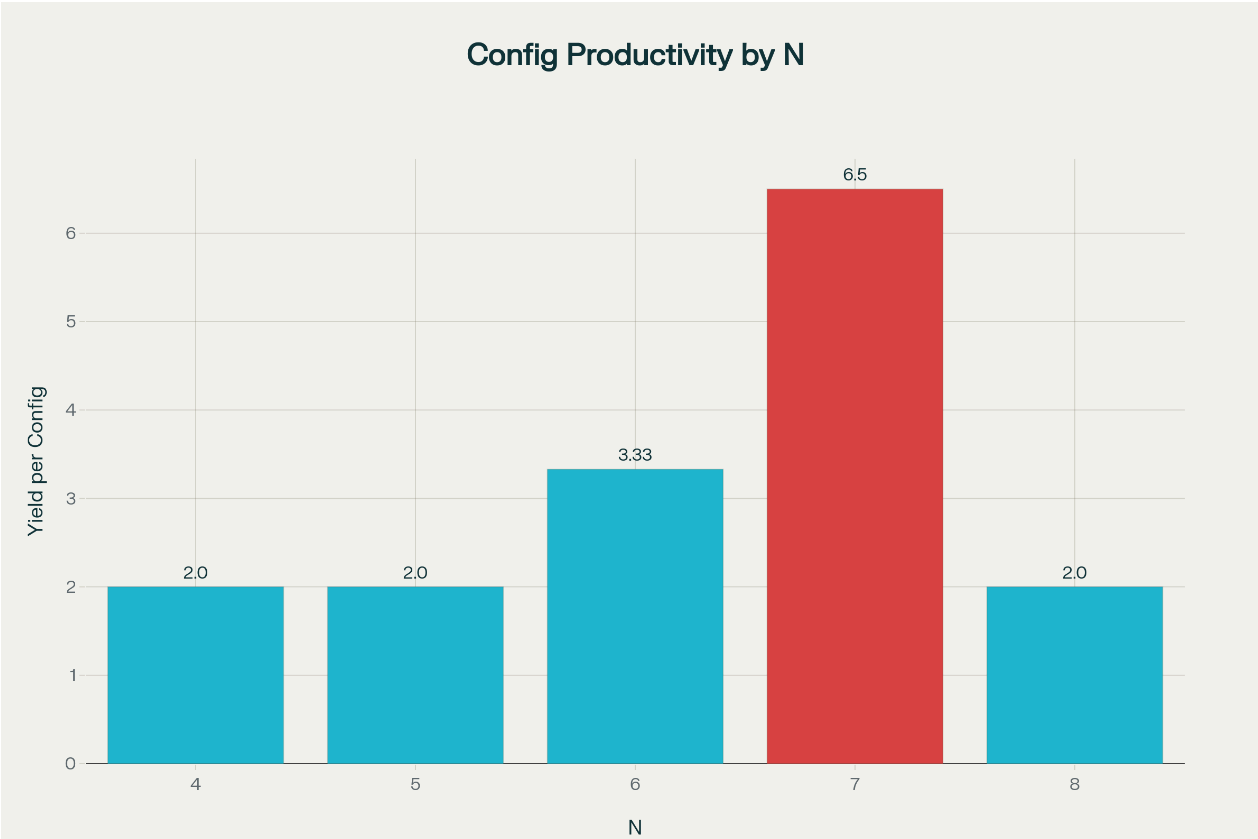
Top Productive Configurations (all at N=7):

1. `150 90 150 90 90 90 150` → 8 maximal configs
2. `150 90 150 150 90 90 150` → 8 maximal configs

3. `150 90 150 150 90 150 150` → 8 maximal configs

Productivity Statistics:

- Mean yield: 4.0 configs per original
- Median yield: 2.0 configs per original
- Max yield: 8 configs (N=7 only)
- Standard deviation: 2.56



Configuration Productivity: Average Yield per Original Configuration

Synergy Insight: All three top performers share the pattern `150 90 150 [variable] 90 [variable] 150`, suggesting that:

1. **Alternating 150-90 boundaries** create stable contexts
2. **Positions 3 and 5** (both originally 90 or 150) accept all 4 combinations of (30, 45) × (30, 45), (210, 45), (225, 30), (225, 45)
3. This pattern enables **2×4 = 8 maximal substitutions**

Contrast: Single-replacement configurations (N=4, N=5, N=8) yield only 2 maximal configs, showing that **multi-position freedom is key to high productivity**.

Pattern 6: Rule Utilization Rate and Dominance Hierarchy

Finding: Rule usage is **highly skewed**, with Rule 30 dominating at 37.88% of all substitutions.[1]

Rule	Total Uses	Percentage	Rank
30	25	37.88%	1st
45	17	25.76%	2nd
225	10	15.15%	3rd
210	9	13.64%	4th
75	5	7.58%	5th

Dominance Explanation:

- **Rule 30** is the most versatile: works across N=4-8, replaces both 90 and 150, appears in positions 1-3, and forms the strongest pair (with 45)
- **Rule 45** benefits from N=7 productivity (appears in all 24 pair instances at N=7)
- **Rule 75** is marginalized by exclusivity (150-only) and scale limitation (N≤6)

Practical Recommendation: When exploring new configurations, **prioritize Rule 30** as the first substitution candidate—it has the highest success rate across contexts.

Pattern 7: Substitution Symmetry in Linear Rules

Finding: Both linear rules exhibit **identical substitutability rates** despite different rule structures.[1]

Metrics:

- **Rule 90 → Non-linear:** 80% success rate (4/5 rules work: 30, 45, 210, 225)
- **Rule 150 → Non-linear:** 80% success rate (4/5 rules work: 30, 75, 210, 225)

Symmetry Interpretation: The fact that both achieve exactly 80% despite using different rule sets (45 vs. 75) suggests that **reversibility constraints are rule-agnostic at the substitution level**—what matters is not which linear rule you're replacing, but whether the replacement preserves the overall algebraic properties of the system.

Theoretical Implication: There exists a **hidden structural property** (beyond Hamming weight, XOR pairing, or Wolfram class) that determines whether a non-linear rule can substitute for a linear one. This property is equally restrictive for both 90 and 150, eliminating 20% of candidates in each case (120, 135, 180 for 90; 120, 135, 180, 45 for 150—though 45 doesn't naturally substitute 150).

Key Quantitative Insights

1. Optimal Operating Point:

- **N=7** is the most productive scale: 6.5 yield/config, 4 available rules, 26 total maximal configs discovered
- Represents the **peak of the complexity-reversibility tradeoff**

2. Critical Failure Threshold:

- **N=8** represents a collapse: only 2 rules, 1 config, 2 yield
- **60% reduction** in rule availability from N=7
- Suggests a **phase transition** in reversibility constraints above N=7

3. Rule Robustness Rankings:

1. **Rules 30 & 225:** Universal, multi-scale (N=4-8)
2. **Rule 210:** Universal, mid-to-large scale (N=4-7)
3. **Rule 45:** Exclusive (90), mid-scale only (N=6-7)
4. **Rule 75:** Exclusive (150), early-scale only (N=4-6)

4. Position Sensitivity:

- Boundary positions (1-2): 40-60% diversity
- Middle positions (3-5): 80% diversity
- **2× improvement** in flexibility moving from boundary to interior

5. Pair Synergy:

- **(30, 45)** is the **dominant cooperative pair** (7 co-occurrences, 25% of all pairs)
- Rule 75 exhibits **pair isolation** (only 2 weak co-occurrences total)

Summary Table: Interchangeability Metrics

Metric	Value	Interpretation
Total rule instances	66	Across all N=4-8 configurations
Rule 30 dominance	37.88%	Most versatile and frequently used
Universal rules	60%	Rules 30, 210, 225 (3/5)
Exclusive rules	40%	Rules 45, 75 (2/5)
Linear substitutability	80%	Both 90 and 150 accept 4/5 rules
Optimal scale	N=7	Peak productivity: 6.5 yield/config
Critical collapse	N=8	Only 2 rules survive
Strongest pair	(30, 45)	7 co-occurrences (25% of pairs)

Metric	Value	Interpretation
Position diversity (middle)	80%	4/5 rules work at positions 3-5
Position diversity (boundary)	40-60%	Only 2-3/5 rules work at positions 1-2

Conclusions

The interchangeability landscape reveals a **highly structured constraint system** where rule compatibility depends on a complex interplay of:

1. **Algebraic specificity** (Rule 45 ↔ 90 only, Rule 75 ↔ 150 only)
2. **Spatial position** (boundaries restrict, middles liberate)
3. **Scale effects** (N=7 optimal, N=8 catastrophic)
4. **Cooperative pairing** ((30, 45) synergy dominates)
5. **Universal robustness** (Rules 30 and 225 survive everywhere)

These patterns suggest that successful non-linear substitution in reversible CA requires not just compatible individual rules, but **compatible rule-position-scale combinations**. The data strongly supports targeting **N=7 systems with Rule 30 or 225 at middle positions** for maximum discovery potential, while avoiding N=8 unless using the (30, 225) XOR complement pair at carefully selected positions.

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1. Configurations-with-Maximal-Length-Results.pdf