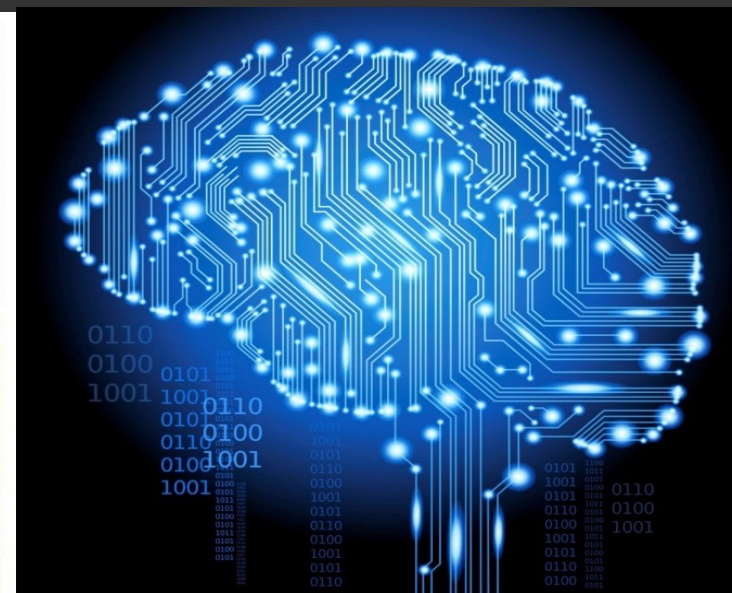
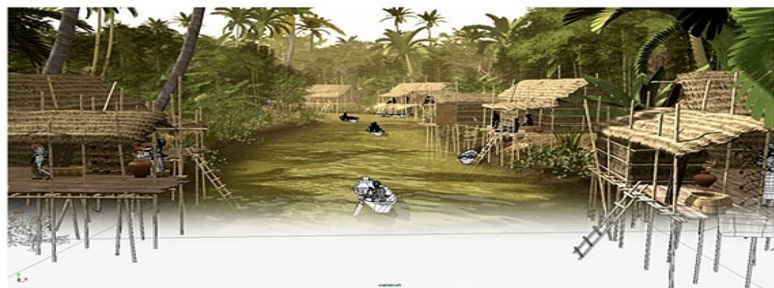
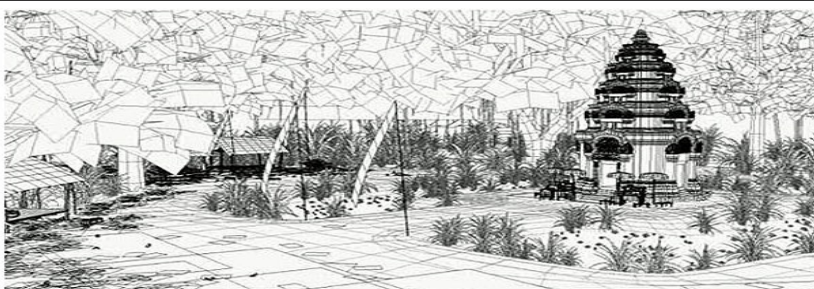




# Conflict Resolution with Linear Probing

Prepared by Maria Garcia de la Banda  
Updated by Brendon Taylor and Alexey Ignatiev



# Objectives for this lesson

- **To understand the main method of conflict resolution:**
  - Open addressing:
    - Linear Probing
- **To understand its advantages and disadvantages**
- **To be able to implement it**

# Conflict Resolution

## Add

# Hash Table operations: Add

- Apply the hash function to get a hash value (position) N
- Try to add key at position N
- Deal with collision / conflict if any

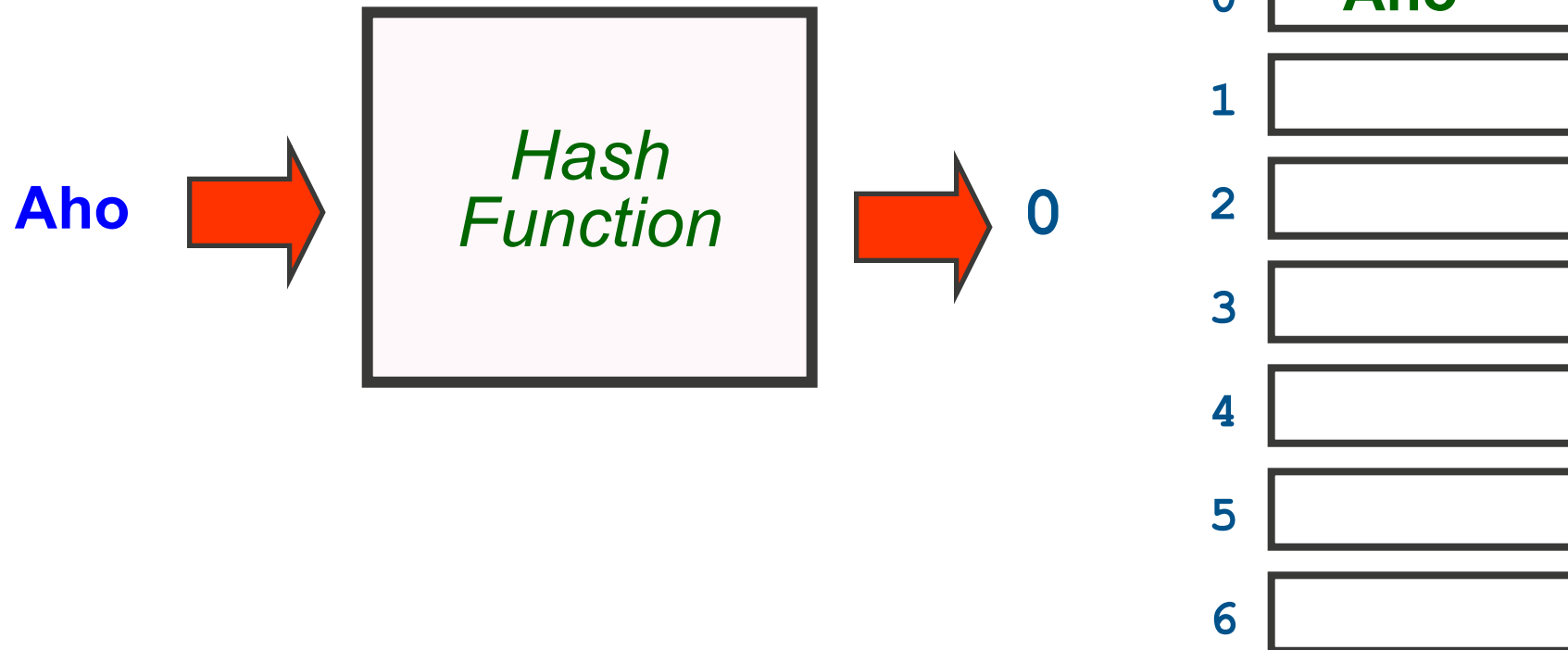
## For clarity:

We shall refer to a situation of  $hash(key1) = hash(key2)$  as a **collision** while a **conflict** is assumed to occur when a position in the hash table *we attempt to use for a given key* is already occupied.

## Example: Add

Aho, Kruse, Standish, Horowitz, Langsam, Sedgewick, Knuth

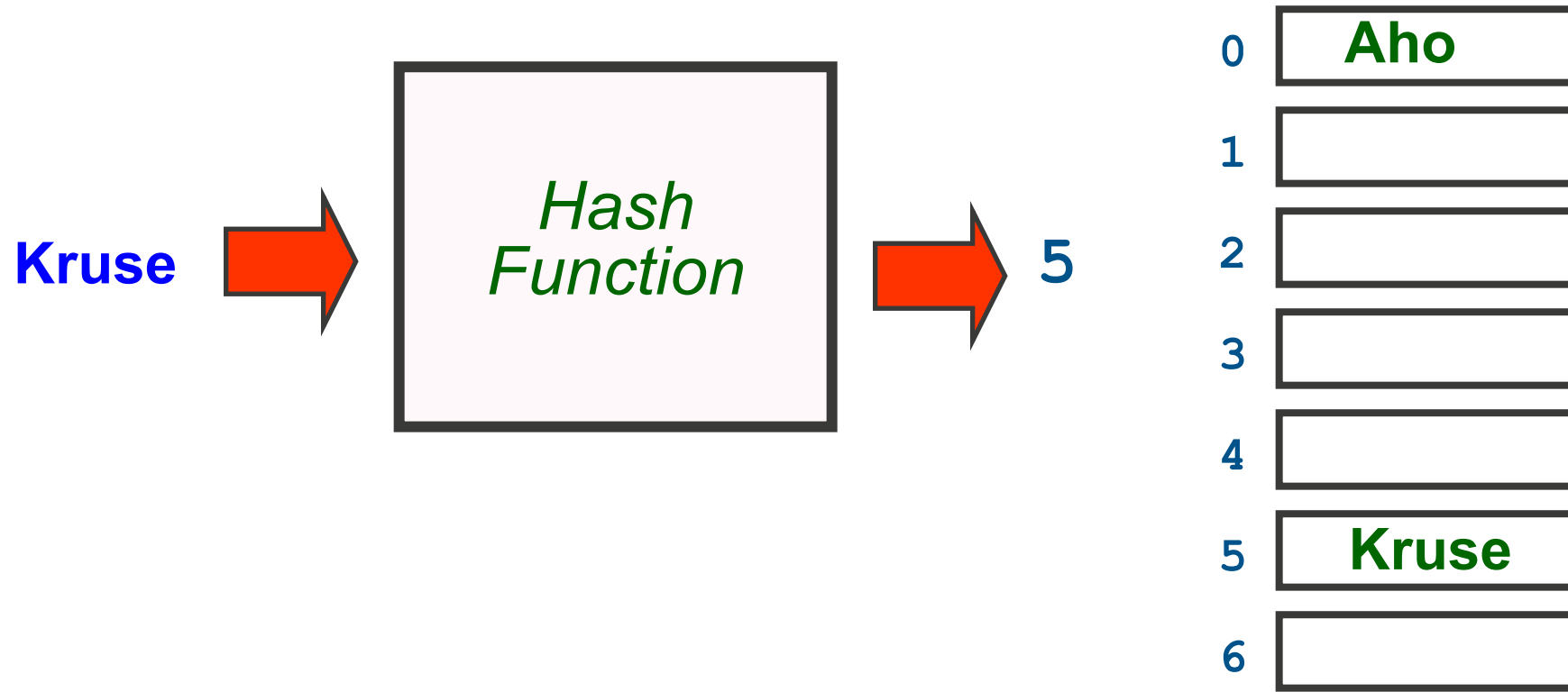
*hash table*



## Example: Add

Aho, Kruse, Standish, Horowitz, Langsam, Sedgewick, Knuth

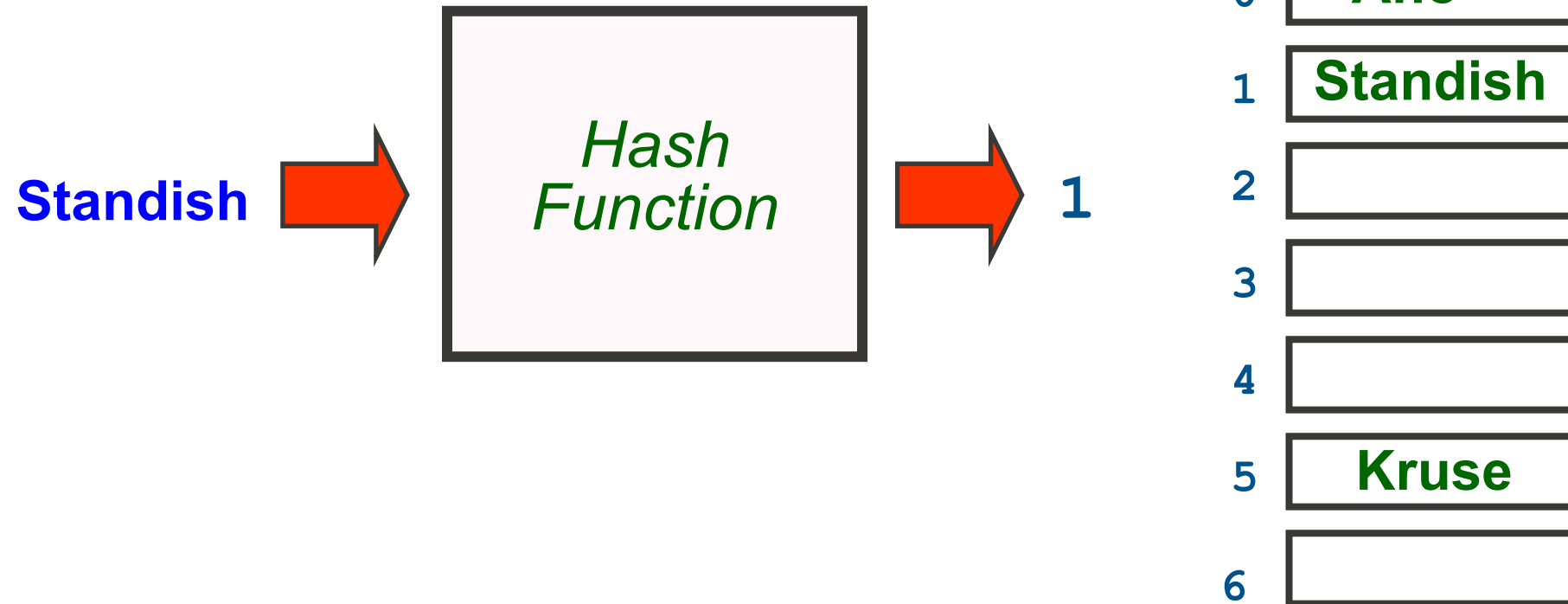
*hash table*



## Example: Add

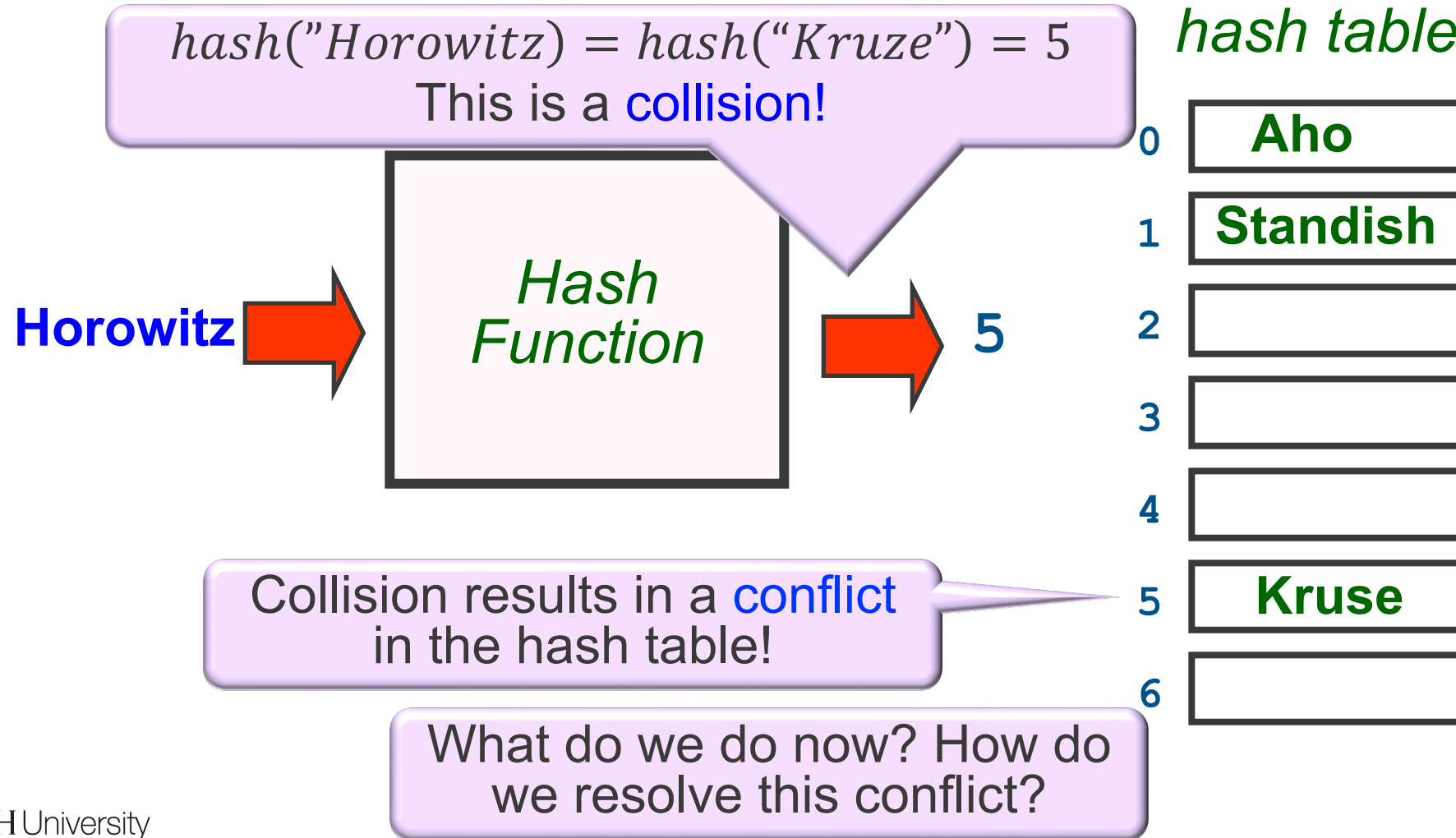
Aho, Kruse, Standish, Horowitz, Langsam, Sedgewick, Knuth

*hash table*



## Example: Add

Aho, Kruse, Standish, Horowitz, Langsam, Sedgewick, Knuth





# Revision: two main approaches to conflict resolution

We've seen this in the previous lesson!

## ▪ Separate chaining:

- Each array position contains a **linked list** of items
- Upon collision, either update (same key) or add the element to the linked list

## ▪ Open addressing:

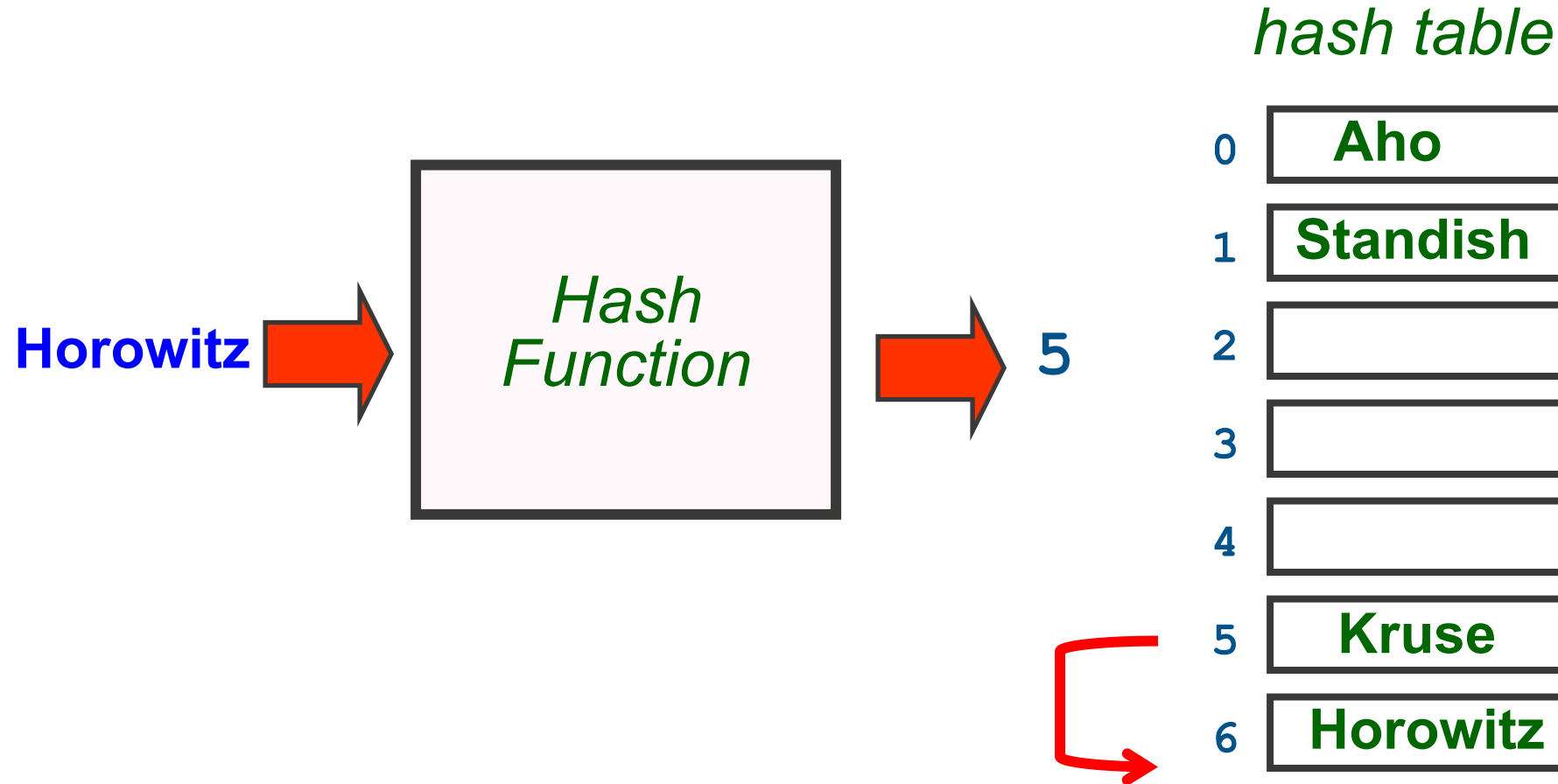
- Each array position contains a **single item**
- Upon collision, either update (same key) or use an empty space to store the new item (which empty space depends on the technique)
- As we will see:
  - Requires an array of at least **double the size** of the number of elements
  - Thus, we must be able to estimate in advance the number of elements (or risk a dynamic resize)

# Open Addressing: Linear Probing

- **Add item with hash value N:**
  - If array[N] is empty: put item there
  - If there is already an item there with:
    - A different key:
      - search for the **first empty** space in the array from N+1
      - add the item there (if any)
    - Same key: update the data associated to the key
- **Basically: linear search from N until an empty slot is found**
- **But careful, you must deal with:**
  - **Full** table (to avoid going into an infinite loop)
  - Restarting from position 0 if **the end of table** is reached

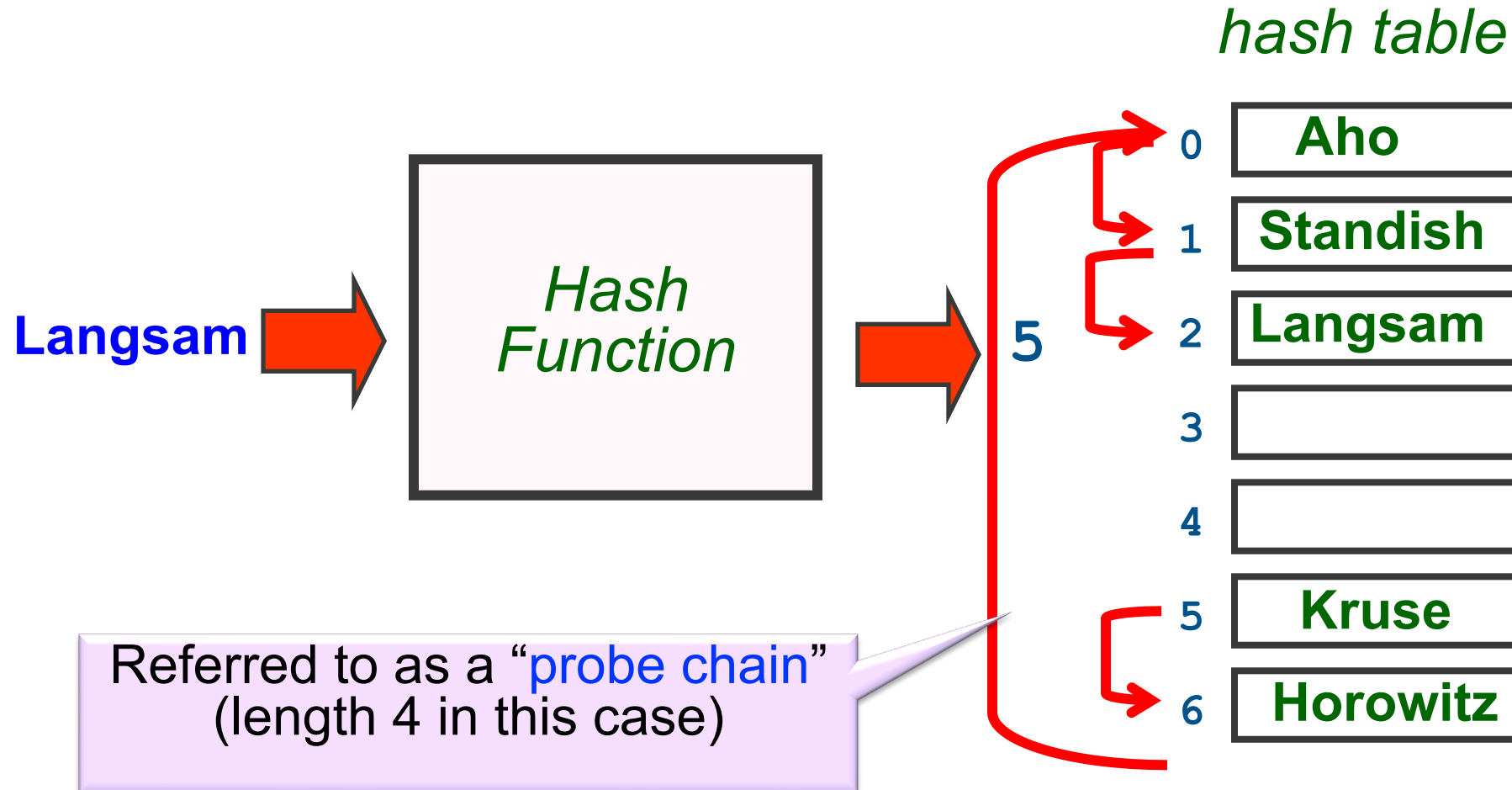
## Example: Add

Aho, Kruse, Standish, Horowitz, Langsam, Sedgewick, Knuth



## Example: Add

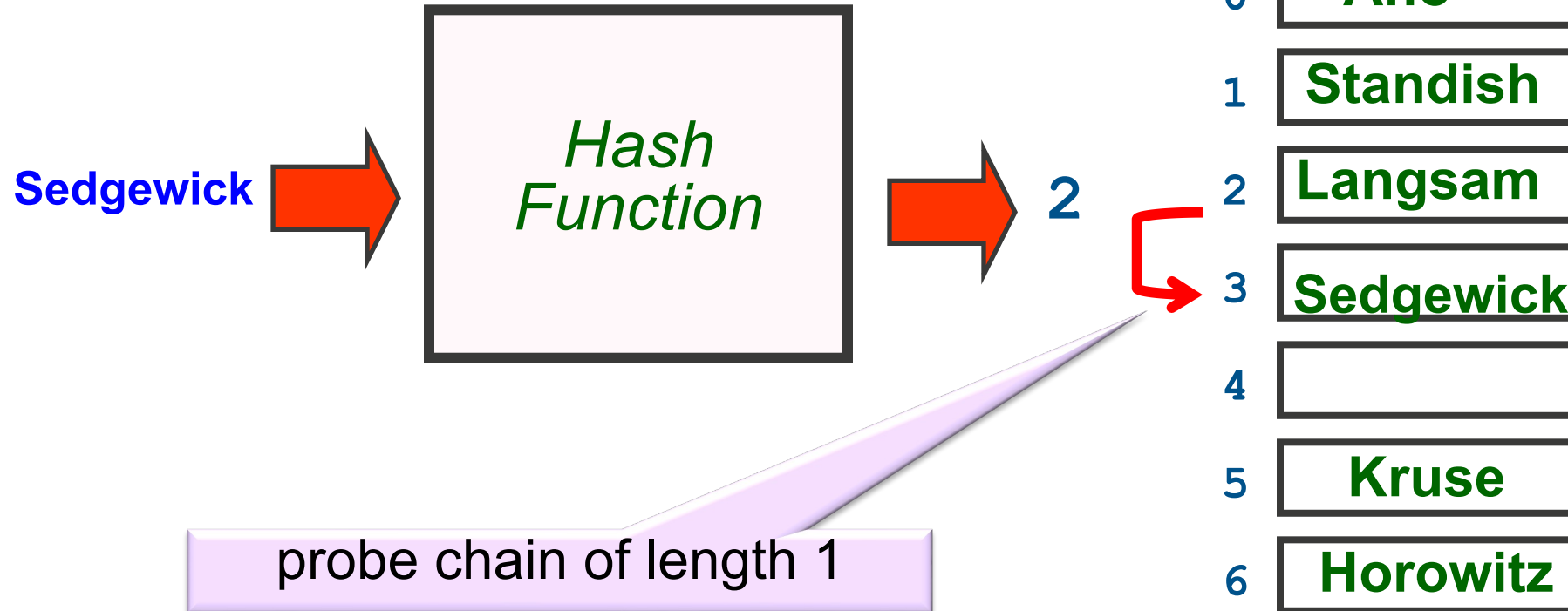
Aho, Kruse, Standish, Horowitz, Langsam, Sedgewick, Knuth



# Let's keep on going

Aho, Kruse, Standish, Horowitz, Langsam, Sedgewick, Knuth

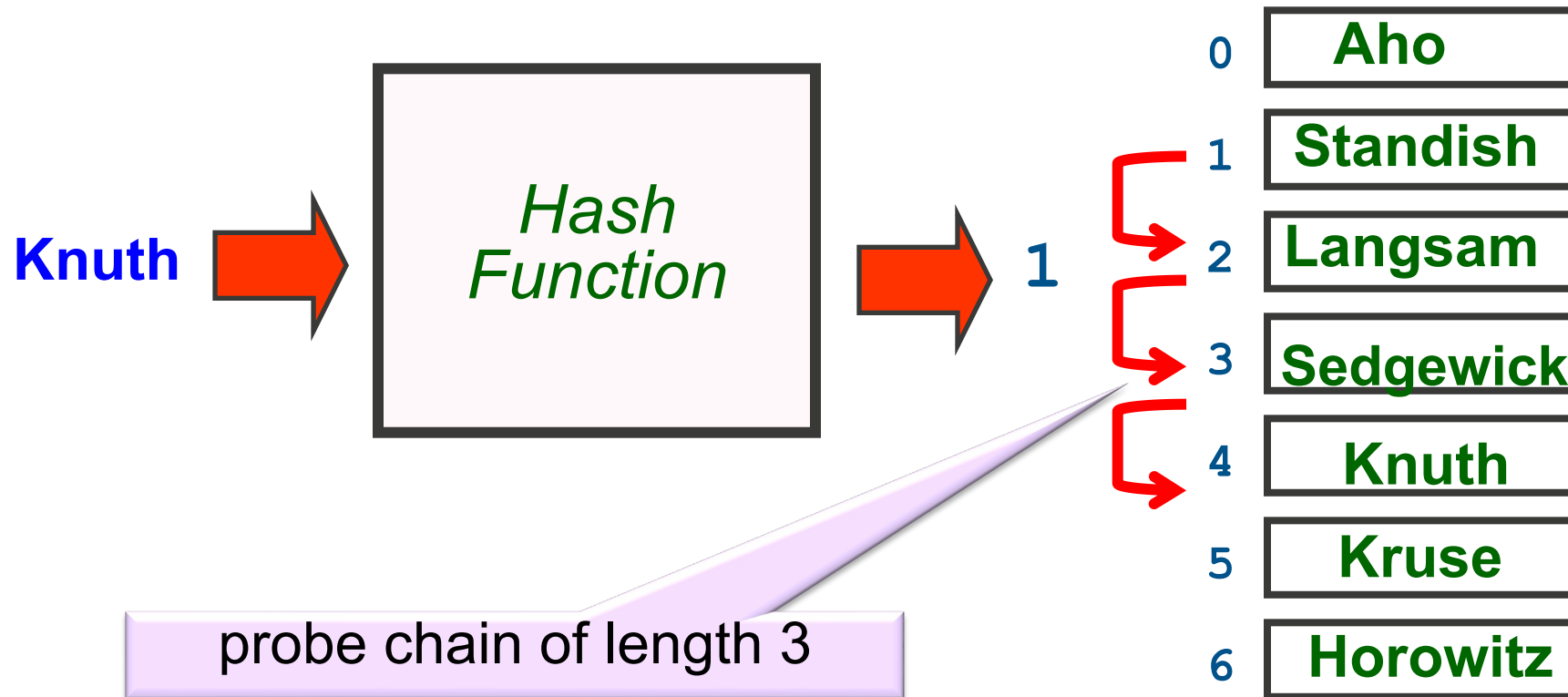
*hash table*



# Let's keep on going

Aho, Kruse, Standish, Horowitz, Langsam, Sedgewick, Knuth

*hash table*



```
from typing import TypeVar, Generic
T = TypeVar('T')
```

```
class LinearProbeTable(Generic[T]):
```

Default size (a prime)

```
    def __init__(self, size: int = 7919) -> None:
        self.count = 0
        self.table = ArrayR(size)
```

How many elements?

The array to store them

```
    def __len__(self) -> int:
        return self.count
```

```
    def hash(self, key: str) -> int:
        value = 0
        a = 31415
        b = 27183
        for char in key:
            value = (ord(char) + a*value) % len(self.table)
            a = a * b % (len(self.table)-1)
        return value
```

Universal hashing

$$h = ((\dots(a_0x + a_1)x + \dots + a_{n-3})x + a_{n-2})x + a_{n-1})x + a_n$$

Base changes for each position pseudo randomly

# Reminder: Adding in Linear Probing

- **Add item with hash value N:**

- If array[N] is empty: put item there
- If there is already an item there with:
  - A different key:
    - search for the **first empty space** in the array from N+1
    - add the item there (if any)
  - Same key: update the data associated to the key

But what is  
an item?

Up to now, we were  
storing only the key  
(the item was the key)

- **Basically: **linear** search from N until an empty slot is found**

- **But careful, you must deal with:**

- **Full** table (to avoid going into an infinite loop)
- Restarting from position 0 if the **end of table** is reached



# We were storing the key only

Key	Hash
Aho	0
Kruse	5
Standish	1
Horowitz	5
Langsam	5
Sedgewick	2
Knuth	1



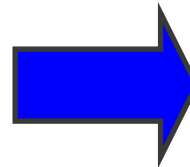
## Hash Table

0	Aho
1	Standish
2	
3	
4	
5	Kruse
6	Horowitz

- In practice we want to store also data associated to each key

# We also need to store the data

Key	Hash	Data
Aho	0	Data structures and algorithms
Kruse	5	Data structures and program design in C++
Standish	1	Data structures in Java
Horowitz	5	Fundamentals of Data Structures
Langsam	5	Data structures using C and C++
Sedgewick	2	Algorithms in C++
Knuth	1	The art of computer programming



## Hash Table

0	<b>Aho</b>	Data structures and algorithms
1	<b>Standish</b>	Data structures in Java
2		
3		
4		
5	<b>Kruse</b>	Data structures and program design in C++
6	<b>Horowitz</b>	Fundamentals of Data Structures

## We also need to store the data (cont)

- How do we store both key and data in the hash table?
- We need an object that stores:
  - Key
  - Data
- Have we seen anything already?
- Tuples! We could use: (key, data)
- We can then access them as usual

```
>>> my_tuple = ("Aho", "Data structures")
>>> my_tuple[0]
'Aho'
>>> my_tuple[1]
'Data structures'
>>>
```

### Hash Table

0	(Aho,	Data structures and algorithms	)
1	(Standish,	Data structures in Java	)
2			
3			
4			
5	(Kruse,	Data structures and program design in C++	)
6	(Horowitz,	Fundamentals of Data Structures	)

# Reminder: Adding in Linear Probing

- **Add item with hash value N:**

- If array[N] is empty: put item there
- If there is already an item there with:
  - A different key:
    - search for the **first empty space** in the array from N+1
    - add the item there (if any)
  - Same key: update the data associated to the key

But what is  
an item?

Now we know: an item  
is a tuple **(key,data)**

- **Basically: **linear** search from N until an empty slot is found**

- **But careful, you must deal with:**

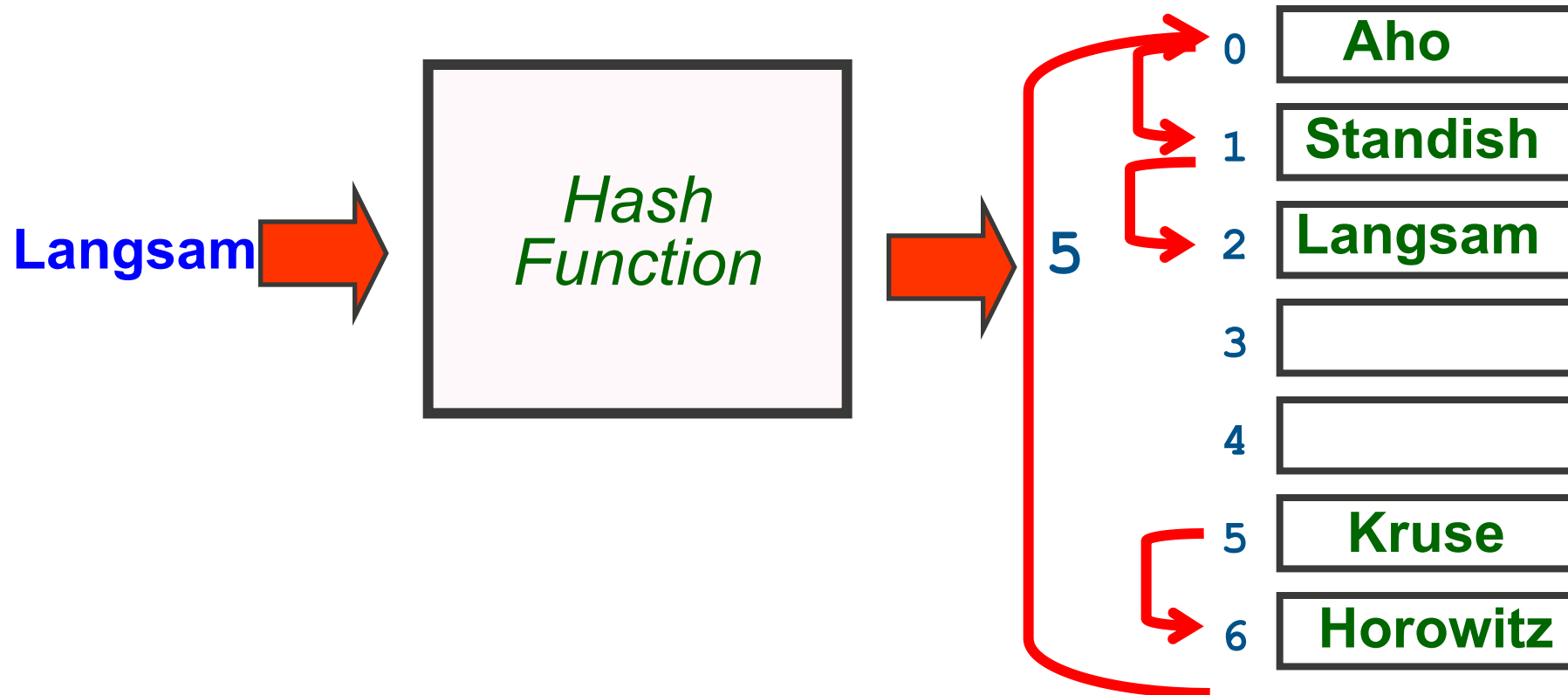
- **Full** table (to avoid going into an infinite loop)
- Restarting from position 0 if the **end of table** is reached

# Adding algorithm for Linear Probing

- The algorithm for `add(key, data)` for linear probing is as follows
- Get the **position**  $N$  using the hash function  $N = \text{hash}(\text{key})$
- If `array[N]` is empty, just put the item (key, data) there
- Else, if there is already an item there:
  - If the item has the **same key**: **update** the data
  - If it has a **different key**, keep looking in next cell (wrapping around)
    - What if we never find it and there is no empty spot?
    - Then we need to **rehash**: create a bigger array and reinsert all items
- **We must traverse the table before we can rehash:**
  - Even if it is known to be full, **in case** the key is already in (we are doing an **update**)
  - Also, as we will see later, rehash in Linear Probing should happen much earlier than when the table is full...

## Example: Add

Aho, Kruse, Standish, Horowitz, **Langsam**, Sedgewick, Knuth  
*hash table*



Allows our container class to provide the `[]` notation

```
def __setitem__(self, key: str, data: T) -> None:
```

```
    position = self.hash(key) # get the position using hash
```

```
    for _ in range(len(self.table)): # start traversing
```

```
        if self.table[position] is None: # found empty slot
```

```
            self.table[position] = (key, data)
```

```
            self.count += 1
```

```
            return
```

```
        elif self.table[position][0] == key: # found key
```

```
            self.table[position] = (key, data)
```

```
            return
```

```
        else: # not found, try next
```

```
            position = (position+1) % len(self.table)
```

```
    self.rehash() # move everything to a new, larger table
```

```
    self.__setitem__(key, data) #try again
```

Traverse each  
item in our hash  
table from  
position

Item already  
exists, overwrite  
the data

Linear Probing

# Your turn..

- Write `__str__` for a Linear Probe hash table, e.g.:

(Aho, Data structures and algorithms)

(Standish, Data Structures in Java)

```
def __str__(self) -> str:
    result = ""
    for item in self.array:
        if item is not None:
            (key, value) = item
            result += "(" + str(key) + "," + str(value) + ")\n"
    return result
```

Hash Table

0	Aho
1	Standish
2	Langsam
3	
4	
5	Kruse
6	Horowitz

But we are traversing the Hash Table! Didn't we say not to do that?

No! We said not to traverse IN A PARTICULAR ORDER



# Conflict Resolution Search

# Searching in Linear Probing

- **Search for an item with hash value N:**
  - Perform a linear search from `array[N]` until either the item or an empty space is found (if so, raise a `KeyError(key)` exception)
- **But careful, you must deal again with:**
  - Full table (to avoid going into an infinite loop)
  - Restarting from position 0 if the end of table is reached

# Searching algorithm for Linear Probing

- The algorithm for `search(key, data)` is as follows
- Get the **position** `N` using the hash function `N = hash(key)`
- If `array[N]` is empty, raise a `KeyError(key)` exception
- Else, if there is already an item there:
  - If the item has the **same key**: return the associated data
  - If it has a **different key**, keep looking
    - What if we never find the key and there is no empty spot?
    - Then we raise a `KeyError(key)` exception
- We used `__setitem__` for adding
- We will use `__getitem__` for searching

```
def __getitem__(self, key: str) -> T:
```

```
    position = self.hash(key) # get the position using hash
```

```
    for _ in range(len(self.table)): # start traversing
```

```
        if self.table[position] is None: # found empty slot
```

```
            raise KeyError(key) # so the key is not in
```

```
        elif self.table[position][0] == key: # found key
```

```
            return self.table[position][1] #return data
```

```
        else: # there is something but not the key, try next
```

```
            position = (position+1) % len(self.table)
```

```
    # At this point, I have gone through the table and not found
```

```
    raise KeyError(key)
```

Stop if we find an empty slot

Traverse each item in our hash table from position

Linear Probing

```
def __linear_probe(self, key: str, is_search: bool) -> int:
```

```
    position = self.hash(key)
```

Traverse each item in our hash table from position

```
    for _ in range(len(self.table)):
```

```
        if self.table[position] is None: # found empty slot
```

If we're searching for an item (eg. \_\_getitem\_\_)

```
            if is_search: # if searching
```

```
                raise KeyError(key) # key is not in
```

```
            else:
```

```
                return position # if adding, return position
```

```
        elif self.table[position][0] == key: # found key
```

```
            return position
```

```
        else: # there is something but not the key, try next
```

```
            position = (position + 1) % len(self.table)
```

Linear Probing

```
    raise KeyError(key)
```

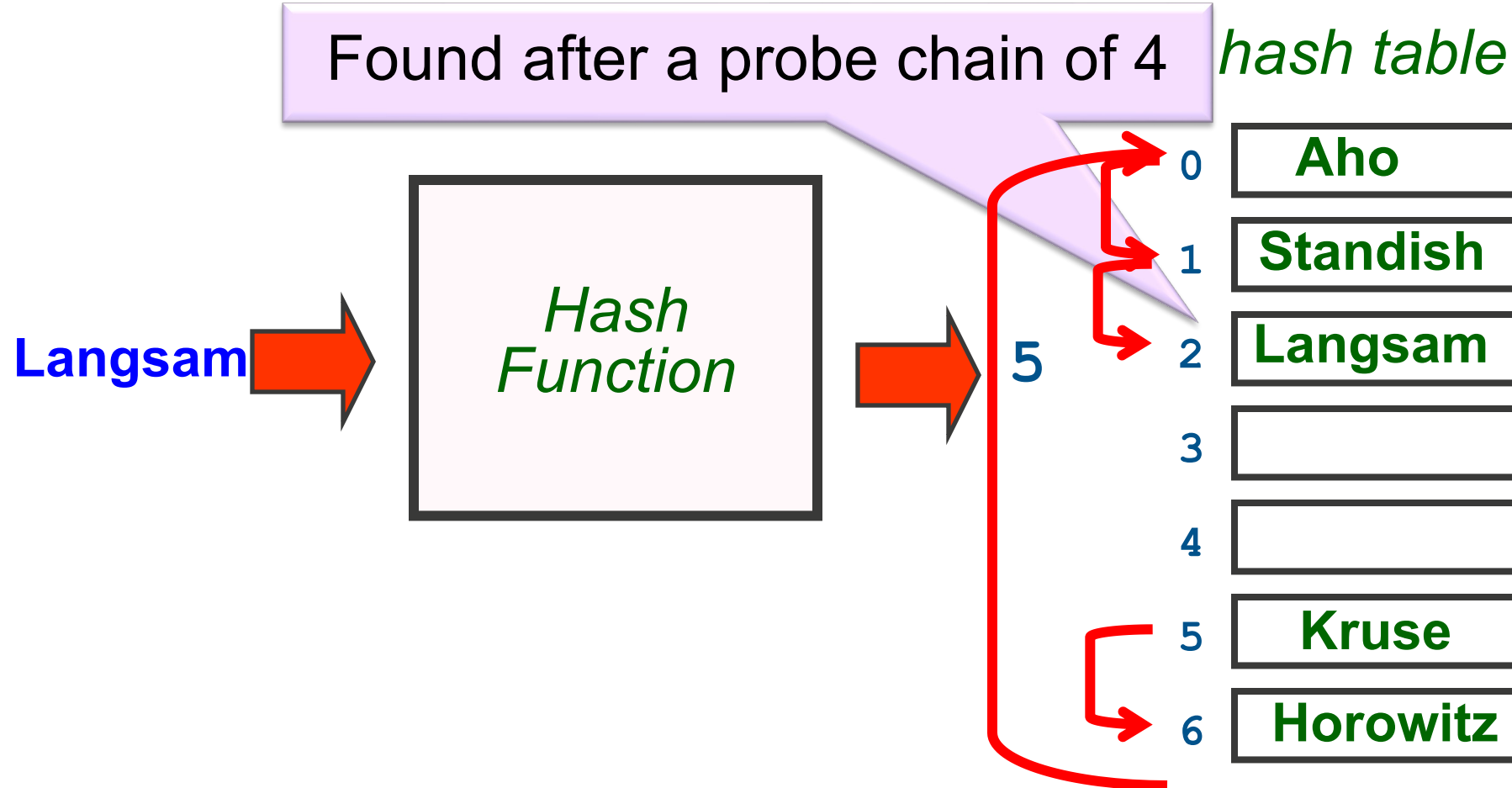
```
def __getitem__(self, key: str) -> T:
    position = self.__linear_probe(key, True)
    return self.table[position][1]
```

Will raise a KeyError if not found

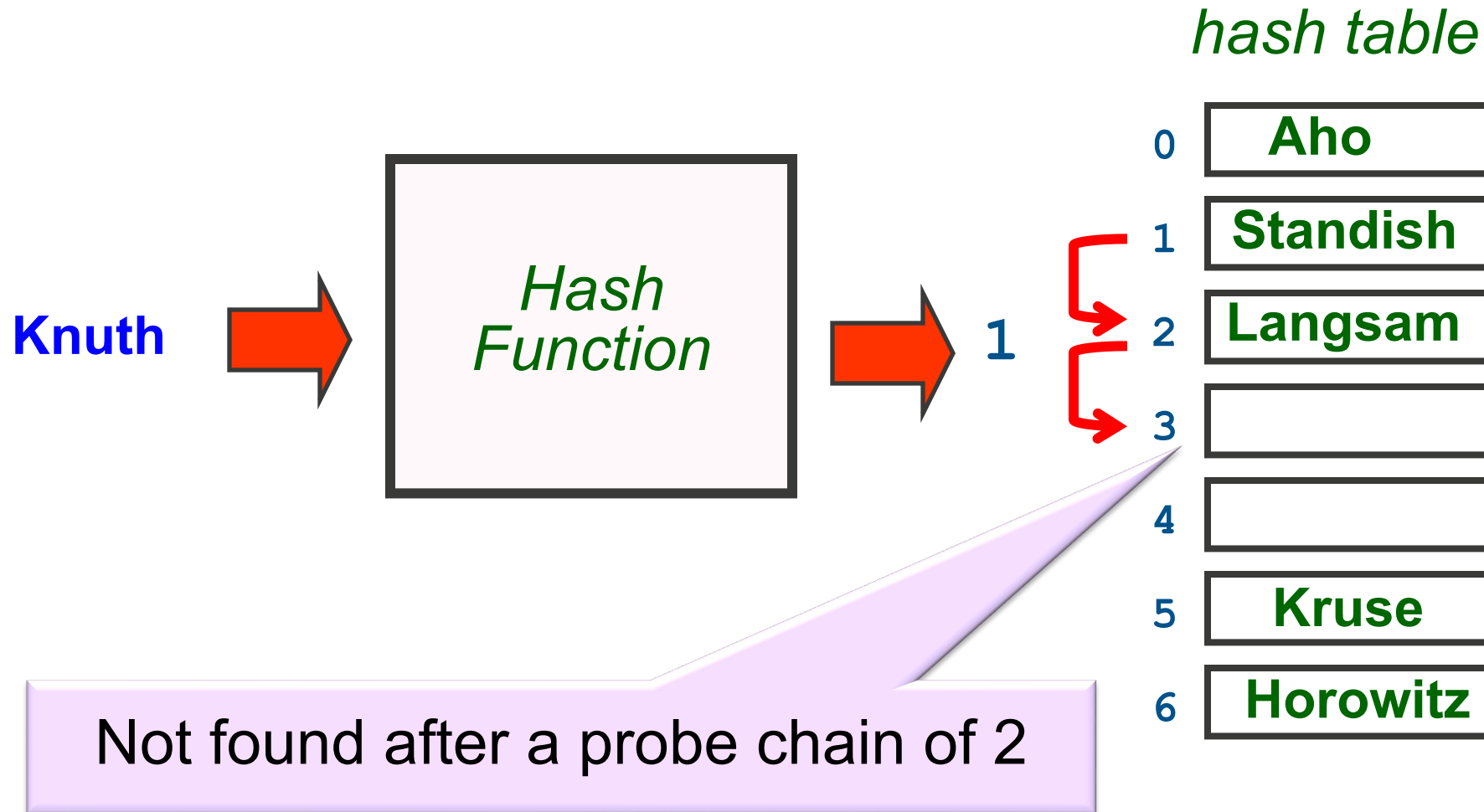
```
def __setitem__(self, key: str, data: T) -> None:
    try:
        position = self.__linear_probe(key, False)
    except KeyError:
        self.__rehash()
        self.__setitem__(key, data) # try again
    else:
        if self.table[position] is None: # if it's a new item
            self.count += 1
        self.table[position] = (key, data)
```

Full Hash Table, need to resize

## Example: search



## Example: search





# Conflict Resolution

## Delete

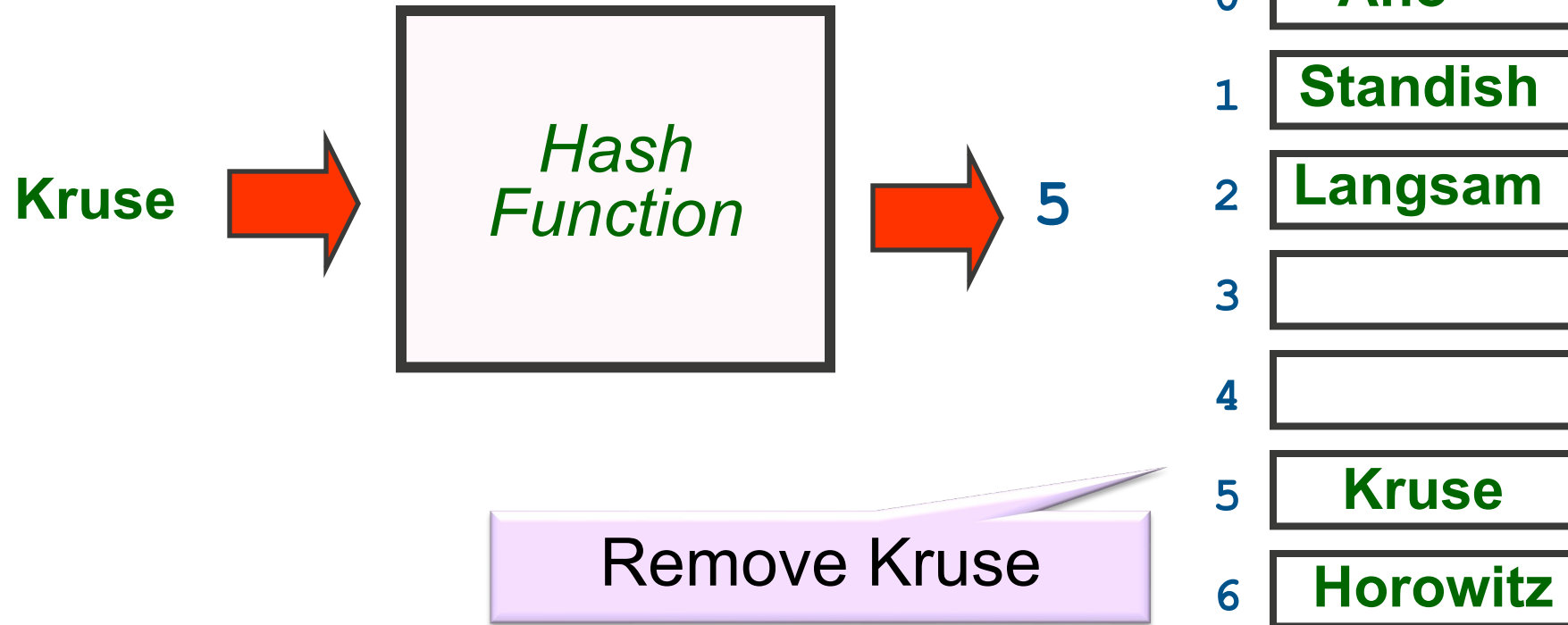
# Deleting in Linear Probing

- **What about delete?**
  - Use the search function to find the item
  - If found at  $N$ , then what?
- **Should we simply delete it and leave it empty?**
  - No, as empty spots have meaning in linear probing...

Invariant in Linear Probing: if an item with  $\text{hash}(\text{key})=N$  is in the table, it will always appear between  $N$  and the first empty position (wrapping around)

## Example: bad delete I

- Assume we delete Kruse and just leave it empty



# Example: bad delete I

- Assume we delete Kruse and just leave it empty
- If we now search for Horowitz (key 5)?
  - It will say it does not find it (raise `KeyError`)

```
def __linear_probe(self, key: str, is_search: bool) -> int:
    position = self.hash(key)

    for _ in range(len(self.table)):
        if self.table[position] is None:
            if is_search:
                raise KeyError(key)
            else:
                return position
        elif self.table[position][0] == key:
            return position
        else:
            position = (position + 1) % len(self.table)
    raise KeyError(key)
```

*hash table*

0	Aho
1	Standish
2	Langsam
3	
4	
5	
6	Horowitz

# Deleting in Linear Probing

- **What about delete?**

- Use the search function to find the item
- If found at  $N$ , then what?

- **Should we simply delete it and leave it empty?**

- No, as empty spots have meaning in linear probing...

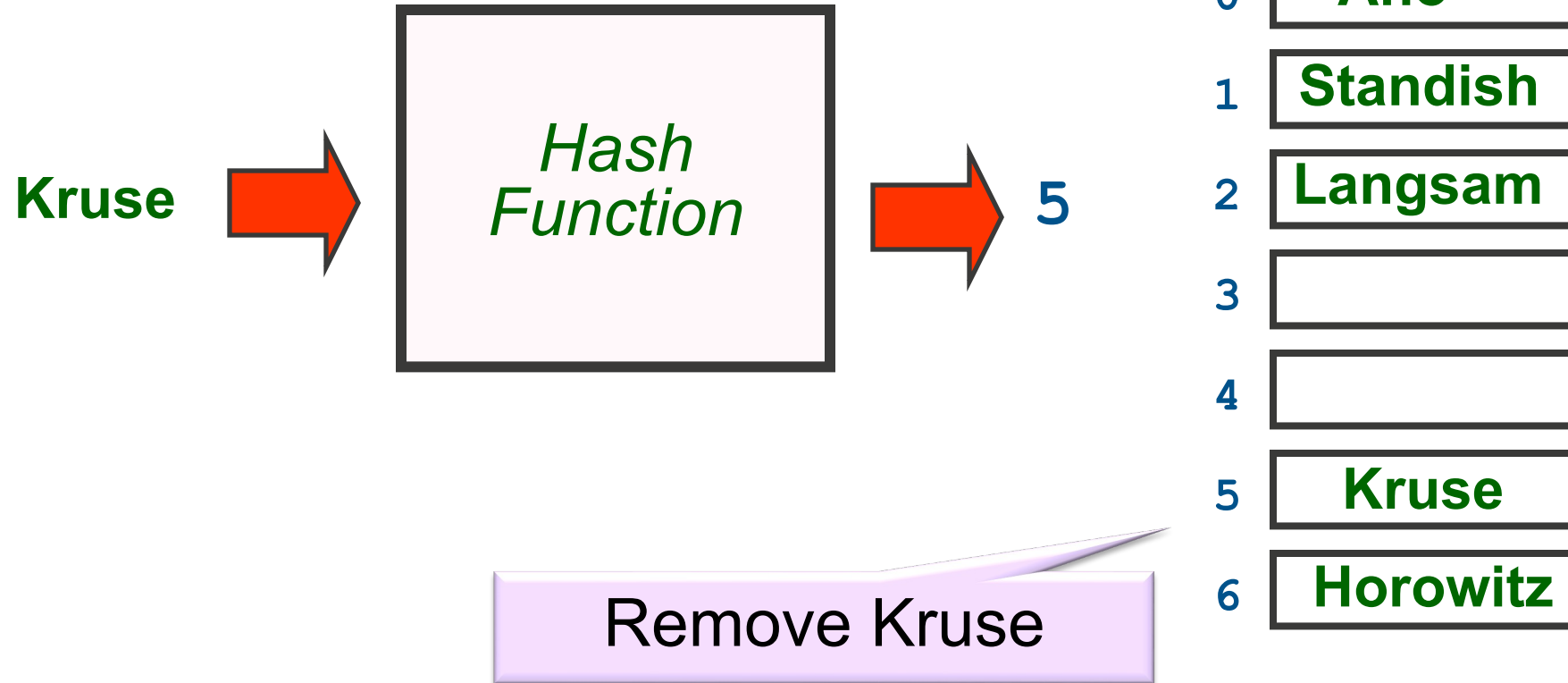
- **Should we shuffle everything from  $N+1$  upwards?**

- From  $N+1$  to what?
  - To the first empty position
- Is shuffling a good idea though?
  - No, we might move items that were in the correct positions!

Invariant in Linear Probing: if an item with  $\text{hash}(\text{key})=N$  is in the table, it will always appear between  $N$  and the first empty position (wrapping around)

## Example: bad delete II

- Assume we delete Kruse and shuffle



## Example: bad delete II

- Assume we delete Kruse and shuffle

*hash table*

0	Aho
1	Standish
2	Langsam
3	
4	
5	Horowitz
6	

Shuffle up until first empty position

## Example: bad delete II

- Assume we delete Kruse and shuffle

*hash table*

0	
1	Standish
2	Langsam
3	
4	
5	Horowitz
6	Aho

Shuffle up until first empty position



## Example: bad delete II

- Assume we delete Kruse and shuffle

*hash table*

0	Standish
1	
2	Langsam
3	
4	
5	Horowitz
6	Aho

Shuffle up until first empty position

## Example: bad delete II

- Assume we delete Kruse and shuffle
- If we now search for Horowitz (key 5)?
  - Will find it without problem
- And if we search for Aho (key 0)?
  - Will not find it
- Shuffling can incorrectly move elements

*hash table*

0	Standish
1	Langsam
2	
3	
4	
5	Horowitz
6	Aho

# Deleting in Linear Probing

- **What about delete?**

- Use the search function to find the item
- If found at  $N$ , then what?

- **Should we simply delete it and leave it empty?**

- No, as empty spots have meaning...

- **Should we shuffle everything from  $N+1$  upwards?**

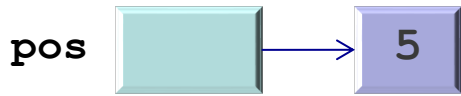
- From  $N+1$  to what?
  - To the first empty position
- Is shuffling a good idea though?
  - No, we might move items that were in the correct positions!

Invariant in Linear Probing: if an item with  $\text{hash}(\text{key})=N$  is in the table, it will always appear between  $N$  and the first empty position (wrapping around)

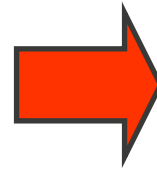
- **One possibility:**

- If found at  $N$ , **reinsert every item from  $N+1$**  to the first empty position
- Time consuming! (though should not be many)

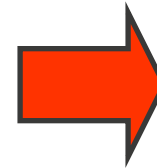
# Example: delete



Kruse



Hash  
Function



5

*hash table*

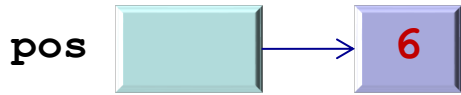
0	Aho
1	Standish
2	Langsam
3	
4	
5	Kruse
6	Horowitz

```
def __delitem__(self, key: str) -> None:
    pos = self.__linear_probe(key, True)
    self.table[pos] = None
    self.count -= 1
```

```
pos = (pos + 1) % len(self.table)
while self.table[pos] is not None:
    item = self.table[pos]
    self.table[pos] = None
    self.count -= 1
    self[str(item[0])] = item[1]
    pos = (pos + 1) % len(self.table)
```

Need to delete it

# Example: delete



```
def __delitem__(self, key: str) -> None:
    pos = self.__linear_probe(key, True)
    self.table[pos] = None
    self.count -= 1

    pos = (pos + 1) % len(self.table)
    while self.table[pos] is not None:
        item = self.table[pos]
        self.table[pos] = None
        self.count -= 1
        self[str(item[0])] = item[1]
        pos = (pos + 1) % len(self.table)
```

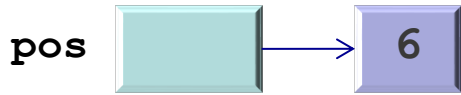
Key	Hash
Aho	0
Kruse	5
Standish	1
Horowitz	5
Langsam	5
Sedgewick	2
Knuth	1

*hash table*

0	Aho
1	Standish
2	Langsam
3	
4	
5	
6	Horowitz

Reinsert Horowitz

# Example: delete



```
def __delitem__(self, key: str) -> None:
    pos = self.__linear_probe(key, True)
    self.table[pos] = None
    self.count -= 1

    pos = (pos + 1) % len(self.table)
    while self.table[pos] is not None:
        item = self.table[pos]
        self.table[pos] = None
        self.count -= 1
        self[str(item[0])] = item[1]
        pos = (pos + 1) % len(self.table)
```

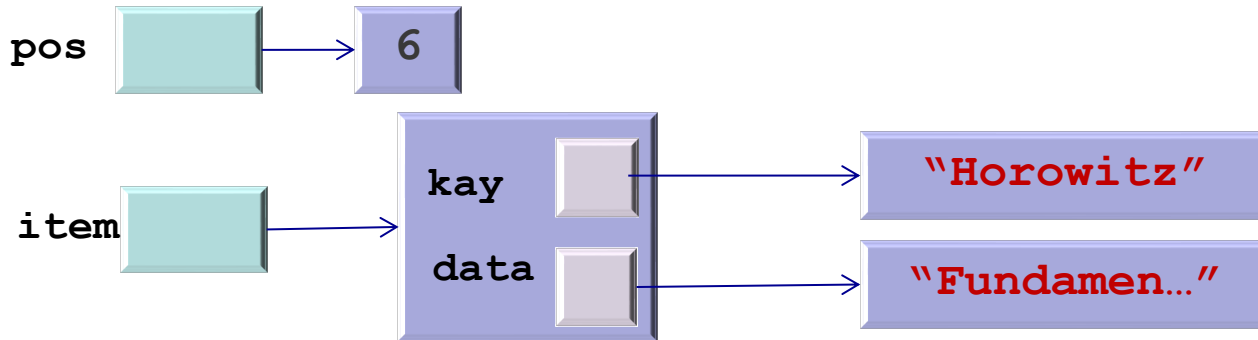
Key	Hash
Aho	0
Kruse	5
Standish	1
Horowitz	5
Langsam	5
Sedgewick	2
Knuth	1

*hash table*

0	Aho
1	Standish
2	Langsam
3	
4	
5	
6	Horowitz

Reinsert Horowitz

# Example: delete



```
def __delitem__(self, key: str) -> None:
    pos = self.__linear_probe(key, True)
    self.table[pos] = None
    self.count -= 1

    pos = (pos + 1) % len(self.table)
    while self.table[pos] is not None:
        item = self.table[pos]
        self.table[pos] = None
        self.count -= 1
        self[str(item[0])] = item[1]
        pos = (pos + 1) % len(self.table)
```

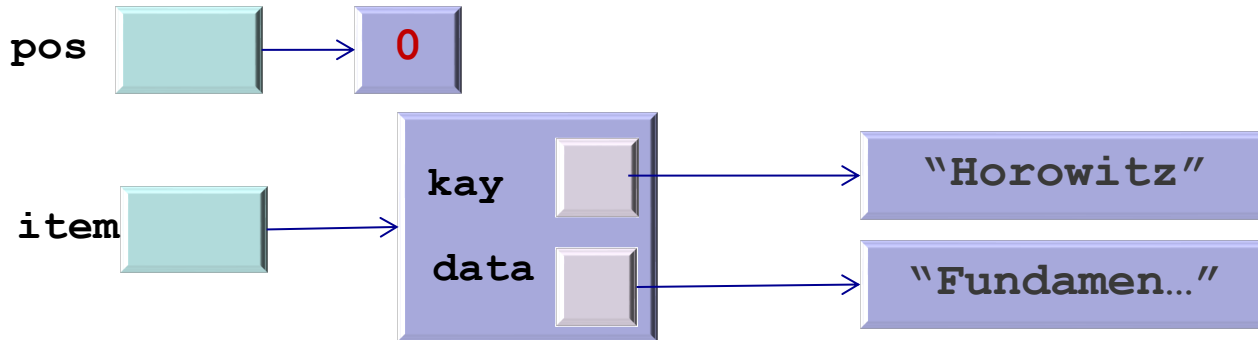
Key	Hash
Aho	0
Kruse	5
Standish	1
Horowitz	5
Langsam	5
Sedgewick	2
Knuth	1

*hash table*

0	Aho
1	Standish
2	Langsam
3	
4	
5	Horowitz
6	

Reuse our `__setitem__`

# Example: delete



```
def __delitem__(self, key: str) -> None:
    pos = self.__linear_probe(key, True)
    self.table[pos] = None
    self.count -= 1

    pos = (pos + 1) % len(self.table)
    while self.table[pos] is not None:
        item = self.table[pos]
        self.table[pos] = None
        self.count -= 1
        self[str(item[0])] = item[1]
    pos = (pos + 1) % len(self.table)
```

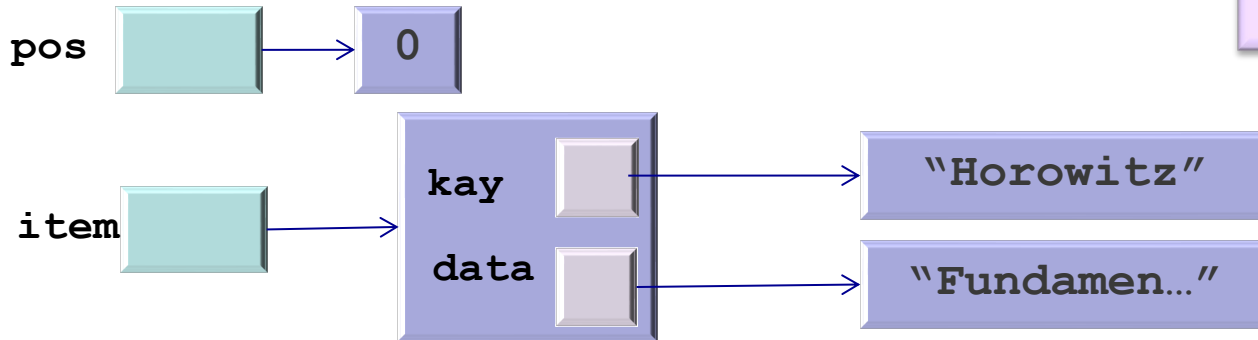
Key	Hash
Aho	0
Kruse	5
Standish	1
Horowitz	5
Langsam	5
Sedgewick	2
Knuth	1

*hash table*

0	Aho
1	Standish
2	Langsam
3	
4	
5	Horowitz
6	



# Example: delete



```
def __delitem__(self, key: str) -> None:
    pos = self.__linear_probe(key, True)
    self.table[pos] = None
    self.count -= 1

    pos = (pos + 1) % len(self.table)
    while self.table[pos] is not None:
        item = self.table[pos]
        self.table[pos] = None
        self.count -= 1
        self[str(item[0])] = item[1]
        pos = (pos + 1) % len(self.table)
```

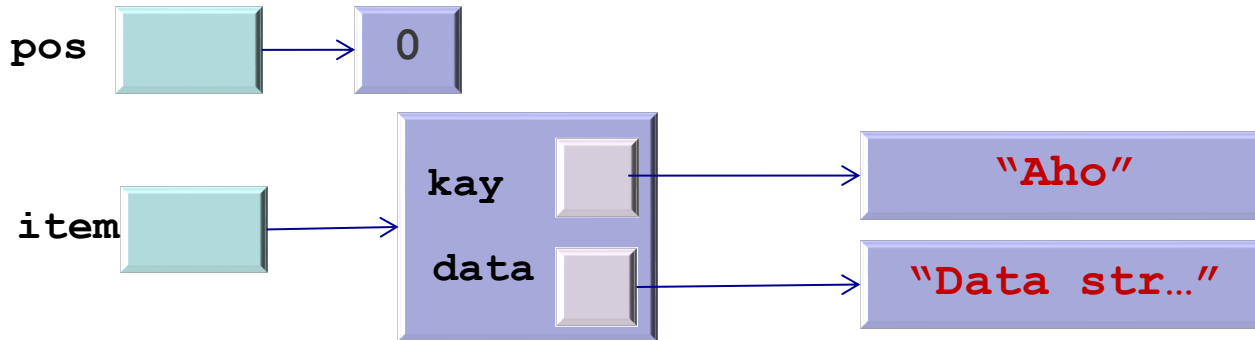
Reinsert Aho

Key	Hash
Aho	0
Kruse	5
Standish	1
Horowitz	5
Langsam	5
Sedgewick	2
Knuth	1

*hash table*

0	Aho
1	Standish
2	Langsam
3	
4	
5	Horowitz
6	

# Example: delete



```
def __delitem__(self, key: str) -> None:
    pos = self.__linear_probe(key, True)
    self.table[pos] = None
    self.count -= 1

    pos = (pos + 1) % len(self.table)
    while self.table[pos] is not None:
        item = self.table[pos]
        self.table[pos] = None
        self.count -= 1
        self[str(item[0])] = item[1]
        pos = (pos + 1) % len(self.table)
```

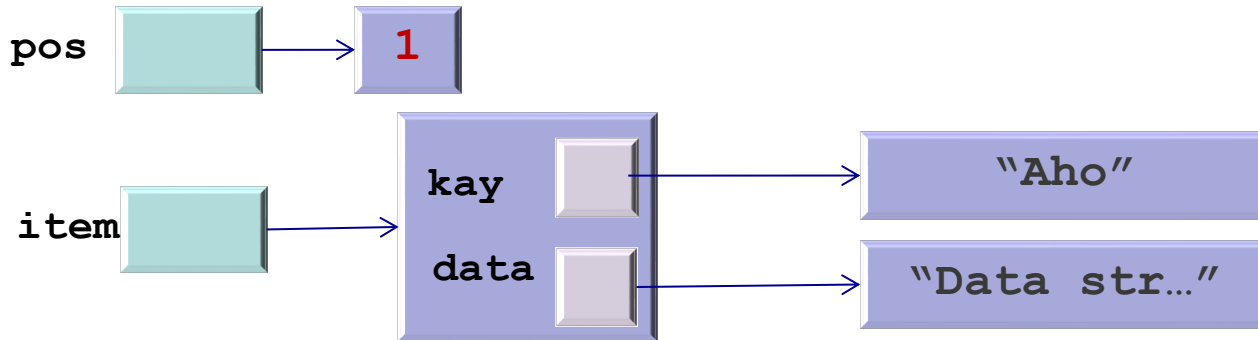
Stays the same

Key	Hash
Aho	0
Kruse	5
Standish	1
Horowitz	5
Langsam	5
Sedgewick	2
Knuth	1

*hash table*

0	Aho
1	Standish
2	Langsam
3	
4	
5	Horowitz
6	

# Example: delete



```
def __delitem__(self, key: str) -> None:
    pos = self.__linear_probe(key, True)
    self.table[pos] = None
    self.count -= 1

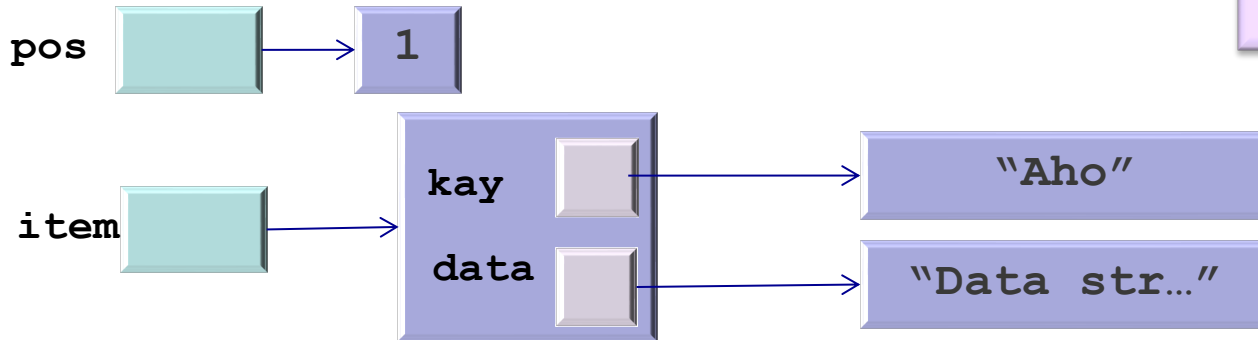
    pos = (pos + 1) % len(self.table)
    while self.table[pos] is not None:
        item = self.table[pos]
        self.table[pos] = None
        self.count -= 1
        self[str(item[0])] = item[1]
    pos = (pos + 1) % len(self.table)
```

Key	Hash
Aho	0
Kruse	5
Standish	1
Horowitz	5
Langsam	5
Sedgewick	2
Knuth	1

*hash table*

0	Aho
1	Standish
2	Langsam
3	
4	
5	Horowitz
6	

# Example: delete



```
def __delitem__(self, key: str) -> None:
    pos = self.__linear_probe(key, True)
    self.table[pos] = None
    self.count -= 1

    pos = (pos + 1) % len(self.table)
    while self.table[pos] is not None:
        item = self.table[pos]
        self.table[pos] = None
        self.count -= 1
        self[str(item[0])] = item[1]
        pos = (pos + 1) % len(self.table)
```

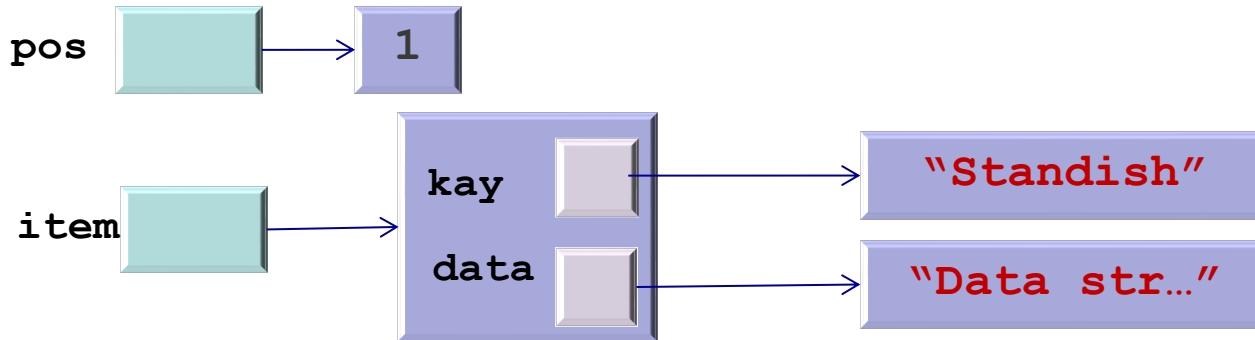
Reinsert Standish

Key	Hash
Aho	0
Kruse	5
Standish	1
Horowitz	5
Langsam	5
Sedgewick	2
Knuth	1

*hash table*

0	Aho
1	Standish
2	Langsam
3	
4	
5	Horowitz
6	

# Example: delete



```
def __delitem__(self, key: str) -> None:
    pos = self.__linear_probe(key, True)
    self.table[pos] = None
    self.count -= 1

    pos = (pos + 1) % len(self.table)
    while self.table[pos] is not None:
        item = self.table[pos]
        self.table[pos] = None
        self.count -= 1
        self[str(item[0])] = item[1]
        pos = (pos + 1) % len(self.table)
```

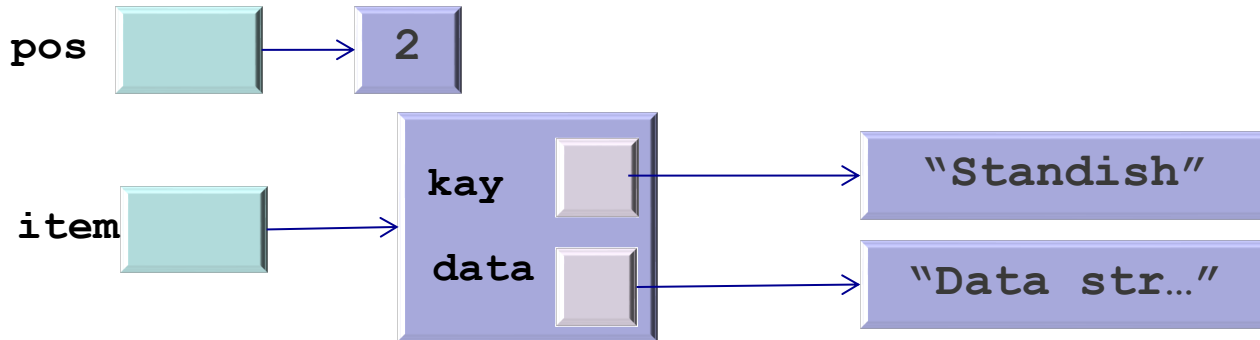
Stays the same

Key	Hash
Aho	0
Kruse	5
Standish	1
Horowitz	5
Langsam	5
Sedgewick	2
Knuth	1

*hash table*

0	Aho
1	Standish
2	Langsam
3	
4	
5	Horowitz
6	

# Example: delete



```
def __delitem__(self, key: str) -> None:
    pos = self.__linear_probe(key, True)
    self.table[pos] = None
    self.count -= 1

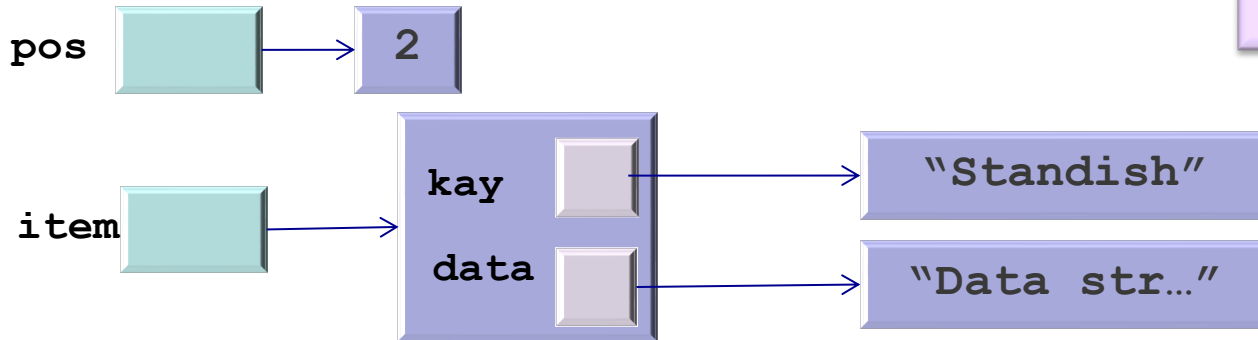
    pos = (pos + 1) % len(self.table)
    while self.table[pos] is not None:
        item = self.table[pos]
        self.table[pos] = None
        self.count -= 1
        self[str(item[0])] = item[1]
    pos = (pos + 1) % len(self.table)
```

Key	Hash
Aho	0
Kruse	5
Standish	1
Horowitz	5
Langsam	5
Sedgewick	2
Knuth	1

*hash table*

0	Aho
1	Standish
2	Langsam
3	
4	
5	Horowitz
6	

# Example: delete



```
def __delitem__(self, key: str) -> None:
    pos = self.__linear_probe(key, True)
    self.table[pos] = None
    self.count -= 1

    pos = (pos + 1) % len(self.table)
    while self.table[pos] is not None:
        item = self.table[pos]
        self.table[pos] = None
        self.count -= 1
        self[str(item[0])] = item[1]
        pos = (pos + 1) % len(self.table)
```

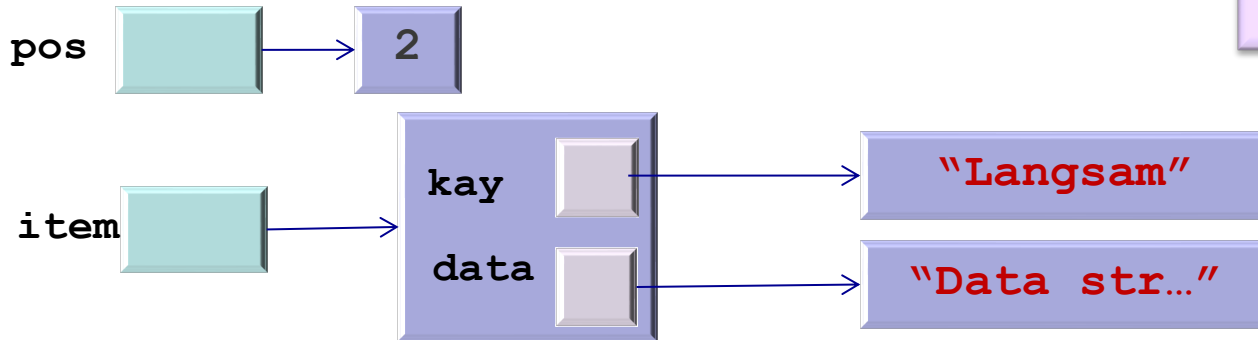
Reinsert Langsam

Key	Hash
Aho	0
Kruse	5
Standish	1
Horowitz	5
Langsam	5
Sedgewick	2
Knuth	1

*hash table*

0	Aho
1	Standish
2	Langsam
3	
4	
5	Horowitz
6	

# Example: delete



```
def __delitem__(self, key: str) -> None:
    pos = self.__linear_probe(key, True)
    self.table[pos] = None
    self.count -= 1

    pos = (pos + 1) % len(self.table)
    while self.table[pos] is not None:
        item = self.table[pos]
        self.table[pos] = None
        self.count -= 1
        self[str(item[0])] = item[1]
        pos = (pos + 1) % len(self.table)
```

Reinsert Langsam

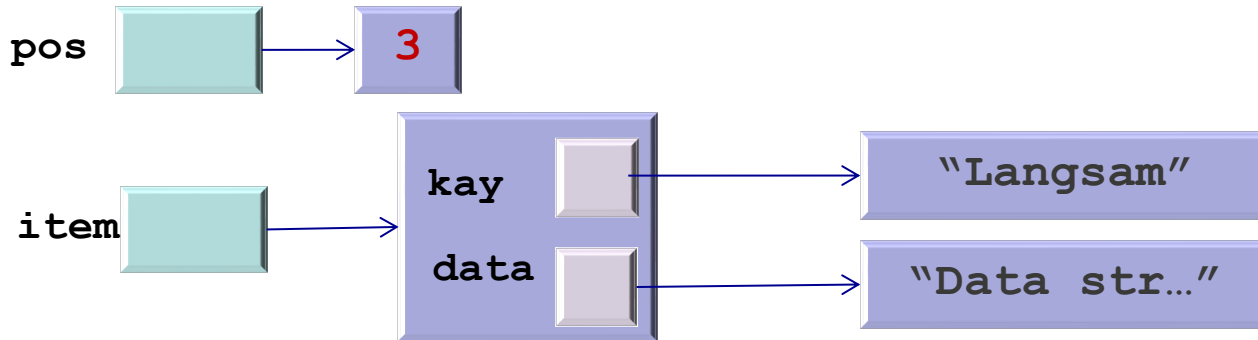
Key	Hash
Aho	0
Kruse	5
Standish	1
Horowitz	5
Langsam	5
Sedgewick	2
Knuth	1

*hash table*

0	Aho
1	Standish
2	
3	
4	
5	Horowitz
6	Langsam



# Example: delete



```
def __delitem__(self, key: str) -> None:
    pos = self.__linear_probe(key, True)
    self.table[pos] = None
    self.count -= 1

    pos = (pos + 1) % len(self.table)
    while self.table[pos] is not None:
        item = self.table[pos]
        self.table[pos] = None
        self.count -= 1
        self[str(item[0])] = item[1]
    pos = (pos + 1) % len(self.table)
```

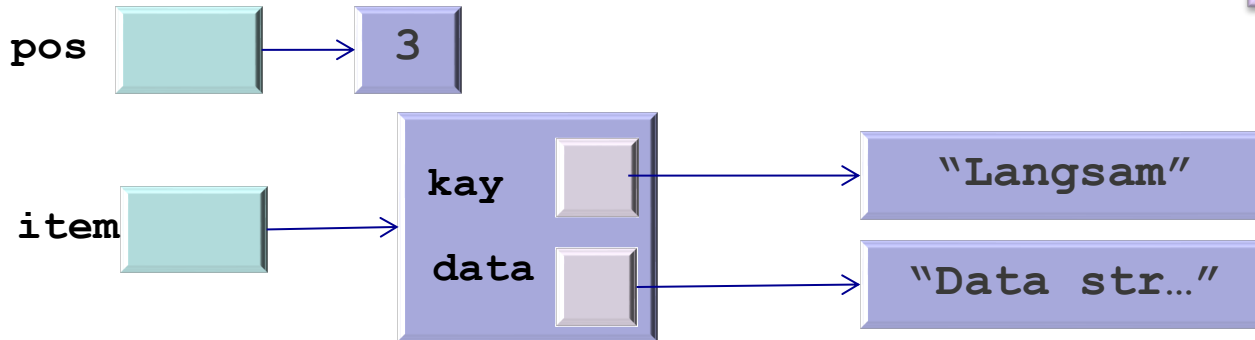
Key	Hash
Aho	0
Kruse	5
Standish	1
Horowitz	5
Langsam	5
Sedgewick	2
Knuth	1

*hash table*

0	Aho
1	Standish
2	
3	
4	
5	Horowitz
6	Langsam

# Example: delete

Reached empty, finish



```
def __delitem__(self, key: str) -> None:
    pos = self.__linear_probe(key, True)
    self.table[pos] = None
    self.count -= 1

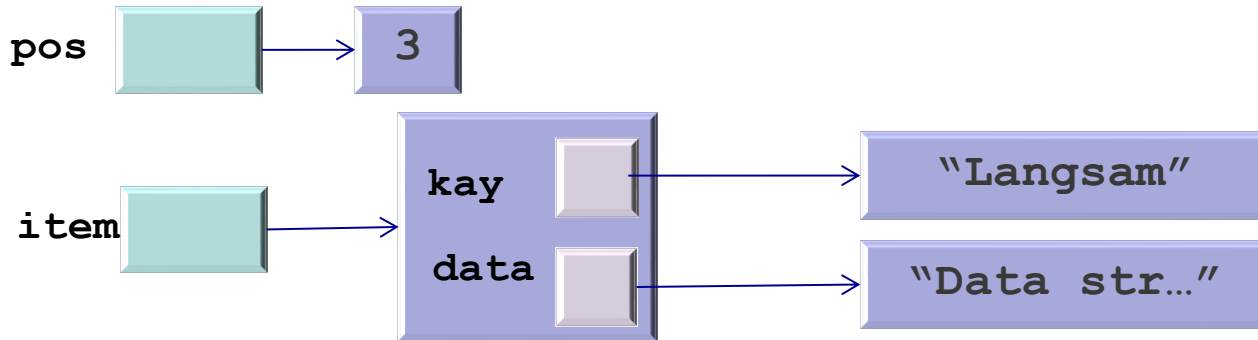
    pos = (pos + 1) % len(self.table)
    while self.table[pos] is not None:
        item = self.table[pos]
        self.table[pos] = None
        self.count -= 1
        self[str(item[0])] = item[1]
        pos = (pos + 1) % len(self.table)
```

Key	Hash
Aho	0
Kruse	5
Standish	1
Horowitz	5
Langsam	5
Sedgewick	2
Knuth	1

*hash table*

0	Aho
1	Standish
2	
3	
4	
5	Horowitz
6	Langsam

# Example: delete



```
def __delitem__(self, key: str) -> None:
    pos = self.__linear_probe(key, True)
    self.table[pos] = None
    self.count -= 1

    pos = (pos + 1) % len(self.table)
    while self.table[pos] is not None:
        item = self.table[pos]
        self.table[pos] = None
        self.count -= 1
        self[str(item[0])] = item[1]
        pos = (pos + 1) % len(self.table)
```

Key	Hash
Aho	0
Kruse	5
Standish	1
Horowitz	5
Langsam	5
Sedgewick	2
Knuth	1

*hash table*

0	Aho
1	Standish
2	
3	
4	
5	Horowitz
6	Langsam

# Conflict Resolution

## Open Addressing

# Open Addressing: Linear Probing

## ▪ Another possibility for delete:

- Use a special symbol (a **sentinel**) to denote delete
- Modify add and search to take that symbol into account
  - For search: treat the sentinel as you would treat a cell with a key different to the one you are searching for (keep on looking)
    - If found, you could move it to the first deleted cell you found
    - What else would need to be done?
  - For add: treat it as empty and add it in the first deleted cell
    - But this only works for new keys (those not already in the table)
    - If you want updates... think about it!

# Open Addressing: Linear Probing

- **Load factor:** total number of items/TABLESIZE
- **Cluster:** sequence of full hash table slots (i.e., without an empty slot)
- **Cluster can form even when the load is small**
- **Once a cluster forms, it tends to grow larger**
  - Items that hash to a value within the cluster, get added at the end making it bigger
  - This might involve more than one hash value

## Example of cluster

- All 4 elements are part of a cluster
- Two of them have the same hash value:
  - Kruse and Horowitz (5)
- The other two have different hash values (0 and 1)
- From then on, any element mapped to 0,1,5 or 6 will be part of the cluster. And adding elements mapped to 0,1,2,4,5, or 6 will make it grow.

*hash table*

0	Aho
1	Standish
2	
3	
4	
5	Kruse
6	Horowitz

# Linear Probing: Problems

- Tendency for **clustering** to occur as the load is  $> 0.5$
- **Low speed on clustering:**
  - Adding a key with hash value  $N$  can drastically increase the search time for keys with values other than  $N$
  - Deletion can also be time consuming, as the entire cluster needs to be rehashed
  - This means we start to under-deliver on the  $O(1)$  promise
- **If implemented in arrays – table may become full fairly quickly, resizing is time and resource consuming**

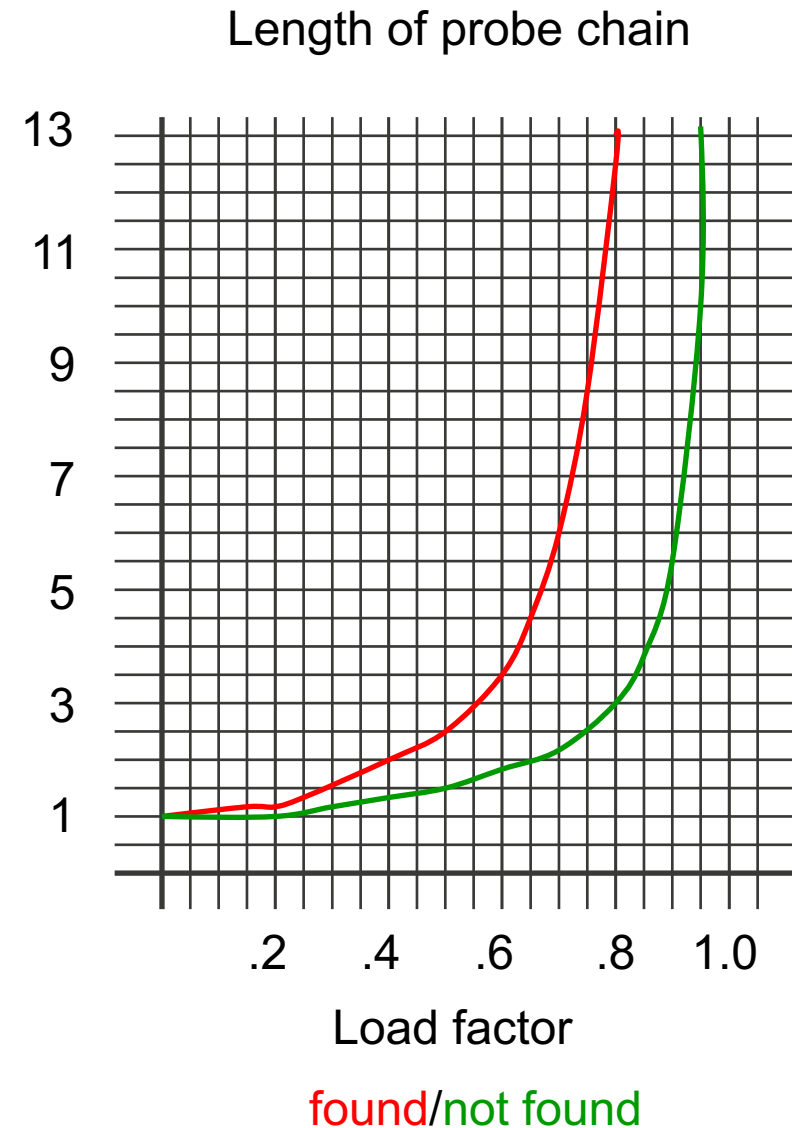
What can we do?

Idea: reduce clustering by taking bigger and bigger steps (rather than always using  $+S$ )



# Open Addressing

- **You must keep the load under  $2/3$** 
  - Otherwise the probe length (i.e., number of items visited before the element is found/not found) is too high
- **Even better: under  $1/2$**



# Conclusion

- Hash Tables are one of the most used data types, as they have expected  $O(1)$  complexity for adding, deleting and searching, if *built properly*

As you've seen, hard to achieve in practice!

- You have a very good chance of using them in your professional career
- They are very simple conceptually
- But they are also very “empirical”:
  - A significant amount of experimental evaluation is usually needed to fine tune the hash function and the TABLESIZE
- A good choice of hash function, collision handling and load factor are crucial to maintaining an efficient hash table (i.e., keep the  $O(1)$  promise)

# Summary

- **Open Addressing**
  - Linear Probing
- **Advantages/Disadvantages**
  - Length of probe
  - Memory usage
  - Resizing