Bayesian Marketing Mix Modeling

Time-Varying Non-Linear Models for Predicting Optical Fiber Sales

R. Montagna, Z. Peracchione, F. Pertusi, A. Riboni, F. Tomasi, D. Vozza

Tutor: Emanuele Fedrigolli

Marketing Mix Modeling (MMM)

What is MMM?

- A statistical technique used to measure the impact of marketing activities on sales.
- Helps identify which advertising channels generate the best ROI.

What is it used for?

- Optimizes marketing budget allocation.
- Improves sales forecasting.
- Supports strategic decisions based on data.

In this industry context the key challenges in budget allocation are:

- **Attribution difficulty:** Multiple advertising channels are used simultaneously.
- **Non-linear effects:** More advertising doesn't always mean more sales (saturation).
- **External factors:** Seasonality, competitor pricing, market trends.

Example: If a TV promotion boosts sales today, was it due to the ad itself or a competitor lowering prices?

Dataset Overview

Dataset Description

- 418 weekly observations (2017-2024).
- Includes sales data, marketing spend, pricing strategies, and competitor activities.
- Response variable: weekly fiber-optic offers sold.
- 89 variables across different categories:
  - **Marketing Exposure**: Digital Ads, TV campaigns, Audio Ads.
  - **Competitor Advertising**: Rival ad spending across channels.
  - **Pricing Information**: Own & competitor pricing.
  - **Consumer Engagement & Promotions**.

Data Preprocessing

Steps Taken to Prepare the Data:

- **Variable Correlation Analysis**
- **Feature Aggregation**
- **Dummy Variables**
  - Christmas & Ferragosto (seasonality effects).
  - COVID impact (shift in consumer demand).
  - Promotional weeks (special offers).
- **Train-Test Splitting**
- **Range-Aware Scaling**
- **Variable Selection**

Bayesian Framework

- Incorporates prior distributions to regularize estimates.
- Quantifies uncertainty via probability distributions.

Non-Linear Effects

Carryover Effect $\rightarrow$ AdStock with Geometric decay:

$$\text{AdStock}(x_t) = \frac{\sum_{l=0}^{L} \theta^l x_{t-l}}{\sum_{l=0}^{L} \theta^l} \qquad \theta \overset{\text{iid}}{\sim} \text{Beta}(\mu = \text{theta\_params}, \sigma = 0.05).$$

Saturation Effect $\rightarrow$ Logistic Function:

$$f(x) = \frac{1 - e^{-\lambda x}}{1 + e^{-\lambda x}} \qquad \lambda \overset{\text{iid}}{\sim} \text{Gamma}(\mu = \text{lambda\_params}, \sigma = 0.05).$$

Model Formulation

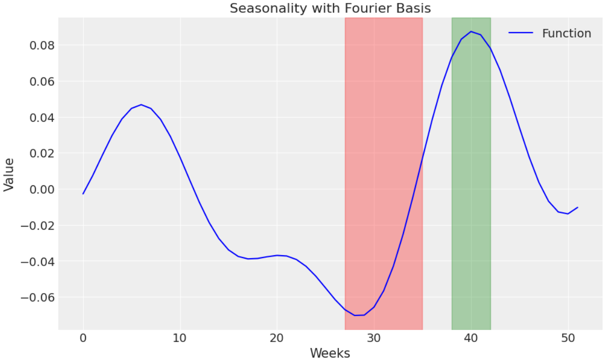
The likelihood of the model is obtained by: $y_t|\mu_t, \sigma \overset{\text{ind}}{\sim} \mathcal{N}(\mu_t, \sigma)$

$$\mu_t = \beta_0 + f_t \cdot \left(\sum_{i=1}^{n_{\text{channels}}} X_{\text{transformed},i,t} \cdot \beta_{\text{channel},i} \right) + \\ + \sum_{i=1}^{n_{\text{price}}} (X_{\text{price},i,t} \cdot \beta_{\text{price},i}) - \sum_{i=1}^{n_{\text{competitors}}} (X_{\text{competitors,i,t}} \cdot \beta_{\text{competitors, i}}) + \\ + \sum_{i=1}^{n_{\text{control}}} (X_{\text{control},i,t} \cdot \beta_{\text{control},i}) + \sum_{i=1}^{n_{\text{fourier}}} (X_{\text{fourier},i,t} \cdot \beta_{\text{fourier},i}) \tag{1}$$

$\sigma \sim \text{HalfNormal}(0.1)$.

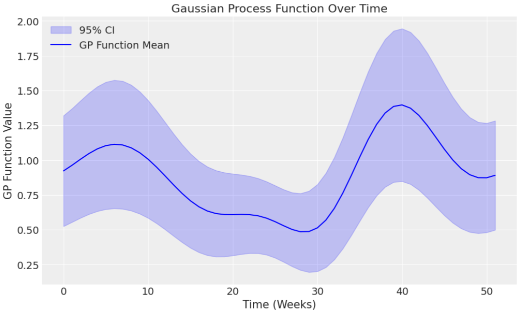
Time-Varying Effects - Fourier Seasonality

$$f(t) = \sum_{k=1}^{3} \left[a_k \cos\left(\frac{2\pi kt}{52}\right) + b_k \sin\left(\frac{2\pi kt}{52}\right) \right]$$



Time-Varying Effects - Gaussian Process

$$f_t \sim \mathcal{GP}(0, k(t, t')) \qquad k(t, t') = \exp\left(-\frac{2\sin^2\left(\frac{\pi|t-t'|}{52}\right)}{\lambda^2}\right)$$



Priors Distributions

The prior distributions of the model parameters are the following:

- $\beta_0 \sim \textit{HalfNormal}(\mu_{\text{intercept}}, 0.1)$
- $\beta_{\text{channel}} \overset{\text{iid}}{\sim} \textit{HalfNormal}(0, 0.08)$.
- $\beta_{\text{price}} \overset{\text{iid}}{\sim} \mathcal{N}(\mu_{\text{price}}, 0.1)$
- $\beta_{\text{competitors}} \overset{\text{iid}}{\sim} \textit{HalfNormal}(0, 0.1)$.
- $\beta_{\text{control}} \overset{\text{iid}}{\sim} \mathcal{N}(\mu_{\text{control}}, 0.1)$
- $\beta_{\text{fourier}} \overset{\text{iid}}{\sim} \mathcal{N}(\mu_{\text{fourier}}, 0.01)$

Models Definition and Evaluation

Three principal models: with Fourier Basis and/or Gaussian Process.
Common pipeline for each model:

1. Priors Identification:
   1.1 Correlation and Plot Analysis
   1.2 Non Bayesian OLS
2. Model Definition and Prior Predictive Check
3. Sampling and Convergence Analysis
4. Posterior Predictive Distribution
5. Rescaling
6. Contribution Analysis
7. Evaluation Metrics

Posterior Predictive Distribution

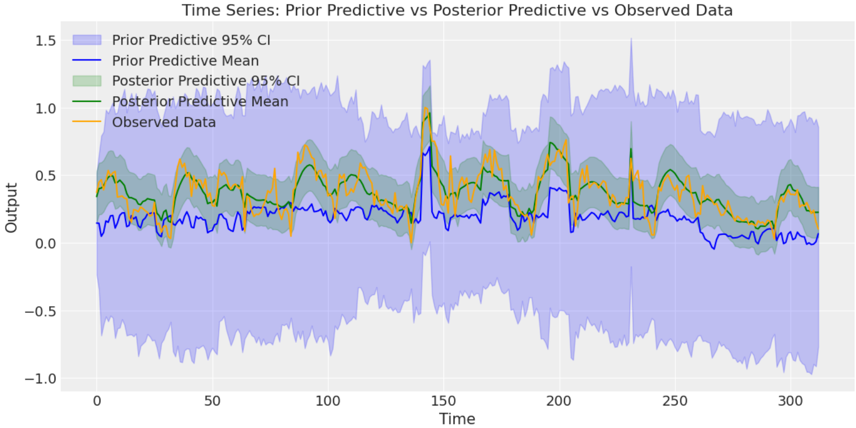


Figure: Prior vs Posterior Predictive Distribution - Model 2

Posterior Predictive Distribution

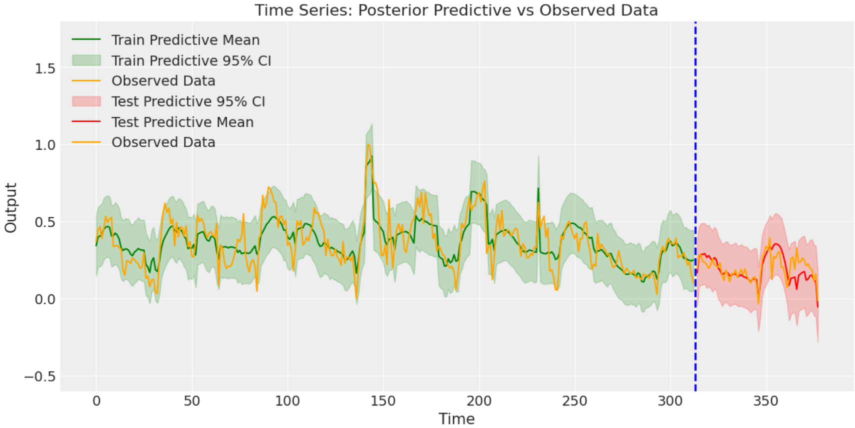


Figure: Posterior Predictive Distribution - Training vs Test - Model 3

Contribution Analysis

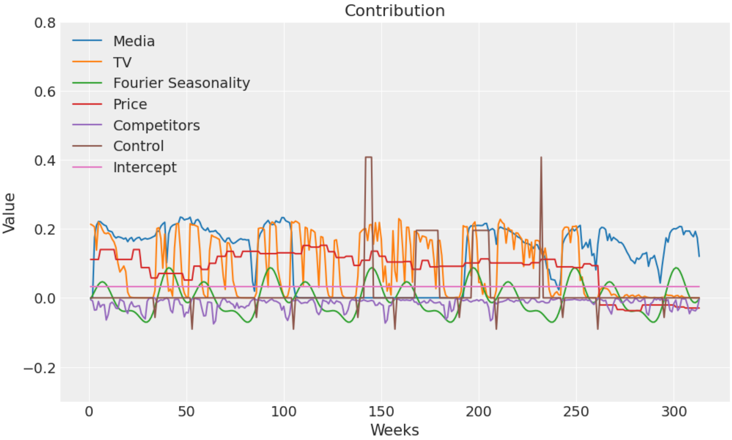


Figure: Contribution Analysis - Model 3

Models Results

Model	Fourier Basis	Gaussian Process	MAPE
Model 1	Yes	No	13.001%
Model 2	No	Yes	13.347%
Model 3	Yes	Yes	11.747%

Table: Results of the final versions of the three model structures.

- The combination of both techniques gives the best performance.
- Good trade-off between desired accuracy and overfitting reduction.
- Needed more informative and restrictive priors.
- Performance reduction in the last timestamps.

Achievements, Limitations

- Variables impact on sales.
- Bayesian framework advantages:
  - Uncertainty quantification.
  - Inclusion of experts knowledge.
  - Identification problem resolution.
- Marketing explainability.

- Challenges in distinguishing the different effects.
- Strategy and market changes.
- Data availability.
- Not available closed-form solutions for the posterior and marginal distributions.

Future Work

- Explore alternative priors and model structures.
- Obtain the ideal threshold: 10%.
- ROI optimization, budget allocation.