I needed to solve the exact same problem, and your diagram has given me the idea for what I think is the exact solution. This will work for varying radiuses, and can be simplified if all radiuses are equal, as in your case.

1. First a special case. If the point is outside of all circles, we have a trivial solution.

2. Find all the circles the point is inside. Calculate the closest point on their circumference (just move out from the original point along the radius).

3. Find all the intersection points between pairs of circles.

4. Combine the sets of points from steps 2 and 3, and filter these by finding the ones that are not covered by any other circle.

5. Pick the closest point from the remaining set. Done!

This seems to be O(n^3), so not terribly fast, but should be doable if your set is not too huge.