To expand on falagar's answer, my favourite proof of Binet's formula:

...Which I was going to post a summary of here, but remembered that everything was awful without Tex, so here is a link to [some notes on it I found on google][1].

The basic idea is to treat pairs of fibonnacci numbers, adjacent in the sequence, as vectors. Moving on to the next adjacent pair induces a linear transformation not unlike that of the matrix falagar posted. Calculating eigenvalues and eigenvectors can give a complete prediction of where an initial vector will find itself, predicting the whole sequence.

It's quite a lot of work but I think it's rather illuminating.

[1]: http://livetoad.org/Courses/Documents/132d/Notes/fibonacci\_matrix.pdf