\begin{aligned}

I&=\_^1\frac{\ln(1+x-x^2)}x\mathrm{d}x\\

&\overset{(1)}{=}\_^1\_{n=1}^\infty\frac{(-1)^{}(x-x^2)^n}{nx}\mathrm{d}x\\

&\overset{(2)}{=}\_{n=1}^\infty\frac{(-1)^{}}n\_^1x^{}(1-x)^n\mathrm{d}x\\

&\overset{(3)}{=}\_{n=1}^\infty\frac{(-1)^{}}n\frac{()!n!}{(2n)!}\\

&=\_{n=}^\infty\frac{(-1)^{n}(n!)^2}{(2n+2)!}\\

&=\_{n=}^\infty\frac{(-1)^{n}(1\times2\times\cdots\times n)(1\times2\times\cdots\times n)}{1\times2\times\cdots\times (2n+2)}\\

&=\_{n=}^\infty\frac{(-1)^{n}(1\times2\times\cdots\times n)}{1\times3\times5\times\cdots\times(2n+1)\times (2n+2)2^n}\\

&=\_{n=}^\infty\frac{(-1)^nn!}{(2n+1)!!(2n+2)2^n}

\end{aligned}

\_\_Explanation\_\_

(1) Using the Maclaurin series of , where .

(2) It is legal to change the position of and .

(3) Integrate by parts times.

[Notice that](https://math.stackexchange.com/questions/534736/evaluate-this-power-series) \_{n=}^\infty\frac{(2n)!!}{(2n+1)!!}x^{2n+1}=\frac{\arcsin x}{\sqrt{1-x^2}},

integrate both sides from , we have \_{n=}^\infty\frac{(2n)!!}{(2n+1)!!(2n+2)}x^{2n+2}=\frac12\arcsin^2x.

Letting leads to

\_{n=}^\infty\frac{(-1)^{n+1}(2n)!!}{(2n+1)!!(2n+2)}2^{-2n-2}=\frac12\arcsin^2\frac i2.

Combining with , we have

or where denotes the golden ratio.