Put

\begin{equation\*}

I=\int\_{0}^1\dfrac{\ln(1+-^2)}{}\,\mathrm{d} = \int\_{0}^1\dfrac{\ln(1+(1-))}{}\mathrm{d}.

\end{equation\*}

If we change to 1- we get

\begin{equation\*}

I=\int\_{0}^1\dfrac{\ln(1+-^2)}{1-}\,\mathrm{d}.

\end{equation\*}

Consequently

\begin{equation\*}

2I = \int\_{0}^1\ln(1+-^2)\left(\dfrac{1}{}+\dfrac{1}{1-}\right)\,\mathrm{d}.

\end{equation\*}

The net step will be integration by parts.

\begin{equation\*}

2I= \underbrace{\left[\ln(1+-^2)\ln\dfrac{}{1-}\right]\_{0}^{1}}\_{=0} -\int\_{0}^1\dfrac{1-2}{1+-^2}\ln\dfrac{}{1-}\, \mathrm{d}

\end{equation\*}

Then\begin{equation\*}

I=\dfrac{1}{2}\int\_{0}^1\dfrac{2-1}{1+-^2}\ln\dfrac{}{1-}\, \mathrm{d}.

\end{equation\*}

If we substitute z=\dfrac{}{1-} we get

\begin{equation\*}

I = \int\_{0}^{\infty}\dfrac{(z-1)\ln z}{2(z+1)(z^2+3z+1)}\,\mathrm{d}z.

\end{equation\*}

In order to evaluate this integral we integrate along a keyhole contour and use residue calculus. We get that

\begin{equation\*}

I = 2\ln^2\varphi

\end{equation\*}

where