Let

\begin{align}

I’(a)&= \int\_0^1\frac{4(1-x)\sinh2a }{1+4 x(1-x) \sinh^2a}dx\\

&= \left[ \frac{\ln[1+4 x(1-x) \sinh^2a]}{\tanh a}

+2\tanh^{-1}\frac{2x-1}{\coth a}\right]\_0^1=4a

\end{align}

Then

\int\_0^1\frac{\ln(1+x-x^2)}xdx

=\int\_0^{\sinh^{-1}\frac12}I’(a)da

= \int\_0^{\ln\phi}4a\ da=2\ln^2\phi