Linear Algebra

Homework 7

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$$A = \begin{bmatrix} 4 & -1 & 6 \\ 2 & 1 & 6 \\ 2 & -1 & 8 \end{bmatrix}$$

Question: 1

Begin by computing the characteristic polynomial of A (i.e. compute $\det(A - \lambda I)$). Use your polynomial to find the eigenvalues of A. (Note, you may use a computer to factor the characteristic polynomial).

Answer to 1: $-\lambda^3 + 13\lambda^2 - 40\lambda + 36$
$-\lambda^3 + 13\lambda^2 - 40\lambda + 36 = 0$
$\lambda = 2, 9$
Eigenvalues are 2 and 9. □

Question: 2

For each eigenvalue from part 1 compute a basis for the associated eigenspace. i.e., Compute a basis for each $\text{null}(A - \lambda I)$.

Answer to 2:

$$\text{null}(\begin{bmatrix} 4 & -1 & 6 \\ 2 & 1 & 6 \\ 2 & -1 & 8 \end{bmatrix} - \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix}) \tag{1}$$

$$\text{null}(\begin{bmatrix} 2 & -1 & 6 \\ 2 & -1 & 6 \\ 2 & -1 & 6 \end{bmatrix}) \tag{2}$$

$$\{\begin{bmatrix} \frac{1}{2} \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} -3 \\ 0 \\ 1 \end{bmatrix}\} \tag{3}$$

and,

$$\text{null}(\begin{bmatrix} 4 & -1 & 6 \\ 2 & 1 & 6 \\ 2 & -1 & 8 \end{bmatrix} - \begin{bmatrix} 9 & 0 & 0 \\ 0 & 9 & 0 \\ 0 & 0 & 9 \end{bmatrix}) \tag{4}$$

$$\text{null}(\begin{bmatrix} -5 & -1 & 6 \\ 2 & -8 & 6 \\ 2 & -1 & -1 \end{bmatrix}) \tag{5}$$

$$\{\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}\} \tag{6}$$

□

Question: 3

Use your answers from parts 1 and 2 to find a diagonal matrix D and an invertible matrix P such that $A = PDP^{-1}$

Answer to 3:

$$D = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 9 \end{bmatrix}$$

$$P = \begin{bmatrix} 1 & -3 & 1 \\ 2 & 0 & 1 \\ 0 & 1 & 1 \end{bmatrix}$$

□

Question: 4

Check the matrices you found in part 3 by computing AP and PD. They should be the same.

Answer to 4:

$$\begin{bmatrix} 4 & -1 & 6 \\ 2 & 1 & 6 \\ 2 & -1 & 8 \end{bmatrix} \begin{bmatrix} 1 & -3 & 1 \\ 2 & 0 & 1 \\ 0 & 1 & 1 \end{bmatrix} \tag{7}$$

$$\begin{bmatrix} -6 & 2 & 9 \\ 0 & 4 & 9 \\ 2 & 0 & 9 \end{bmatrix} \tag{8}$$

and,

$$\begin{bmatrix} 1 & -3 & 1 \\ 2 & 0 & 1 \\ 0 & 1 & 1 \end{bmatrix} \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 9 \end{bmatrix} \tag{9}$$

$$\begin{bmatrix} -6 & 2 & 9 \\ 0 & 4 & 9 \\ 2 & 0 & 9 \end{bmatrix} \tag{10}$$

which shows that P and D are correct. □

Question: 5

What is D^n?

Answer to 5:

$$D^n = \begin{bmatrix} 2^n & 0 & 0 \\ 0 & 2^n & 0 \\ 0 & 0 & 9^n \end{bmatrix}$$

□

Question: 6

Use your answer from parts 3 and 5 to compute a formula for A^n.

Answer to 6:

$$A^n = \begin{bmatrix} 1 & -3 & 1 \\ 2 & 0 & 1 \\ 0 & 1 & 1 \end{bmatrix} \begin{bmatrix} 2^n & 0 & 0 \\ 0 & 2^n & 0 \\ 0 & 0 & 9^n \end{bmatrix} \begin{bmatrix} 1 & 2 & 0 \\ -3 & 0 & 1 \\ 1 & 1 & 1 \end{bmatrix}^{-1}$$

□