Вдв
Решение дз №3

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Вопрос 1

Посчитать производные следующих функций

1. $\operatorname{arctg} x$
2. $\dfrac{1}{1 + x^2 \sin^2 x}$
3. $\dfrac{\ln x}{\sin x}$
4. $\ln(\ln x)$
5. $\exp^{x + \cos x}$

Решение:

1. $\operatorname{arctg}' x = \dfrac{1}{\operatorname{tg}' \operatorname{arctg} x} \Rightarrow \operatorname{tg}' \operatorname{arctg} x = \dfrac{1}{\cos^2 \operatorname{arctg} x} = \dfrac{1}{x^2 + 1}$
2. $\left(\dfrac{1}{1 + x^2 \sin^2 x}\right)' = -\dfrac{2x \sin^2 x + x^2 2 \sin x \cos x}{(1 + x^2 \sin^2 x)^2}$
3. $\left(\dfrac{\ln x}{\sin x}\right)' = \dfrac{\frac{1}{x} \sin x - \ln x \cos x}{\sin^2 x}$
4. $\ln(\ln x)' = \dfrac{1}{\ln x \cdot x}$
5. $(\exp^{x + \cos x})' = (1 - \sin x) \exp^{x + \cos x}$

Ответ:

1. $\dfrac{1}{x^2 + 1}$

2. $-\dfrac{2x \sin^2 x + x^2 2 \sin x \cos x}{(1 + x^2 \sin^2 x)^2}$

3. $\dfrac{\frac{1}{x} \sin x - \ln x \cos x}{\sin^2 x}$

4. $\dfrac{1}{\ln x \cdot x}$

5. $(1 - \sin x) \exp^{x + \cos x}$

Вопрос 2

Производные ~~от которые хочеться плакать~~ посложнее

1. $\arcsin(\cos(\ln(x + x^2))) \dfrac{1}{x^2 + \cos x}$

2. $\exp(\arcsin(\operatorname{arctg}(\exp x))) \dfrac{\exp(x) - \ln x}{2^{\cos x} - 1}$

Решение:

1. $\left(\arcsin(\cos(\ln(x + x^2))) \dfrac{1}{x^2 + \cos x}\right)' = \arcsin(\cos(\ln(x + x^2))) \cdot -\dfrac{2x - \sin x}{(x^2 + \cos x)^2} +$ что-то

   что-то $= \dfrac{1}{x^2 + \cos x} \cdot \left(\arcsin(\cos(\ln(x + x^2))\right)'$

$\left(\arcsin(\cos(\ln(x+x^2))\right)' = \dfrac{1}{\sqrt{1-(\cos(\ln(x+x^2)))^2}} \cdot \left(\cos(\ln(x+x^2))\right)' = -\dfrac{1}{\sqrt{1-(\cos(\ln(x+x^2)))^2}} \cdot \sin(\ln(x+x^2)) \cdot$
$\left(\ln(x+x^2)\right)' = -\dfrac{1}{\sqrt{1-(\cos(\ln(x+x^2)))^2}} \cdot \sin(\ln(x+x^2)) \cdot \dfrac{1}{x+x^2} \cdot (1+2x) \Rightarrow \left(\arcsin(cos(\ln(x+x^2)))\dfrac{1}{x^2+\cos x}\right)' =$
$-\arcsin(\cos(\ln(x+x^2))) \cdot \dfrac{2x-\sin x}{(x^2+\cos x)^2} - \dfrac{1}{\sqrt{1-(\cos(\ln(x+x^2)))^2}} \cdot \sin(\ln(x+x^2)) \cdot \dfrac{1}{x+x^2} \cdot (1+2x) \cdot \dfrac{1}{x^2+\cos x}$

2. $\left(\exp(\arcsin(\operatorname{arctg}(\exp x)))) \dfrac{\exp(x)-\ln x}{2^{\cos x}-1}\right)' = (\exp(\arcsin(\operatorname{arctg}(\exp x))))' \cdot \dfrac{\exp(x)-\ln x}{2^{\cos x}-1}) + \exp(\arcsin(\operatorname{arctg}(\exp x))) \cdot$
$\left(\dfrac{\exp(x)-\ln x}{2^{\cos x}-1}\right)'$
$(\exp(\arcsin(\operatorname{arctg}(\exp x))))' = \exp(\arcsin(\operatorname{arctg}(\exp x))) \cdot \dfrac{\exp x}{\sqrt{1-\operatorname{arctg}^2 \exp x}(1+\exp 2x)}$
$\left(\dfrac{\exp(x)-\ln x}{2^{\cos x}-1}\right)' = \dfrac{(\exp x - \ln x)' \cdot (2^{\cos x}-1) - (\exp x - \ln x) \cdot (2^{\cos x}-1)'}{(x^{\cos x}-1)^2} =$
$\dfrac{(\exp x - 1/x) \cdot (2^{\cos x}-1) + (\exp x + \ln x) \cdot (\ln x \cdot 2^{\cos x} \cdot \sin x)}{(x^{\cos x}-1)^2} \Rightarrow$
$\left(\exp(\arcsin(\operatorname{arctg}(\exp x)))) \dfrac{\exp(x)-\ln x}{2^{\cos x}-1}\right)' = \exp(\arcsin(\operatorname{arctg}(\exp x))) \cdot \dfrac{\exp x}{\sqrt{1-\operatorname{arctg}^2 \exp x}(1+\exp 2x)} \cdot \dfrac{\exp(x)-\ln x}{2^{\cos x}-1}) +$
$\exp(\arcsin(\operatorname{arctg}(\exp x)))) \cdot \dfrac{(\exp x - 1/x) \cdot (2^{\cos x}-1) + (\exp x + \ln x) \cdot (\ln x \cdot 2^{\cos x} \cdot \sin x)}{(x^{\cos x}-1)^2}$