Detailed Formulations of the Branch Flow Models

SOCBF:

$$P_{n,t}^{\mathrm{ac}} = \sum_{m\in\pi(n)} \left(H_{mn,t}^{\mathrm{ac}} - r_{mn}^{\mathrm{ac}} I_{mn,t}^{\mathrm{ac}}\right) - \sum_{d\in\delta(n)} H_{nd,t}^{\mathrm{ac}}, \forall t \tag{A1}$$

$$Q_{n,t}^{\mathrm{ac}} = \sum_{m\in\pi(n)} \left(G_{mn,t}^{\mathrm{ac}} - x_{mn}^{\mathrm{ac}} I_{mn,t}^{\mathrm{ac}}\right) - \sum_{d\in\delta(n)} G_{nd,t}^{\mathrm{ac}}, \forall t \tag{A2}$$

$$u_{n,t}^{\mathrm{ac}} = u_{m,t}^{\mathrm{ac}} - 2\left(r_{mn}^{\mathrm{ac}} H_{mn,t}^{\mathrm{ac}} + x_{mn}^{\mathrm{ac}} G_{mn,t}^{\mathrm{ac}}\right) + \left(\left(r_{mn}^{\mathrm{ac}}\right)^2 + \left(x_{mn}^{\mathrm{ac}}\right)^2\right) I_{mn,t}^{\mathrm{ac}} \tag{A3}$$

$$\left(2H_{mn,t}^{\mathrm{ac}}\right)^2 + \left(2G_{mn,t}^{\mathrm{ac}}\right)^2 + \left(I_{mn,t}^{\mathrm{ac}} - u_{m,t}^{\mathrm{ac}}\right)^2 \le \left(I_{mn,t}^{\mathrm{ac}} + u_{m,t}^{\mathrm{ac}}\right)^2 \tag{A4}$$

$$H_{mn}^{\mathrm{ac,min}} \le H_{mn,t}^{\mathrm{ac}} \le H_{mn}^{\mathrm{ac,max}} \tag{A5}$$

$$G_{mn}^{\mathrm{ac,min}} \le G_{mn,t}^{\mathrm{ac}} \le G_{mn}^{\mathrm{ac,max}} \tag{A6}$$

$$\left(U_{n}^{\mathrm{ac,min}}\right)^2 \le u_{n,t}^{\mathrm{ac}} \le \left(U_{n}^{\mathrm{ac,max}}\right)^2 \tag{A7}$$

$$0 \le I_{mn,t}^{\mathrm{ac}} \le \left(I_{mn}^{\mathrm{ac,max}}\right)^2 \tag{A8}$$

$$P_{n,t}^{\mathrm{dc}} = \sum_{m\in\pi(n)} \left(H_{mn,t}^{\mathrm{dc}} - r_{mn}^{\mathrm{dc}} I_{mn,t}^{\mathrm{dc}}\right) - \sum_{d\in\delta(n)} H_{nd,t}^{\mathrm{dc}}, \forall t \tag{A9}$$

$$H_{mn}^{\mathrm{dc,min}} \le H_{mn,t}^{\mathrm{dc}} \le H_{mn}^{\mathrm{dc,max}} \tag{A10}$$

$$0 \le I_{mn,t}^{\mathrm{dc}} \le \left(I_{mn}^{\mathrm{dc,max}}\right)^2 \tag{A11}$$

Here, Equations (A1)-(A8) are the constraints for ACM, while Equations (A9)-(A11) are the constraints for DCM.

For ACM, (A1) and (A2) denote the active and reactive power balance at each node, respectively. The Ohm's law over branch *mn* is expressed as (A3). The current magnitude of each line can be determined by (A4). (A5)-(A8) bound the limits for active flow power, reactive flow power, voltage magnitude and current magnitude, respectively.

For DCM, (A9) denotes the active power balance at each node. (A10) bounds the limits for active flow power, while (A11) limits the upper bounds for current magnitude. Note that, since reactive power is not considered in DC grids, the constraints associated with reactive power and voltage magnitude are not in the DCM.

LBF:

$$P_{n,t}^{\mathrm{ac}} = \sum_{m\in\pi(n)} H_{mn,t}^{\mathrm{ac}} - \sum_{d\in\delta(n)} H_{nd,t}^{\mathrm{ac}}, \forall t \tag{A12}$$

$$Q_{n,t}^{\mathrm{ac}} = \sum_{m\in\pi(n)} G_{mn,t}^{\mathrm{ac}} - \sum_{d\in\delta(n)} G_{nd,t}^{\mathrm{ac}}, \forall t \tag{A13}$$

$$U_{n,t}^{\mathrm{ac}} = U_{m,t}^{\mathrm{ac}} - \left(r_{mn}^{\mathrm{ac}} H_{mn,t}^{\mathrm{ac}} + x_{mn}^{\mathrm{ac}} G_{mn,t}^{\mathrm{ac}}\right) \Big/ U_{0,t}^{\mathrm{ac}} \tag{A14}$$

$$H_{mn}^{\mathrm{ac,min}} \le H_{mn,t}^{\mathrm{ac}} \le H_{mn}^{\mathrm{ac,max}} \tag{A15}$$

$$G_{mn}^{\mathrm{ac,min}} \le G_{mn,t}^{\mathrm{ac}} \le G_{mn}^{\mathrm{ac,max}} \tag{A16}$$

$$U_{n}^{\mathrm{ac,min}} \le U_{n,t}^{\mathrm{ac}} \le U_{n}^{\mathrm{ac,max}} \tag{A17}$$

$$P_{n,t}^{\mathrm{dc}} = \sum_{m \in \pi(n)} H_{mn,t}^{\mathrm{dc}} - \sum_{d \in \delta(n)} H_{nd,t}^{\mathrm{dc}}, \forall t \tag{A18}$$

$$H_{mn}^{\mathrm{dc,min}} \le H_{mn,t}^{\mathrm{dc}} \le H_{mn}^{\mathrm{dc,max}} \tag{A19}$$

Here, Equations (A12)-(A17) are the constraints for ACM, while Equations (A18)-(A19) are the constraints for DCM.

For ACM, (A12) and (A13) denote the active and reactive power balance at each node, respectively. The Ohm's law over branch *mn* is expressed as (A14) through linearized forms. (A15)-(A17) bound the limits for active flow power, reactive flow power and voltage magnitude, respectively.

For DCM, (A18) denotes the active and reactive power balance at each node. (A19) bounds the limits for active flow power. Similarly, since reactive power is not considered in DC grids, the constraints associated with reactive power and voltage magnitude are not in the DCM.

Table A1
Notation Applied for the Branch Flow Models

Indices and sets	
d/*m*/*n*	Node in networked microgrids
t	Operation period
$\pi(n)$	Set of all parents of node *n*
$\delta(n)$	Set of all children of bus
mn	Branch *mn*
Parameters	
r_{mn}^{ac}	Resistance of branch *mn* in networked ACM
x_{mn}^{ac}	Reactance of branch *mn* in networked ACM
$U_n^{\mathrm{ac,max}}$ / $U_n^{\mathrm{ac,min}}$	Upper/lower bound of voltage magnitude at node *n* in networked ACM
$H_{mn}^{\mathrm{ac,max}}$ / $H_{mn}^{\mathrm{ac,min}}$	Upper/lower bound of active power flow from node *m* to *n* in networked ACM
$H_{mn}^{\mathrm{dc,max}}$ / $H_{mn}^{\mathrm{dc,min}}$	Upper/lower bound of active power flow from node *m* to *n* in networked DCM
$I_{mn}^{\mathrm{ac,max}}$ / $I_{mn}^{\mathrm{dc,max}}$	Current capacity limit of branch *mn* in networked ACM/DCM
$G_{mn}^{\mathrm{ac,max}}$ / $G_{mn}^{\mathrm{ac,min}}$	Upper/lower bound of reactive power flow from node *m* to *n* in networked ACM
Variables	
$U_{n,t}^{\mathrm{ac}}$	Voltage magnitude of node *n* in networked ACM
$I_{mn,t}^{\mathrm{ac}}$ / $I_{mn,t}^{\mathrm{dc}}$	Square of current of branch *mn* in networked ACM/DCM
$u_{n,t}^{\mathrm{ac}}$	Square of voltage magnitude of node *n* in networked ACM
$H_{mn,t}^{\mathrm{ac}}$ / $H_{mn,t}^{\mathrm{dc}}$	Active power flow from node *m* to *n* in networked ACM/DCM
$G_{mn,t}^{\mathrm{ac}}$	Reactive power flow from node *m* to *n* in networked ACM
$P_{n,t}^{\mathrm{ac}}$ / $P_{n,t}^{\mathrm{dc}}$	Active injection power of node *n* in networked ACM/DCM
$Q_{n,t}^{\mathrm{ac}}$	Reactive injection power of node *n* in networked ACM