Detailed Proof of the Proposition 1

Proposition 1: A solution to model (3), where $P_{l,t}^{\mathrm{acdc}}$, $P_{l,t}^{\mathrm{dcac}} > 0, \exists l, t$, is suboptimal if any one of the following conditions hold: **1)** $o_{l,1}^2 < 1$ and $0 \le \rho_t^{\mathrm{w}} < P_t^{\mathrm{w}}$; **2)** $o_{l,1}^2 > 1$ and $0 < \rho_t^{\mathrm{w}} \le P_t^{\mathrm{w}}$; **3)** $o_{l,1}^2 = 1$.

Proof: This proposition can be proven based on the reduction to absurdity method. First, we assume $\exists l, t$, $P_{l,t}^{\mathrm{acdc}}, P_{l,t}^{\mathrm{dcac}} > 0$. Let ℓ be the Lagrange function of model (3) in the manuscript. Then, applying the Karush–Kuhn–Tucker (KKT) conditions for (3) yields the following.

$$\partial \ell / \partial P_{l,t}^{\mathrm{acdc}} = c_l - \underline{\lambda}_{l,t}^{\mathrm{ad}} + \overline{\lambda}_{l,t}^{\mathrm{ad}} - \lambda_t^{\mathrm{ac}} + o_{l,1} \lambda_t^{\mathrm{dc}} = 0 \tag{4a}$$

$$\partial \ell / \partial P_{l,t}^{\mathrm{dcac}} = c_l - \underline{\lambda}_{l,t}^{\mathrm{da}} + \overline{\lambda}_{l,t}^{\mathrm{da}} + o_{l,1} \lambda_t^{\mathrm{ac}} - \lambda_t^{\mathrm{dc}} = 0 \tag{4b}$$

$$\partial \ell / \partial \rho_t^{\mathrm{w}} = -\underline{\lambda}_t^{\mathrm{w}} + \overline{\lambda}_t^{\mathrm{w}} - \lambda_t^{\mathrm{ac}} = 0 \tag{4c}$$

The condition $P_{l,t}^{\mathrm{acdc}}$, $P_{l,t}^{\mathrm{dcac}} > 0, \exists l, t$ ensures that $\underline{\lambda}_{l,t}^{\mathrm{ad}} = \underline{\lambda}_{l,t}^{\mathrm{da}} = 0$ and $\overline{\lambda}_{l,t}^{\mathrm{ad}}, \overline{\lambda}_{l,t}^{\mathrm{da}} \ge 0$. Introducing these conditions into (4a) and (4b), and merging (4a) and (4b) by eliminating the variable λ_t^{dc} yields the following.

$$\vartheta := \left(1 + o_{l,1}\right) c_l + \overline{\lambda}_{l,t}^{\mathrm{ad}} + o_{l,1} \overline{\lambda}_{l,t}^{\mathrm{da}} + \left(o_{l,1}^2 - 1\right) \lambda_t^{\mathrm{ac}} = 0 \tag{4d}$$

Under **condition 1)**, we obtain $\overline{\lambda}_t^{\mathrm{w}} = 0$ and $\underline{\lambda}_t^{\mathrm{w}} \ge 0$, which ensures that $\lambda_t^{\mathrm{ac}} = -\underline{\lambda}_t^{\mathrm{w}} \le 0$ based on (4c). However, $\left(1 + o_{l,1}\right) c_l + \overline{\lambda}_{l,t}^{\mathrm{ad}} + o_{l,1} \overline{\lambda}_{l,t}^{\mathrm{da}} > 0$. Therefore, these conditions produce a contradiction in (4d) because the value of ϑ is always positive.

Under **condition 2)**, we obtain $\overline{\lambda}_t^{\mathrm{w}} \ge 0$ and $\underline{\lambda}_t^{\mathrm{w}} = 0$, which ensures that $\lambda_t^{\mathrm{ac}} = \overline{\lambda}_t^{\mathrm{w}} \ge 0$. Again, these conditions produce a contradiction in (4d) because the value of ϑ is always positive.

Under **condition 3)**, we obtain the relationship $\vartheta > 0$ since $\left(1 + o_{l,1}\right) c_l > 0$, $\overline{\lambda}_{l,t}^{\mathrm{ad}} \ge 0$ and $o_{l,1} \overline{\lambda}_{l,t}^{\mathrm{da}} \ge 0$, which is contrary to (4d).