

Statistics Project 1 :

Statistics - Math 2606

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1 Experiment

The assignment was to estimate the height of Searles. Although it would have been ideal to estimate the height to the top of the spire, we did not do this. Instead, we measured to the point at the top of the front facade above the door. This approach has the advantage of not requiring that one estimate the horizontal distance from the spire to the front facade, which would introduce more error.

Let the x -axis be the imaginary line passing through the door, parallel to the walkway that passes in front of the building, and let the y -axis be the line passing through the door, perpendicular to the walkway. For all measurements, we maintained a constant distance from the x -axis. We chose this value to be the distance from the door to a point directly in front of it, at the far edge of the walkway, where the pavement meets the grass. Call this value a . To vary the distance from the door to the mirror, we marked out 10 positions at 2m-intervals along the line defined by a that is parallel to the x -axis. Call that distance to the point directly in front of the door b . Then our direct distance to the base of the building directly below the top of the front facade above the door is $\sqrt{a^2 + b^2}$. This is a simple application of the Pythagorean theorem, as our distance from the point directly in front of the door, and our distance from the point on the line parallel to the x -axis are perpendicular.

Note that the error obtained in measuring the distance from the point directly in front of the door, and the error in measuring distance from the mirror to that point do **not** add up linearly. That is, call them ϵ_1 and ϵ_2 , and the distance from the point perpendicular to the building a , the distance from the point perpendicu [FOR KEVIN TO FINISH]

Will acted as the observer for all trials. His height (h_1) was measured 10 times with a tape measure, once for each trial. The distances a and b (together determining d_2), as well as the distance from Will to the mirror (d_1), were determined by extending a length of string the desired distance, marking the string, and then measuring the string with the tape measure. Measurements for b and d_1 were also taken 10 times, but we used the same value of a throughout.¹ All measurements are shown in Fig. 1. The value obtained for d_2 in measurement 1 shown in the figure was the value we used for a .

2 Results

From our 10 trials, we inferred the value of h_2 to be about 16.64m. Our mean values for h_1 , d_1 , and d_2 can also be found in Fig. 1. Our estimates for the standard deviation of the measurements,

¹We recognize that this is a mistake and that we should have measured a 10 times as well.

up to an order of magnitude, are as follows:

$$\begin{aligned}\sigma_{h_1} &= 1\text{cm} \\ \sigma_{d_1} &= 1\text{dm} \\ \sigma_{d_2} &= 1\text{m}\end{aligned}$$

To determine the impact of the error on the inferred value of h_2 , we need to understand the change in the height of Searles with regard to a change in the distance from the mirror to the base of Searles, the mirror to Will, and the height of Will. We simply take the partial derivatives with respect to each quantity to understand how the height of Searles depends on these measurements.

$$\begin{aligned}\frac{\partial}{\partial H_w} &= \frac{D_s}{D_w} \\ \frac{\partial}{\partial H_s} &= \frac{H_w}{D_w} \\ \frac{\partial}{\partial D_s} &= -D_w^{-2} D_s H_w = -\frac{D_s H_w}{D_w^2}\end{aligned}$$

Now we can calculate the values of these derivatives using an average value for D_s, H_w, D_w and multiply these derivatives with their estimated standard deviations, respectively, and the largest value should be the error that has the greatest impact on our calculated height [FOR KEVIN TO FINISH].

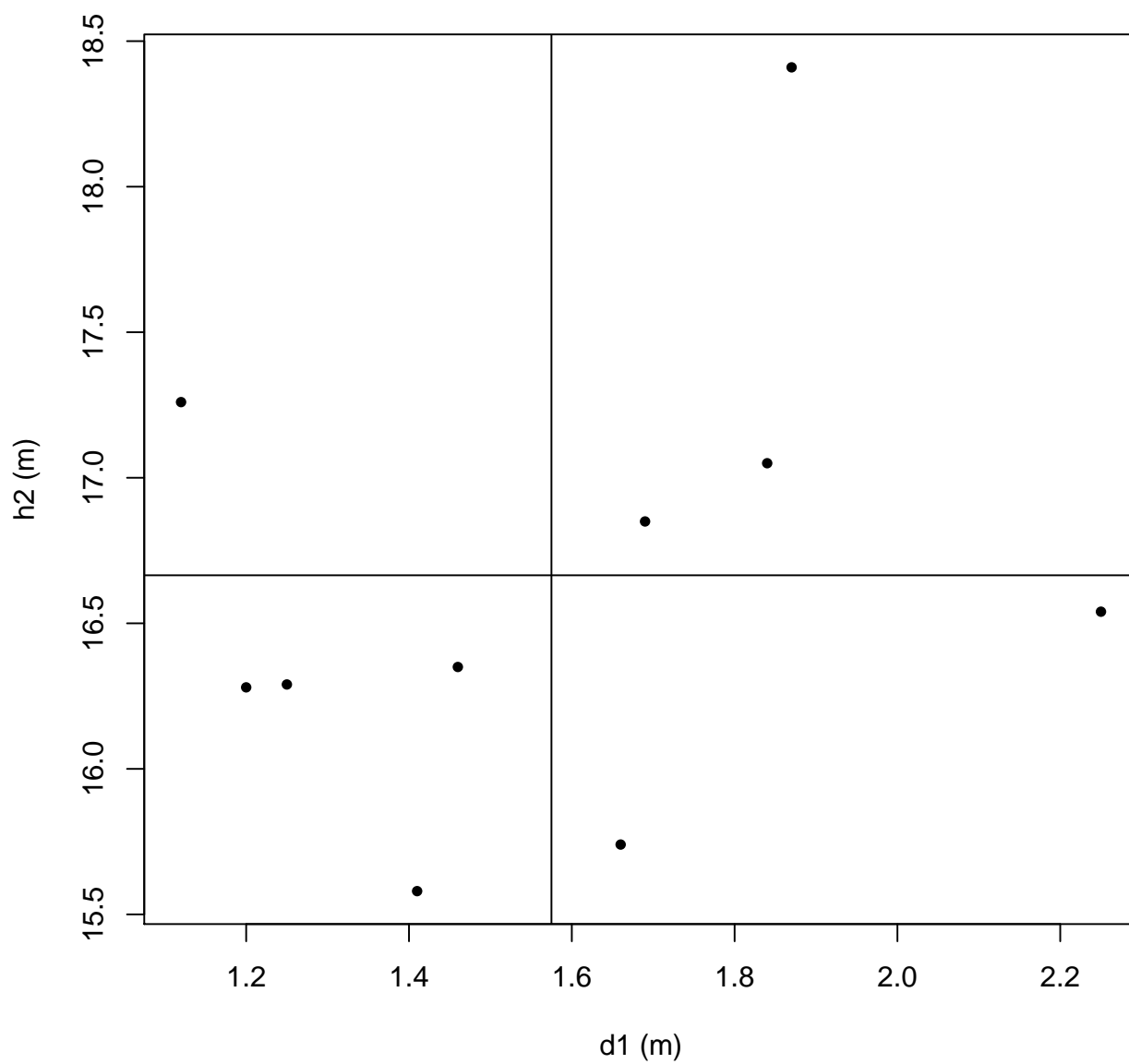
The plots below show, respectively, d_1 , d_2 , and h_1 graphed against inferred values for h_2 . The vertical line in each plot represents the mean value of the parameter on the x -axis (d_1 , d_2 , or h_1), and the horizontal line represents the mean height of Searles (16.64m). These lines allow us to visualize the deviation of each measurement from the mean value of the corresponding parameter.

The curious thing is that the graphs show no clear relationship between any of d_1 , d_2 , and h_1 on the one hand, and h_2 on the other. For all of our calculations, we posited a model of additive error. Moreover, we showed that, based on that model, the error in d_1 should correlate most strongly with the error in h_2 . (To frame it another way: a change in d_1 should produce a greater change in h_2 than either h_1 or d_2 would produce.) And yet, the graphs provide no evidence for such a conclusion. There seem to be two inferences we could draw from these results:

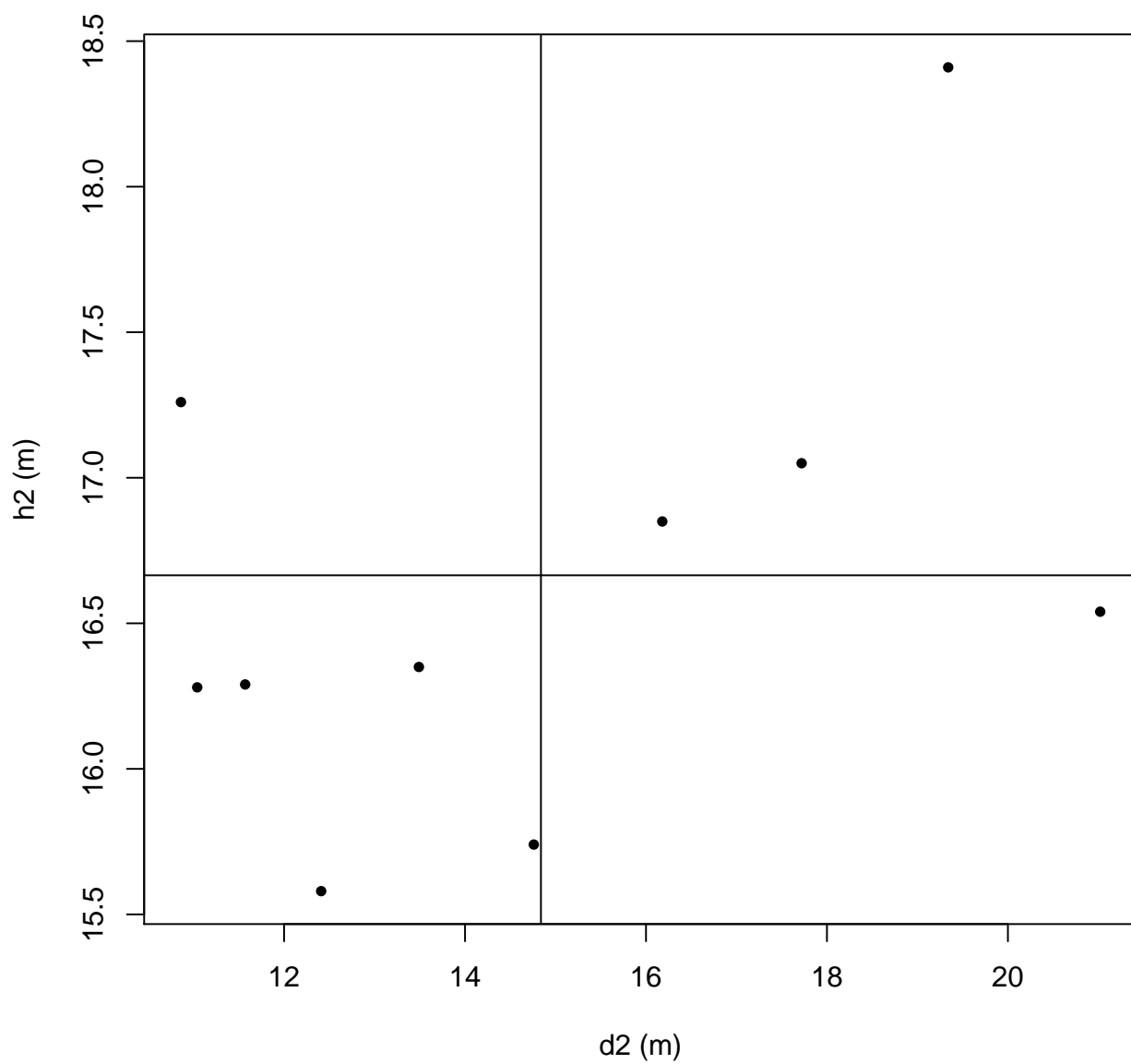
1. Our data collection was sufficiently imprecise to obscure a relationship between the measurements that does in fact exist.
2. The model of additive error that we posited is not an accurate representation of reality.

We believe that although there were problems with our data collection, it is likely the case that the model of additive error is not a good one for the purposes of this experiment.

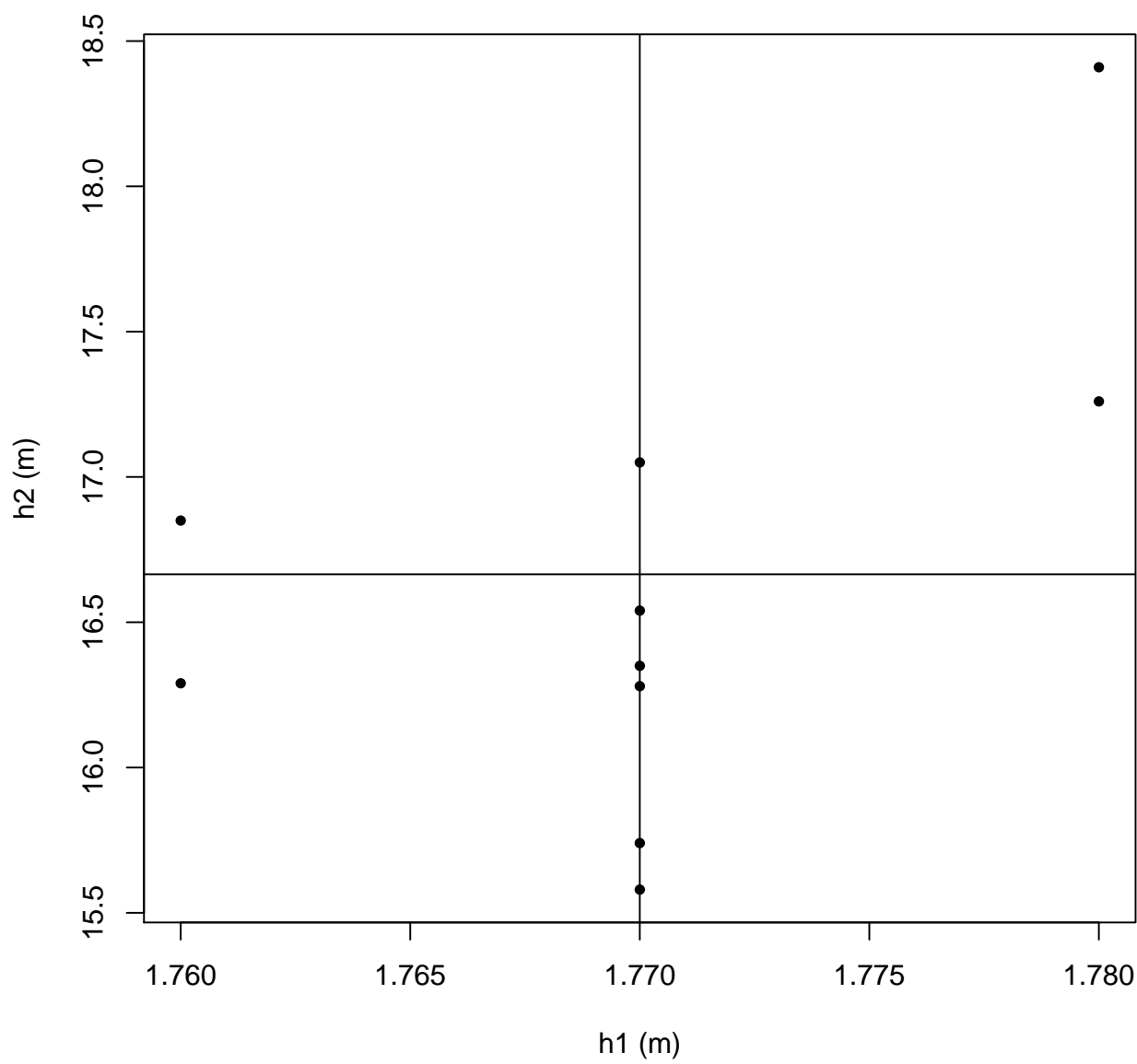
Distance from Will to Mirror vs. Searles Height



Distance from Mirror to Searles vs. Searles Height



Iris Height vs. Searles Height



Measurement #	h_1	d_1	d_2	h_2
1	1.78	1.12	10.86	17.26
2	1.77	1.2	11.04	16.28
3	1.76	1.25	11.57	16.29
4	1.77	1.41	12.41	15.58
5	1.77	1.46	13.49	16.35
6	1.77	1.66	14.76	15.74
7	1.76	1.69	16.18	16.85
8	1.77	1.84	17.72	17.05
9	1.78	1.87	19.34	18.41
10	1.77	2.25	21.02	16.54
mean	1.77	1.575	14.839	16.635

Figure 1: Measurements for iris height (h_1), the distance from the mirror to Will (d_1), the distance from Searles to the mirror (d_2), and the resultant estimate of the height of Searles (h_2). All units are in meters.