For office use only	Team Control Number	For office use only
T1 ________________	**44509**	F1 ________________
T2 ________________		F2 ________________
T3 ________________		F3 ________________
T4 ________________	Problem Chosen	F4 ________________
	A	

Having a sensible and enjoyable hot bath

Summary

A spa-style tub with a secondary heating system can give person an excellent feeling of having a hot bath, but what if the user only have a bathtub with a single faucet and an overflow drain? This time, having a good bath is always combined with the loss of time, comfort as well as the water. It is time to find a strategy to have a sensible and enjoyable hot bath.

First of all, we confirm the key factors that our strategy want to improve. In other words, having what kind of bath means sensible and enjoyable? Actually, there are three key factors, which respectively are keeping the temperature even, keeping the initial temperature as well as not wasting too much water. In order to evaluate these indexes, we make a quantification analysis to find a specific index to assess. Finally, we use ‘even time’, ‘suffering degree’ and ‘wasting water’ to evaluate whether a strategy is good or efficient to have a sensible and enjoyable hot bath.

Next, we use three hierarchical models to solve the problem. The first model is called the ‘even loop model’, including the process of cooling and looping, which extremely simplifies and clarifies the problem to describe the change of the temperature. We regard the floating water as a river to model the movement of the liquid and take the heat conduction and the time as well as the space into consideration. The second model is called the ‘suffering degree model’. Based on the theory of Weber-Fechner law, we successfully use temperature difference and integration for curved surface to show whether people suffers from cold water. The last model is the comprehensive evaluation model. Three key factors are weighted to form a final index to evaluate whether the strategy is efficient or not. Based on the data and simulation, we use our model to find the best strategy of having a hot bath.

Besides, we need to consider many additional factors such as the bathtub, the person and the bubble. We use controlling variables method and sensitivity analysis to describe and analyze the influences additional factors have on the strategy. We surprisingly find that many additional factors will have huge impacts on the initial best strategy. This reminds us of the difficulty to evenly maintain the initial temperature without wasting too much water, which proves that our model works but some parameters are greatly sensitive to the model, so it may be wise for the user to have a spa-style tub with a secondary heating system if he often gets a bath.

Moreover, we make improvements about the process of cooling to revise the rough detail. We deeply know that our model approximatively simulate the scene of the problem and strike out the main body of the problem. However, we also notice that we make too many and even some subjective assumptions and deal with some details roughly, which can be improved much better.

Last but not least, having a sensible and enjoyable hot bath is not only responsible for oneself, but also responsible for the sustainable society.

Keywords: a hot bath, loop model, Weber-Fechner law, quantification analysis

Contents

1. Introduction

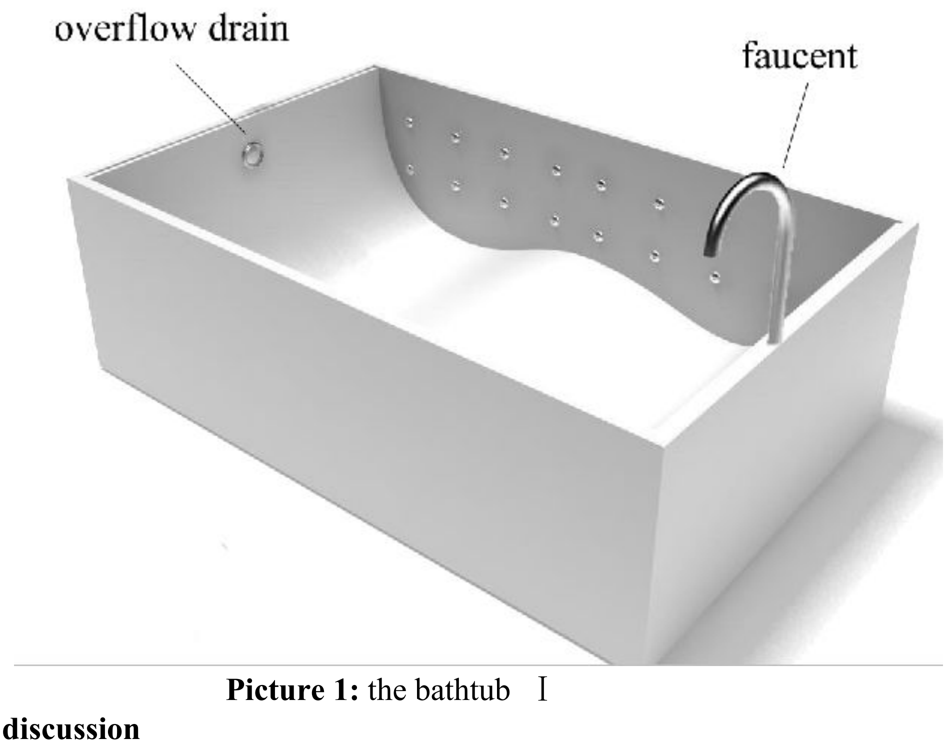
1.1Background

With the development of science technology, people are chasing a much more sensible and enjoyable life. Having a hot bath gradually and globally becomes a popular way for most people to cleanse and relax in order to get rid of the uncleanness and tiredness. However, comparing to taking a shower, having a hot bath not only costs much water as well as time, but also needs more essential preparations.

To some extent, a spa-style tub with a secondary heating system, which is the creature of the age, is a good answer to the demerits of having a hot bath when compared with taking a shower. Nevertheless, what if a person just has a bathtub with a single faucet and an overflow drain but also wants to have a hot bath?

To most people, a common bathtub is what they are used to in their daily lives because someone can't afford a spa-style tub or even doesn't know it. What comes to mind immediately when mentioning having a hot bath in a common bathtub is the miserable scene that one person is continuously adding hot water into the bathtub and stirring the water again and again with distressed feeling about the wasting water, that is why what really matters when people have a hot bath is about how to have a both sensible and enjoyable hot bath.

Our aim is to find a comparatively efficient strategy for people to have a hot bath in a common bathtub advisably and comfortably. Besides, based on the reality, we have to consider some influencing factors from the tub itself, the person who has a hot bath and some additions into the water such as bubbles so as to make some adjustments to face different situations.



Picture 1: the bathtub Ⅰ

1.2 Further discussion

Firstly, there is a question about how to evaluate whether our strategy is efficient or not, which means, in other words, whether you are having a sensible and enjoyable hot bath or not. Therefore, quantification analysis is a necessary and important tool to assess the efficiency of the strategy.

Talking about the specific of being sensible and enjoyable, the main problem we would like to solve is about keeping the temperature even throughout the whole bathtub and as close as possible to the initial temperature without wasting too much water. Hence, there are three key factors that have a deciding impact on the strategy. The first one is the ‘even time’, which means the time from adding the hot water into the bathtub to feeling that the temperature of water becomes even. The second one is the ‘suffering degree’, which means the degree a person suffers when the temperature is lower than its initial temperature. The last one is undoubtedly the ‘wasting water’, which means the volume of the wasting water.

Above all, we use quantification analysis to combine three key factors to evaluate the efficiency of the strategy. Moreover, we use controlling variables method and sensitivity analysis to describe and analyze the influences other factors have on the strategy, so we can give some conclusions and advices about the strategy and make some changes to deal with the different situations.

2. Model 1: Even loop model

2.1 Cooling model

2.1.1 Variable declaration

Symbol	Definition
Q	The heat of the water in the bathtub
m	The weight of the water which fills the bathtub
c	The specific heat capacity of the water in the bathtub
t	The time
T_0	The initial temperature(average) of the water in the bathtub
T	The temperature(average) of the water in the bathtub at the moment t (the time t mentioned above)
θ	The temperature of the air above the bathtub
k_0	A parameter related to the area and properties of the interface

Table 1: variable declaration

2.1.2 Hypothesis of model

- During the process of water being cool in the bathtub, the temperature of the water is the same throughout the whole bathtub and changes at the same pace.
- The temperature of the air does not change as the time flows. In other words, during the process of heat exchange between the water and the air, the temperature of the air is literal.
- k_0 is only related to the area and properties of the interface while other factor’s impacts can be ignored.

2.1.3 Newton's law of cooling

Newton's law of cooling states that when the object, which means the thermodynamic system, cools in the natural condition, and the temperature of the experimental system T is higher than the temperature of the outside θ while $T-\theta$ is small, the speed of the lost heat of the system by surface radiation is proportional to $(T-\theta)$. In other words, $\frac{dQ}{dt}=k_0(T-\theta)$. k_0 is related to the area and smoothness of the system’s surface[1].

Newton's law of cooling has its applicable condition[2]. Newton's law of cooling can be used only when $(T-\theta)$ is to an extent. Some literatures make comments that when the object cools in the natural condition, Newton's law of cooling can be used when $\left(T-\theta\right)\le 25^{\circ}C$.

2.1.4 The application of the Newton's law of cooling

Due to the Newton's law of cooling,

$$\frac{dQ}{dt}=k_0\left(\theta-T\right),$$

$$dQ=k_0\left(\theta-T\right)dt,$$

But $dQ=cmdT$, so $dT=\frac{k_0}{cm}\left(\theta-T\right)dt$,

$$dt=\frac{cm}{k_0\left(\theta-T\right)}dT,$$

$$\int_0^t dt=\int_{T_0}^{T}\frac{cm}{k_0\left(\theta-T\right)}dT=\int_{T_0}^{T}\frac{cm}{k_0\left(\theta-T\right)}d(T-\theta)=\frac{cm}{k_0}\ln\frac{T_0-\theta}{T-\theta},$$

Therefore $t=\frac{cm}{k_0}\ln\frac{T_0-\theta}{T-\theta}$.

As we can see, in the environment where the temperature of the air is θ, the time enabling the water, whose weight is m and specific heat capacity is c, in the bathtub to cool from T_0 to T in a natural condition $t=\frac{cm}{k_0}\ln\frac{T_0-\theta}{T-\theta}$. The water is in the appointed bathtub, and the properties of the interface between water and air will not change.

2.1.5 Brief explanation about influencing factors

2.1.5.1 The area of the interface

The volume and shape of the bathtub will influence the area of the interface between water and air so as to influence the value of k_0. It is not very hard to find that the bigger the area of the interface is, more heat will be lost in the same time, which means the value of k_0 is bigger.

2.1.5.2 The properties of the interface

The properties of the interface will also influence the value of k_0. For example, when bubbles are floating on the water, this will greatly hinder water vapor from evaporation, radiation, convection and many other forms of heat conduction as well as hinder the heat of the water from the conduction. At this time, the value of k_0 will decrease.

2.1.6 Brief explanation about precondition

From one essay, we know that the temperature of the most part of the room keeps at about $12^{\circ}C$ in winter[3].It is common sense that the temperature will be higher in summer. However, the best temperature of the water in the bathtub that fits people well is $38^{\circ}C$, so $\left(T-\theta\right)$ will be about $26^{\circ}C$. Actually in most situations, $\left(T-\theta\right)$ will be less than or equal to $25^{\circ}C$. Moreover, we can reason that the bathroom's condition won't be too bad if there is a bathtub, so somehow the bathtub will have a bath heater. Based on this, if the temperature of the room is much lower, we can use the bath heater to ensure that the temperature of the room is higher than $15^{\circ}C$, so that $\left(T-\theta\right)\leq 38^{\circ}C-15^{\circ}C=23^{\circ}C\leq 25^{\circ}C$. That is why the Newton's law of cooling can be used to analyze the problem in such a situation.

2.2 Even loop model

2.2.1 Variable declaration

Symbol	Definition
$u=u(x,y,\mathrm{z},\mathrm{t})$	The temperature of the water at the position $\left(x,y,z\right)$ in the bathtub
$c(x,\mathrm{y},\mathrm{z})$	The specific heat capacity of the water at the position $\left(x,y,z\right)$ in the bathtub
$k(x,\mathrm{y},\mathrm{z})$	The heat conduction modulus at the position $\left(x,y,z\right)$ in the bathtub
u_{01}	The temperature of the water in the bathtub that fits people best
u_{02}	The lowest temperature of the water in the bathtub that people can tolerant（It usually can be 35° according to most people.）
u_1	The temperature of adding hot water
u_2	The temperature of the water in the bathtub when adding hot water into the bathtub($u_2\geq u_{02}$)
u_3	The final temperature distribution function when adding hot water into the bathtub
u_4	The final temperature of the water in the bathtub during the fusion
v_1	The volume of adding hot water for one time
v_2	The initial volume of the water in the bathtub before
t_1	The time of adding hot water into the bathtub
t_2	The time of the fusion in a natural condition
n	The times of adding hot water into the bathtub
v	The speed of adding hot water into the bathtub(less than or equal to a constant)
t_e	‘Even time’
l	The length of the bathtub
w	The width of the bathtub
h	The height of the bathtub
h'	The height of the overflow drain
r	The radius of the water column
E_x	Hydrodynamic diffusion coefficient in the direction of x coordinate axis
E_y	Hydrodynamic diffusion coefficient in the direction of y coordinate axis
K_s	Average water coefficient of heat transfer
C_p	Specific heat of water at constant pressure

S	The source intensity
t	Time coordinate
v_x	Average flowing speed in the direction of x coordinate axis
v_y	Average flowing speed in the direction of y coordinate axis
q	The source intensity
g	Gravitational acceleration
f	Cauchy coefficient
m'	Roughness coefficient of the river bed
n'	Viscidity coefficient

Table 2: variable declaration

2.2.2 Constant declaration

Symbol	Value
k	$0.6w/(m \cdot K)$
c	$1000kg/m^3$
ρ	$4200J/(\text{kg} \cdot K)$
a^2	$a^2 = \frac{k}{c\rho}$

Table 3: constant declaration

2.2.3 Model process

The model can be divided into the following three stages:

(1) The stage of adding water: the time from adding water to stopping adding water, which is as long as t_1;

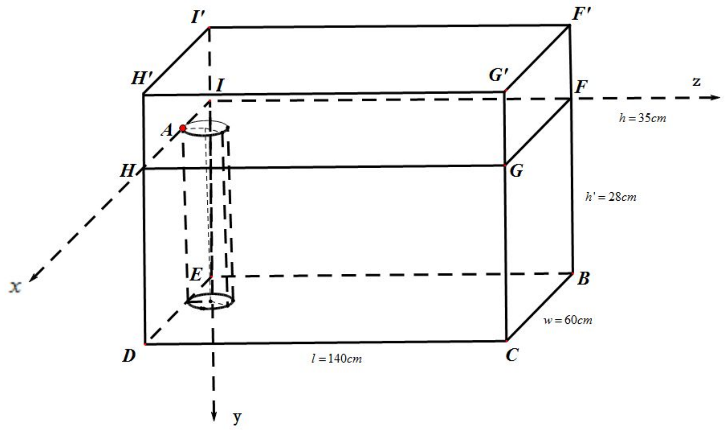
(2) The stage of fusion: the time from stopping adding water to the temperature of the water in the bathtub becoming even, which is as long as t_2;

(3) The stage of becoming cool: the time from the temperature of the water in the bathtub becoming even to the temperature of the water in the bathtub getting as cool as u_2.

2.2.4 Model assumptions

- The overflow drain is on the opposite side to the single faucet
- Generally speaking, the time people take to have a hot bath usually ranges from half an hour to one and a half hour. Due to the change of the water temperature and the reality, we can roughly deduce that the times of adding hot water into the bathtub range from 1 to 4.
- Due to thermodynamic principles, the time that the even hot water takes to become cool is very long, so it is much longer than the 'even time' we have set before. Therefore, the lost heat during the 'even time' can be ignored. That is to say, heat only gets lost during the stage of becoming cool.
- Considering about adding hot water into the bathtub and draining away cold water from the bathtub, because the water is flowing from the single faucet to the bathtub almost in the shape of column, a block of water below the single faucet is in the shape of column and keeps the temperature. The hot water drops into the cold water with high energy, so the height of the block of water is very close to the height of the bathtub, which is shown in the picture below. What is clear is

that the cube $BCDEFGHI$ is the water in the bathtub while the cube $BCDEF'G'H'I'$ is the bathtub.



graph 1： bathtub when added with hot water

2.2.5 Model analysis

From Equations of Mathematical Physics, we can know the equation:

$$\frac{\partial u}{\partial t} = a^2 \left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2}\right)$$

With the initial conditions, we know the temperature of every single position in the bathtub with filled water, which means knowing the 3D temperature distribution in the bathtub. Through the equation above, we can figure out the temperature of any position in the bathtub filled with water at any moment.

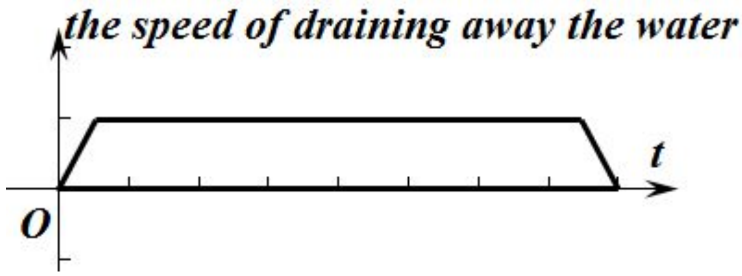
The bathtub is filled with water. Once the hot water from the single faucet strikes the cold water in the bathtub, the kinetic energy that emerges will force the cold water to float into the overflow drain.

Adding hot water can be divided into three stages:

(1) At first, it is the initial stage of adding hot water. The speed of draining away water will increase from zero to a stable number. During this stage, the speed of draining away water will increase as the hot water is added into the bathtub continuously.

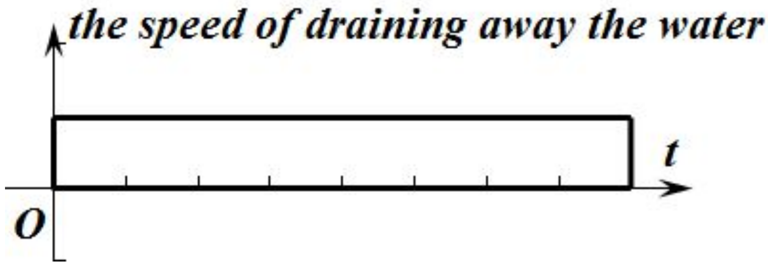
(2) Immediately, the speed of draining away water keeps stable. During this stage, the speed of draining away water is the same as the speed of adding hot water into the bathtub.

(3) Finally, the person stops adding hot water into the bathtub. The speed of draining away water decreases from the stable number to zero. During this stage, although no more water will be added into the bathtub, the liquid level is still a little higher than the height where the overflow drain is. Hence, the process of draining away water is still going on and will finally decrease to zero.



graph 2： v-t actual graph

From the picture above, the time of the first stage and the final stage is very short, so we can approximatively regard that draining away the water goes on at the same time as adding the water and the speed of draining away the water is always the same as the speed of adding water. Now we can make some changes to the picture above to describe the relationship between the time and the speed of draining away the water.



graph 3：v-t ideal graph

Combined with two steps above, including both adding water and draining away water, we can come to the fact that there are two processes happening in the bathtub:

(1) The flow of the water: In this process, hot water is added into the bathtub, while the cold water, which is the same volume as the hot water, will overflow out of the bathtub. As a result, this can be understood that we can regard the whole bathtub as a riverbed, and the flow of the water in the bathtub is like the fluxion of a river with the single faucet as a water inlet, the overflow drain as a water outlet and the casing wall as a river bank. At the time water is added into the bathtub through the faucet, all the other water will flow because of the interaction between water molecules, making the water float to the outlet entirely and move in all directions partly. Accordingly, the move of the water in the bathtub can be dealt as “the model of a river”.

(2) Heat conduction: Heat of the hot water will spread into the other part of the water in the bathtub, accompanied by the flow and diffusion of water naturally, leading to the water exchange. According to assumptions above, the height of hot water column almost occupies the whole water column. Therefore, we can assume that temperature of the same vertical line is the same, so the flow and the heat conduction of water which is actually three-dimensional can be seen as that of water on a plane.

2.2.6 The model of draining away the warm water

As a result, the whole process is that the continuous hot water column enters into the flowing water, realizing the flow and the conduction of water. At the same time, the same volume of cold water will run out of the bathtub through the overflow drain. Consequently, the total process can be regarded as “the model of draining away the warm water”.

We also consider that the room of bathtub is big. Combined with the two simultaneous processes mentioned above, if we adopt two-dimensional equation[4] of flowing water to describe the flow field of water and adopt two-dimensional equation of temperature to simulate the change of water temperature in space, the governing equation can be described as following:

(1) The equation of flowing water:

$$\begin{cases} \dfrac{\partial h'}{\partial t}+\dfrac{\partial (v_x h')}{\partial x}+\dfrac{\partial (v_y h')}{y}=q \\ \dfrac{\partial (h'v_x)}{\partial t}+\dfrac{\partial (h'v_x v_x)}{\partial x}+\dfrac{\partial (h'v_x v_y)}{\partial y}=\dfrac{\partial}{\partial x}(\mathrm{n}'h'\dfrac{\partial v_x}{\partial x})+\dfrac{\partial}{\partial y}(\mathrm{n}'h'\dfrac{\partial v_x}{\partial y})-gm'^2\dfrac{\sqrt{{v_x}^2+{v_y}^2}}{h'^{\frac{1}{3}}}v_x-gh'\dfrac{\partial h'}{\partial x}+fh'v_y \\ \dfrac{\partial (h'v_y)}{\partial t}+\dfrac{\partial (h'v_x v_y)}{\partial x}+\dfrac{\partial (h'v_y v_y)}{\partial y}=\dfrac{\partial}{\partial x}(\mathrm{n}'h'\dfrac{\partial v_y}{\partial x})+\dfrac{\partial}{\partial y}(\mathrm{n}'h'\dfrac{\partial v_y}{\partial y})-gm'^2\dfrac{\sqrt{{v_x}^2+{v_y}^2}}{h'^{\frac{1}{3}}}v_x-gh'\dfrac{\partial z}{\partial y}+fh'v_x \end{cases}$$

(2) The equation of heat conduction:

$$\frac{\partial (h'u)}{\partial t}+\frac{\partial (v_x h'u)}{\partial x}+\frac{\partial (v_y h'u)}{\partial y}=\frac{\partial}{\partial x}(E_x h'\frac{\partial u}{\partial x})+\frac{\partial}{\partial y}(E_y h'\frac{\partial u}{\partial y})-\frac{K_s u}{\rho C_p}+S(x,y)$$

2.2.7 Problem solving

2.2.7.1 Analysis of process

(1) The process of cooling

This step refers to: Firstly, add water into the bathtub until the bathtub is full. Secondly, start to have a bath until the temperature reaches the predesigned water temperature u_2. This period of time is t_0.

(2) The Process of Looping

This part is a process of cycle. The first step is the three stages of adding water into the bathtub. The moment the user stop adding water, the fusion of water starts until the temperature becomes even. The final step is to take the bath until the water temperature drops to u_2.

2.2.7.2 Analysis of model

Assuming that $n=2$, which means adding water twice, the whole process needs to go through the loop twice.

First step is the process of cooling, which lasts for t_0. In the end, the water's temperature in the bathtub is u_2. Then it goes through the loop twice. The whole process can be divided into these stages as follows:

(1) The process of adding Water

This stage can be divided into two processes which happen in the same time: The flow of water and the transfer of heat.

The condition in the beginning of this stage is that the temperature of the water in the bathtub is u_2, so by using the two sets of equations above, we can get the distribution of water temperature in the bathtub at any time. Assuming the time of this process is t_1, and the temperature at the end of this process is u_3, and $u_3=u_3(x,y,z)$, which means that it is a function about the distribution of the temperature in space.

(2) The process of fusion

The beginning of this stage is also the end of the process of adding water. At this very moment, the temperature of every part of the water in the bathtub is influenced by the adding water, but this stage is a natural process of water fusion. Regarding u_3 as the original state, we can get the temperature of every point in the development of this process by using two sets of equations above. Assuming this process continues for t_2, the temperature in the end of this process is u_4, so u_4 is a constant function about space, which means the temperature of every point is u_4.

(3) The process of cooling

This part is the same as the process of cooling above.

2.2.7.3 Analysis of answer

(1) Even time

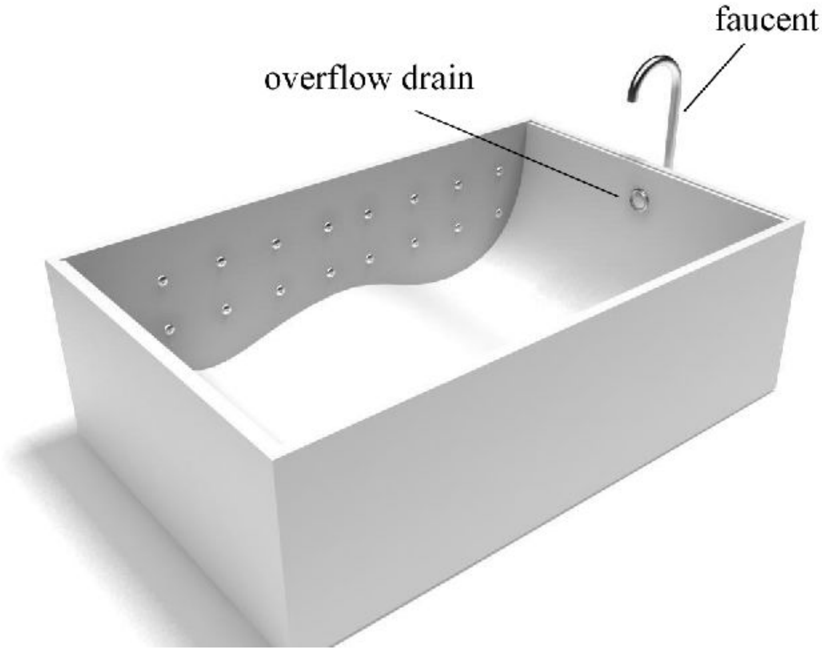
In the model above, $t_e = t_1 + t_2$ is the time we spend in the whole process of looping.

(2) Wasting water

When it comes to the whole processes, the volume of wasting water is $Q = v \cdot t_1 \cdot n$.

2.2.8 Further discussion

The position of the faucet and overflow drain only affects the process of adding water in the whole loop. When the overflow drain is in the opposite side of the faucet, it starts to drain away the water the moment the user adds water into the bathtub and all the water drained away is the cold water without heat exchange with the adding water.



Picture 1: the bathtub Ⅱ

However, when the overflow drain is in the same side of the faucet, our assumptions no longer establish. Now the water drained away is the warm water that exchanges heat with the hot adding water. Such kind of heat exchange is sufficient because the overflow drain is close to the faucet. In this circumstance, more heat will be lost. We can imagine adding hot water with the temperature of $\widetilde{u_1}$ ($\widetilde{u_1} < u_1$), which means the heat between the cold water and the hot water that is drained away is just the lost heat.

It is a great pity that it is almost impossible to confirm the value of u_1, so we venture to think that $\widetilde{u_1} = u_1 - \frac{1}{4}(u_1 - u_2)$ where u_1 stands for the initial temperature of adding water and u_2 stands for the initial temperature of the water in the bathtub. It is obvious that $\widetilde{u_1}$ is related to u_1 and u_2, so our new assumptions conform to our experience.

Finally, we regard $\widetilde{u}_1$ as the temperature of adding water to operate the whole model.

3. Model 2: Body feeling model

3.1 Suffering degree

3.1.1 Weber-Fechner law

Weber-Fechner law tells us that the increase in a geometric ratio of the stimulation will only result in the increase in a linear ratio of the visual response. The formulation is $\frac{\Delta\Phi}{\Phi}=C$, where $\Delta\Phi$, Φ , C respectively stand for the difference threshold, the standard stimulus intensity and the Weber rate.

Fechner, a German psychologist, proposed the law of logarithm to revise the Weber-Fechner law. The law of logarithm describes the quantity relationship between material stimulants and mental impression, whose formulation is $S=K\lg R$, where R , S , K respectively stands for the stimulus intensity, the feeling intensity and the constant. This law states that all the feeling of people including visual sense, auditory sense, cutaneous sensation, gustation, smell, shock sense and so on are not proportional to the corresponding physical quantity, but proportional to the common logarithm of the corresponding physical quantity.

3.1.2 Suffering degree

Trying to maintaining the best temperature as long as possible, which is also the initial temperature, aims to let people have the most comfort. Once the temperature is away from the best temperature, which is also the initial temperature, people will feel less comfortable or even bad. For this, we use ‘Suffering degree’ to describe the level people feel uncomfortable with the temperature of the water in the bathtub. There is one thing we have to mention is that ‘suffering degree’ does not mean that the user feels uncomfortable and bad. Once the user’s feeling is away from the best comfort, the ‘suffering degree’ will be positive instead of zero.

From Weber-Fechner law, we know that the feeling intensity is proportional to the common logarithm of stimulus intensity. Hence, we use the common logarithm of temperature difference to evaluate the ‘suffering degree’. We will use D to represent the ‘suffering degree’.

3.2 Body feeling model

3.2.1 Variable declaration

Symbol	Definition
t_{31}	The time of the first cooling process in the natural condition
t_{32}	The time of all cooling process in the natural condition except the first time
D_1	The ‘suffering degree’ during the first cooling process
D_2	The ‘suffering degree’ during the adding and fusion process
D_3	The ‘suffering degree’ during all the cooling process except the first time
D	The total ‘suffering degree’

Table 4: variable declaration

3.2.2 Model assumption

- The user doesn’t suffer during the first second when the water in the bathtub cools after fusion. That is to say the user suffers from a second after the time when the water in the bathtub cools to t_{31}.

3.2.3 Body feeling model

3.2.3.1 The 'suffering degree' during the first cooling process

From cooling model, during the cooling process, we have $t=\frac{cm}{k_0}\ln\frac{T_0-\theta}{T-\theta}$. When it is the first time for the water to cool, the temperature when the cooling process begins is the initial temperature. That is to say, $T_0=u_{01}$. When the temperature of the water in the bathtub reaches u_2, the user starts to add hot water into the bathtub, so that the time of the first cooling process in the natural condition $t_{31}=\frac{cm}{k_0}\ln\frac{u_{01}-\theta}{u_2-\theta}$.

During the first cooling process, we have the time $t=\frac{cm}{k_0}\ln\frac{u_{01}-\theta}{T-\theta}$, so $T=\theta+\left(u_{01}-\theta\right)\cdot exp(\frac{cm}{k_0 t})$. Then during the first cooling process, we can calculate

$$D_1=K\int_1^{t_{31}}\lg(u_{01}-T)dt=K\int_1^{t_{31}}\lg(\left(u_{01}-\theta\right)\cdot(1-exp(\frac{cm}{k_0 t})))dt,$$

where K and u_2 respectively stand for the constant and the temperature of the water in the bathtub when adding hot water into the bathtub and $t_{31}=\frac{cm}{k_0}\ln\frac{u_{01}-\theta}{u_2-\theta}$.

3.2.3.2 The 'suffering degree' during the adding and fusion process

From even loop model, the temperature of the water in the bathtub at any time after adding water into the bathtub and any position in the bathtub can be confirmed by the function $u(x,y,z,t)$. Hence, the 'suffering degree' during this time can be defined as

$$D_2=\frac{1}{\left|\Sigma\right|}\int_0^{t_1+t_2}K\int_\Sigma\lg(u_{01}-u(x,y,z,t))d\sigma dt,$$

where Σ, $\left|\Sigma\right|$, K respectively stand for an approximate smooth curve made by the surface of the person, the area of the curve and a constant.

3.2.3.3 The 'suffering degree' during all the cooling process except the first time

From the cooling model, during the cooling process, we have $t=\frac{cm}{k_0}\ln\frac{T_0-\theta}{T-\theta}$.

When it is the cooling process but not the first, the temperature when the cooling process begins is the temperature after fusion. That is to say $T_0=u_4$. When the temperature of the water in the bathtub reaches u_2, the user starts to add hot water into the bathtub, so that the time of the cooling process in the natural condition can be $t_{31}=\frac{cm}{k_0}\ln\frac{u_{01}-\theta}{u_2-\theta}$.

During the cooling process, we have the time $t=\frac{cm}{k_0}\ln\frac{u_4-\theta}{T-\theta}$,

so $T=\theta+\left(u_4-\theta\right)\cdot exp(\frac{cm}{k_0 t})$. Then during the cooling process, we can calculate

$$D_3 = K\int_1^{t_{32}} \lg(u_{01} - T)dt = K\int_1^{t_{32}} \lg(u_{01} - \theta - (u_4 - \theta)\cdot exp(\frac{cm}{k_0 t}))dt\ ,$$

where K and u_2 respectively stand for the constant and the temperature of the water in the bathtub when adding hot water into the bathtub and $t_{32} = \frac{cm}{k_0}\ln\frac{T_0 - \theta}{u_2 - \theta}$.

3.2.4 The 'suffering degree'

By D_1、D_2、D_3, we can calculate total 'suffering degree'

$$D = D_1 + n(D_2 + D_3),$$

where n stands for the times of adding hot water into the bathtub.

4. Model 3: Comprehensive evaluation model

4.1 Data processing

Due to the various types of data we calculate, such as the 'even time', the 'suffering degree' and the 'wasting water', which are all in different quantities and units, we need a consistent index to evaluate the strategy.

Since all the data is quantitative, we transfer it into index that belongs to [0, 1]. Different data has different influences on the final evaluation. For example, some factors are positive while some are negative. Considering our three main indexes, they all have negative impacts on the efficiency of the strategy, so the smaller the index is, the more efficient the strategy is. Due to the different properties the index show, we should choose different functions that best fit the character of the index to process the data. Owing to the monotonous negative impact, we establish an extreme membership function to transfer the data into an index that belongs to [0, 1].

For example, the upper limit of wasting water is Q_0 and the lower limit of wasting water is zero, so we assign M the value of zero. Then we use the extreme model to process the data to calculate the final index that shows the level of wasting water, which belongs to [0, 1].

Type	Corresponding function
Extreme	$f(x) = 1 - \frac{2}{\pi * \arctan(\lvert x - M \rvert)}$
Intermediate	$f(x) = \begin{cases} \frac{2(x-m)}{M-m}, m \le x \le \frac{1}{2}(M+m) \\ \frac{2(M-x)}{M-m}, \frac{1}{2}(M+m) \le x \le M \end{cases}$
Interval	$f(x) = \begin{cases} 1 - \frac{a-x}{c}, x < a \\ 1, a \le x \le b \\ 1 - \frac{x-b}{c}, x > b \end{cases}$
Fuzzy	$f(x) = \begin{cases} \left[1 + \alpha(x-\beta)^{-2}\right]^{-1}, 1 \le x \le 3 \\ \gamma \ln x + \eta, 3 < x \le 5 \end{cases}$

Table 5: Data type and corresponding function

4.2 Comprehensive evaluation model

4.2.1 Variable declaration

Symbol	Definition
t_e	'Even time'
D	'Suffering degree'
Q	'Wasting water'
f_1	The extreme function for 'Even time'
f_2	The extreme function for 'Suffering degree'
f_3	The extreme function for 'Wasting water'
W	The weight vector of the three key indexes
E_n	The less efficiency of the strategy with n times of adding water
E	The less efficiency of the strategy

Table 6: variable declaration

4.2.2 Comprehensive evaluation model

Since we have already processed the data, we need to balance the weight of three key indexes according to their influences to the efficiency of the strategy as well as the social factors. We assume that $W=\left(\omega_1,\omega_2,\omega_3\right)$, where $\omega_1,\omega_2,\omega_3$ respectively stand for the weight the corresponding index have.

Finally we use the comprehensive evaluation model to calculate the less efficiency of the strategy:

$$E_n = f_1\left(t_e\right)\omega_1 + f_2\left(D\right)\omega_2 + f_3\left(Q\right)\omega_3$$

$$E=\min\left\{E_n\right\}$$

The smaller E is, the more efficient our strategy is. Hence, what really matters is the minimum of E. That is to say, what we should do is to find the extreme point of function E and confirm the minimum of E so that we could find the most efficient strategy to have a both sensible and enjoyable hot bath.

4.3 Simulation and Strategy

u_{01} he initial temperature of the water	$40^\circ C$	θ The temperature of the air around the tub	$18^\circ C$
l The length of the bathtub	$140cm$	w The width of the bathtub	$60cm$
h The height of the bathtub	$35cm$	m Mass of the water full of the tub	$294kg$
v_2 The initial volume of the water in the tub	$294L$	c Specific heat capacity of the water in the tub	$4.2\times10^3 J/(kg\cdot {}^\circ C)$
k_0 A Parameter	0.03	n The times of adding hot water into the bathtub	2

h' The height of the overflow drain	$28cm$	K_s Average water coefficient of heat transfer	1.4
E_x Hydrodynamic diffusion coefficient in the direction of x coordinate axis	0.8	E_y Hydrodynamic diffusion coefficient in the direction of y coordinate axis	0.6
S' The source intensity	0.12	C_p Specific heat of water at constant pressure	$1Kcal \cdot kg^{-1} \cdot K^{-1}$
q The source intensity	2.9	g Gravitational acceleration	$9.8m/s^2$
f Cauchy coefficient	0.68	m' Roughness coefficient of the river bed	0.17
n' Viscidity coefficient	3.1	Σ an approximate smooth curve made by the surface of the person	Regard the person as a cuboid $(170\times52\times15)cm^3$
$\lvert\Sigma\rvert$ the area of the curve	$2.434m^3$	W The weight of the three key indexes	$15:21:4$
e_a Vapor pressure of the air above the water	$9.81kpa$	S The contact area of water and air	$8.4m^2$

Table 7: data table for simulation

Put the data above into our models and we get the value of E is 1.929 and the model shows that it's best to keep the water in the speed of for 7 minutes when the temperature is down to $36.3^\circ C$. And water is added only once. We regard the strategy as $(36.3, 7, 7, 1)$.

5. Additional factors

Our model emphasizes three elements: even time, wasting water, and suffering degree. In the model above, we ignore some minor factors to simplify the model to some extent. These include that we regard the shape of bathtub as a rectangle, while hot water column is a cylinder. When we back to the model mentioned above, we can obtain the temperature at any time and position easily and gain the three elements we focus on.

However, is this model sensitive and firm enough? How this kind of model affects to conclusions? As a result, we will research the following seven elements in seven different aspects to study sensitivity and reliability of our model. What's more, the third model---Analytic hierarchy process model---is based on the previous two models, and we can abstract and evaluate three elements, so when thinking about effects, we can only take the influence of previous two models into consideration.

5.1 The shape of the tub

(1)Even time:

In the first place, we analyze effects caused by the shape of the tub. According to the three model above, shape of bathtub will have an effect on the previous two models (the even loop model and the body feeling model), especially on the first one. On the other hand, when we turn to the equitation of the model, the main influence focuses on the first two periods (the period of cooling and fusion).

When we are adding water, according to the model of draining away the warm water, the shape of bathtub has no influence on the process of water movement and heat conduction. However, different shapes of bathtub will lead to the change of bathtub's length, width and height. Try to think about it. If the longest side of bathtub changes with water flows as original methods, we need more time to conduct heat. This will lead to the change of , and will also result to the difference of temperature at the end state.

In the period of fusion, due to the difference of , we can also see the change of , if we start from the equation.

Accordingly, the shape of bathtub actually has some impact on even time, but it can be evaluated that these impacts will not be very huge. More importantly, normally there are regular models to describe the shape of bathtub, and little difference varies from each side of bathtub. Consequently, the shape of bathtub will not have great impact on this element.

(2)Suffering degree:

The shape of bathtub will affect the surface between water and air, and we name this area as S. We can learn from the modification of cooling model that relates to S, while the cooling time *t* affects the value of and which appears in suffering degree. As a result, S affects E indirectly.

In simulation, we change the value of S and calculate E. We get some useful information below.

S	8	8.4	9	9.5
E	2.034	1.929	1.969	1.993
The most efficient strategy	（36.7，6.7，7.1,1）	（36.3，7，7,1）	（37.3，8.2，7.4,1）	（37.5，8.9，7.8,1）

Table 8: data information

We find that when S changes, E changes, and the best strategy varies from each model. That is to say that if the shape of bathtub changes, our strategy changes. What's more, these changes are not the same.

(3)Wasting water:

Because the speed of adding water *v,* the time of draining away water *t* and the frequency of adding water all determines the volumes of wasting water. However, we can control these factors, so there is no effect on wasting water.

(4)Conclusion:

The shape of bathtub has little impact on the previous strategy, so it is suitable for different shaped bathtubs.

5.2 The volume of the tub

In the similar way, when it comes to volume, it is the same.

(1)Even time:

Different volume of bathtubs relate to different lengths, widths and heights. Because we assume in the previous strategy that the height of water cylinder is the depth of water, this is based on the hypothesis that the height of bathtub is not very high. However, different bathtubs relate to different heights. If the height is too high, our assumption will be incorrect, and the strategy will not work. The factors of width and length are the same as that discussed in shape before.

(2)Suffering degree:

We consider that the impact of volume is the same as the impact of shape on suffering degree. They both affect the value of E through the area of surface between water and air.

(3)Wasting water:

Similarly, different volume of bathtub means different area of water surface with its affect mainly focuses on the heat lose. To promise the time duration and the lowest temperature of hot water , the more heat loses, the more times we have to add water into bathtub, which we name this variable as *n.* On the other hand, the less heat loses, the less frequency we have to add water. Due to the equation, volume has an impact on wasted water. However, the demand of wasted water is not very strict, it is only required to lower than the predetermined upper bound. Accordingly, there is little influence caused by volume.

(4)Conclusion:

The volume of bathtub impacts largely on the pervious strategies. If volume changes a lot, especially heights, to a large extent, our strategies will be affected.

Modification is to increase the speed and the time of adding water appropriately.

5.3 The shape of the person

(1)Even time：

According to the previous two models, people’s size affects little on water movement because the flow of water is slow. For the same reason, people’s size won’t influence heat conduction a lot.

Therefor as long as a person doesn’t prevent water from flowing, there is little effect on water movement and heat conduction caused by people’s size.

(2)Suffering degree:

In the similar way, the process of cooling mainly depends on the surface of water, while people’s shape doesn’t matter a lot. In other words, people’s shape makes little difference on the change of water when a person is taking a shower.

Whereas temperature varies from part to part in the bathtub, especially before water gets a balanced temperature. If people are in different postures, people’s feeling will be various because of the different position of each point of the body. As a result, to some extent, people’s shape does affect suffering degree.

(3)Wasting water:

Wasting water is only up to the speed of adding hot water, the time of adding hot water and the times of adding hot water, but these factors can be limited by the person, so the wasting water is almost not disturbed by the shape of the person.

(4)Conclusion:

The shape of the person will have some impacts on our strategies before to some extent.

Modification is to adopt a more even posture in the bath.

5.4 The volume of the person

(1)Even time：

Different people have different volumes. Some are fatter while others are thinner.

When different person is having a bath, different volumes of water will be drained away because the position of the overflow drain is fixed. The volume of the water in the bathtub actually influences the process of adding water and fusion. In terms of the formulation, these two processes have little relationship with the factor whether there is a new object or not, so the even time keeps comparatively constant. Considering the decrease of the volume of the water, we can have an elementary conclusion that the bigger the volume of the person is, the smaller the volume of the rest of the water is, which leads to the decrease of the time that the water's temperature becomes even.

(2)Suffering degree:

Generally speaking, the difference of the volume of the person results in the difference of the surface area of the person. People's feeling is usually composed of the addition of the feeling of the different part of the body. That is why the bigger the volume of the person is, the bigger the surface area of the person is, which leads to a deeper suffering degree.

(3)Wasting water:

Wasting water is only up to the speed of adding hot water, the time of adding hot water and the times of adding hot water, but these factors can be limited by the person, so the wasting water is almost not disturbed by the volume of the person.

(4)Conclusion:

People's volume will have some impacts on our strategies before to some extent.

Modification is to increase the temperature of adding water moderately, increase the speed of adding water and decrease the time of draining away water.

5.5 The temperature of the person

(1)Even time:

Different people have different body temperature. Even the same person has different body temperature at different times. There is a difference if different people have a bath. For example, people with high body temperature will speed up the increase of the temperature of the water in the bathtub so as to decrease the time it takes to reach the even temperature, which is just the even time.

(2)Suffering degree:

When the person with high body temperature has a bath, it will shorten the even time a little bit and extend the time of process of cooling a little bit. From this perspective, it increases the person's comfort.

However, considering about the temperature of adding water, when the person's body temperature gets higher, the temperature will falls to the person's body temperature and then falls to u_2. The later time will extend, which means people will feel cold for a longer time. From this perspective, it decreases the person's comfort.

Comprehensively, the two ways are opposite. .The later one is much dominant because the temperature of water decreases slowly which will keeps a long time. Hence, the higher the body temperature is, the higher the suffering degree is.

(3)Wasting water:

Wasting water is only up to the speed of adding hot water, the time of adding hot water and the times of adding hot water, but these factors can be limited by the person, so the wasting water is almost not disturbed by the body temperature.

(4)Conclusion:

The body temperature will have some impacts on our strategies before to some extent.

Modification is to increase the temperature of adding water if the person's body temperature is high and decrease the temperature of adding water if the person's body temperature is low.

5.6 The motions made by the person in the bathtub

(1)Even time:

Person in the bathtub is not motionless because he will make a series of motions which may be very tiny and subtle. These motions, to some degree, will have the influence on water same as stirring the water so as to speed up the balance of the water, which contains both the balance during adding water into the bathtub and the fusion time. As a consequence, the motions made by the person will decrease the even time and the more motions a person make, the less time even time will be.

(2)Suffering degree:

Not only the even time, but also the fusion time will decrease due to the series of motions. Once the time of every loop decreases, the time people suffers will correspondingly decrease, which shows that the suffering degree will correspondingly decrease.

(3)Wasting water:

The time of the process of the loop decreases, but in order to ensure the time of having a bath, the times of adding water may increase. Thus, the quantities of the wasting water correspondingly increase.

(4)Conclusion:

The motion of the people will have some impacts on the strategies before.

Modification is to increase the speed of adding water and stir the water continuously during the process of adding water to enable the water to be even much more quickly and add the times of adding water moderately.

5.7 The person uses a bubble bath additive

(1)Even time:

The thermal conductivity of the water is $0.599w/(m \cdot K)$, while the thermal conductivity of the bubble is $0.025w/(m \cdot K)$. As a result, the appearance of the bubble will influence the thermal conductivity of the water so that the thermal process of the water will be disturbed, resulting in the decrease of its moving efficiency and the increase of its fusion time. Above all, even time will increase.

(2)Suffering degree:

Due to the influence of the bubble, the temperature of the water will not be so easy to get lost, which indirectly results in the time of being uncomfortable increasing so that the suffering degree increases.

(3)Wasting water:

The time during the process of fusion and cleaning increases, so the times of adding water decreases. That is to say, wasting water decreases.

(4)Conclusion:

Bubbles will have some impacts on our strategies before to some extent.

Modification is to increase the speed of adding water and stir the water continuously during the process of adding water to enable the water to be even much more quickly.

6. Improvement of model

Actually, our cooling model is still rough and ideal so that there is a lot of space to improve our cooling model. This time, we are trying to improve our cooling model from a new perspective. Using Weber-Fechner law to solve the problem of water being cool in a natural condition will have some errors, so we specifically analyze the heat exchange between the water in the bathtub and the outside from four aspects.

6.1 Variable declaration

Symbol	Definition

φ_{aw}	Heat flux made by the air's long-wave radiation to water and water's reflection to air's long-wave radiation
φ_{wa}	Heat flux made by the radiation of the water's long-wave return
φ_e	Heat flux made by the evaporation of the water in the bathtub
φ_c	Heat flux made by the heat conduction between water and air
T_a	The temperature of the air
T	The temperature of the water in the bathtub
e_a	Vapor pressure of the air above the water
m	The weight of the water filled the bathtub
c	The specific heat capacity of the water in the bathtub
S	The contact area of water and air

6.2 Model assumption

- Considering the process of cooling in the natural condition, the temperature of the water in the bathtub is equal and changes at the same time consistently regardless of its position.
- The temperature of the air doesn't change as the time flows. That is to say, during the process of exchanging heat with the water, the temperature of the air is constant.

6.3 Heat exchange between the water and outside

6.3.1 Air's long-wave radiation to water and water's reflection to air's long-wave radiation

φ_{aw} is related to the temperature, and its formulation is:

$$\varphi_{aw} = (1-\gamma)\cdot\sigma\cdot\varepsilon_a\cdot\left(273+T_a\right)^4,$$

where γ stands for reflectivity of the long wave and we usually take $\gamma = 0.03$. σ is Stefan-Boltzman constant, which is $5.67\times10^{-8} W/m^2\cdot K^4$.

ε_a is the emissivity of the air, which is related to the temperature. Besides, $\varepsilon_a = 1-0.261\cdot exp(-0.74\times10^{-4}T_a^{\ 2})$ can be given by the formulation of Idso and Jackson. Therefore,

$$\varphi_{aw} = (1-0.03)\cdot 5.67\times10^{-8}\cdot\left[1-0.261\cdot exp(-0.74\times10^{-4}T_a^{\ 2})\right]\cdot\left(273+T_a\right)^4,$$

$$\varphi_{aw} = 5.5\times10^{-8}\cdot\left[1-0.261\cdot exp(-0.74\times10^{-4}T_a^{\ 2})\right]\cdot\left(273+T_a\right)^4$$

6.3.2 The radiation of the water's long-wave return

Water will return radiation to the atmosphere after absorbing long-wave radiation from the atmosphere, which is comparatively important part of the loss of the heat. When we regard water as absolute black body , φ_{wa} can be calculated by $Stefan-Boltzman$ fourth-power law, which is

$$\varphi_{wa} = \sigma\cdot\varepsilon_w\cdot\left(T+273\right)^4 。$$

where σ is Stefan-Boltzman constant, which is $5.67\times10^{-8} W/m^2\cdot K^4$. Since the water is not totally black body, ε_w can be 0.97, less than 1.

$$\varphi_{wa} = 5.67\times10^{-8}\cdot0.97\cdot\left(T+273\right)^4,$$

$$\varphi_{wa} = 5.5\times10^{-8}\cdot\left(T+273\right)^4.$$

6.3.3 The evaporation of the water

Estimation formula about evaporation is usually based on experience, so the transfer formula usually can be $\varphi_e = f(w)\left(e_s - e_a\right)$, where stands for saturate evaporation pressure of the air which is close to the water in response to the temperature of the water which is close to t,

$$e_s = exp(20.85 - \frac{5278}{T+273.3});$$

$f(w)$ is the function of the wind, including the influences of free convection and forced convection on evaporation. w stands for the speed of the wind which is ten meters high, having the formulation that[5]

$$f(w) = 9.2 + 0.46w^2 。$$

6.3.4 The radiation of heat conduction between water and air

When the temperature of the air is not equal to the temperature of the water, the interface between water and air will experience a heat conduction. Thermal conductivity is proportional to the temperature difference between water and air, so the heat flux usually satisfies the following equation:

$$\varphi_c = 0.47f(w)\cdot\left(T - T_a\right).$$

6.3.5 Heat flux made by heat conduction between water and air

Heat flux made by heat conduction between water and air satisfies

$$\varphi = \varphi_{aw} - \varphi_{wa} - \varphi_e - \varphi_c;$$

Therefore,

$$\begin{aligned}\varphi = {} & 5.5\times10^{-8}\cdot\left[1-0.261\cdot exp(-0.74\times10^{-4}T_a{}^2)\right]\cdot\left(273+T_a\right)^4 \\ & -5.5\times10^{-8}\cdot\left(T+273\right)^4 \\ & -f(w)\left(e_s-e_a\right) \\ & -0.47f(w)\cdot\left(T-T_a\right)\end{aligned}$$

We assume $\omega = 0$, that is to say the function of wind values $f(0) = 9.2$.

$$\begin{aligned}\varphi = {} & 5.5\times10^{-8}\cdot\left[1-0.261\cdot exp(-0.74\times10^{-4}T_a{}^2)\right]\cdot\left(273+T_a\right)^4 \\ & -5.5\times10^{-8}\cdot\left(T+273\right)^4 \\ & -9.2\left(exp(20.85-\frac{5278}{T+273.3})-e_a\right) \\ & -4.324\cdot\left(T-T_a\right)\end{aligned}$$

6.4 Heat exchange between the bathtub and outside

The width of most bathtubs is higher than eight centimeters, and the material of the bathtub is usually acrylic and has good insulation effect. We find that when the bathtub is filled with water, the temperature of the inner casing wall is the same as the temperature of the water in the bathtub while the temperature of the outer casing wall is the same as the temperature of the air. That is why we think the bathtub has no heat exchange with the outside.

6.5 Calculating the time of cooling

Due to the relationship between heat flux and heat, wo can know that

$$\varphi = \frac{dQ}{S \cdot dt} = \frac{cm \cdot dT}{S \cdot dt},$$

$$dt = \frac{cm}{S\varphi} \cdot dT,$$

$$t = \int_{T_0}^{T} \frac{cm}{S\varphi} \cdot dT,$$

where

$$\begin{aligned}\varphi = {} & 5.5 \times 10^{-8} \cdot \left[1 - 0.261 \cdot exp(-0.74 \times 10^{-4} T_a{}^2)\right] \cdot \left(273 + T_a\right)^4 \\ & - 5.5 \times 10^{-8} \cdot \left(T + 273\right)^4 \\ & - 9.2\left(exp(20.85 - \frac{5278}{T + 273.3}) - e_a \right) \\ & - 4.324 \cdot \left(T - T_a\right)\end{aligned}.$$

7. Strengths and weaknesses

7.1 Strengths

- We consider the loss of heat comparatively accurately. Many kinds of the loss of heat are under our consideration including the radiation of heat conduction between water and air, the heat flux made by heat conduction between water and air, the radiation of the water’s long-wave return and the evaporation of the water.
- With the help of Weber-Fechner law, we consider the feeling instead of the heat stimulation so that we can ensure which strategy is better.
- We use the comprehensive evaluation model to reduce the influence by the assumed proportion.

7.2 Weaknesses

- We discussed the situation that the overflow drain and the faucet are on the opposite side carefully. When the overflow drain is near the faucet, we only estimate the loss of the heat roughly not accurately. This might affect out strategy.
- The proportion is considered by us,not supported by some study, so it is a little subjective. There may be difference between the ideal proportion and that, even leading to an error.
- During the period of draining away, we think the water is cold. That is to say the water is not influenced by the adding water but it’s not actual. It may be not

good.

8. Conclusion

Firstly we discuss the situation that the overflow drain is on the opposite side of the faucet. Under this circumstances and some other limits, we get the best strategy including the best speed, time, temperature and the times to add the water by three hierarchical models including the 'Even loop model', the 'Body feeling model' and the 'Comprehensive evaluation model'.

A person who lies in the bathtub could adopt it to keep the temperature even throughout the bathtub and as close as possible to the initial temperature without washing too much water in a natural condition.

Besides, based on the reality, we discuss the situation that the overflow drain is on the same side of the faucet. We find that a person can also achieve his goal to have a sensible and enjoyable bath, but he will waste much water with a less efficient way compared to the situation before.

Last but not least, we explore the effects seven additional factors have on our original strategy. We find that all of them somehow have impacts on the strategy to have a sensible and efficient bath. That is why there are no strategies that can be supplied to any situations.

A cover letter for a hot bath

Dear Madam/Sir

This is a cover letter. I am going to introduce and explain our strategies for you to have a sensible and enjoyable hot bath.

Our strategy is mainly based on three points. They are respectively keeping the temperature of the water even, ensuring users' enjoyment and not wasting too much water.

Input the relevant parameters about the bathtub, the environment and the user into the models above and you will get an optimal strategy. If you get $(36.3, 7, 7, 1)$, that means the optimal strategy is to keep the water in the speed of $7L/min$ for 7 minutes when the temperature goes down to $36.3^{\circ}C$. To be specific, users are advised to open the faucet to a great extent but not entirely for about 7 minutes, maybe a period of two songs, when feeling a little cold. Only by doing that can users have the most sensible and enjoyable hot bath.

During the bathing, we advise users to take a bubble bath to save water. Bubble can prevent the heat exchange so that water can be hot longer. In addition, motions in the water are not suggested because they can highly accelerate the water movement so that much heat is lost.

We suggest that users can do some motions only when they think that even temperature of water counts more, especially for those whose skins are sensitive to the temperature. Motions can get water in different temperature mixed together well so that they can reach the same.

Even temperature can make users more enjoyable but that is difficult to reach. When water is added, it spreads to different directions to heat other water but as well as break the even temperature. What's more, water in surface can exchange heat with the air so that the upper water is colder than the rest. That means the temperature is not even as well. Moreover, different parts of body are in different temperature and they can also affect the water.

If you want to enjoy the bath, we strongly recommend you a spa-style tub with a secondary heating system and circulating jets. Use that and you will get rid of all worries about the traditional bathtub.

Best regards

Reference

[1]Liu Zhihua, Liu Ruijin, The study on the regular rule of cooling in Newton's law of cooling[J]. The Reasearch on Science and Technology of SUT,2005,06:23-27.

[2].Zhan Shichang, The research on the applicable range of Newton's law of cooling[J]. College Physics, 2000 , 19(5):36-37.

[3].On the coefficient of the shape in residence building and the energy saving in building,[D].Chongqing University,2007.

[4]. Chen Xinyong, Zhu Jing, Han Longxi, The approach to identifying the water evaporation coefficient and analyzing its sensitivity[J].Water Science and Engineering Technology,2007,05:9-13.

[5]. Chenyongcan, Zhang Baochang, Li Yuliang, The Study on the mathamatical model of the thermal distribution vertically in the Miyun Reservoir[J], Journal of Hydraulic Engineering,1998,09:15-21.