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2015 Mathematical Contest in Modeling (MCM) Summary Sheet

(Attach a copy of this page to your solution paper.)

The recent Ebola Virus Disease (EVD) broke out in West Africa from February 2014.This disease spreads quickly and has claimed 8,981 lives up to 1 February 2015.

In this article, we build Spread Model to forecast the development of EVD and the demand of medication in the future, Production Model to forecast the output of the medication and Delivery Model to give a most effective distribution scheme. Through these three models, the EVD can be under control in the shortest time.

Since West Africa has suffered most from Ebola and countries in this region have many similarities, we use Sierra Leone as an example to show the accurate value of the results in our models.

We set up our Spread Model on the basis of SIR Model, adding the influence of both the drugs and vaccines. In other words, the parameters in the original SIR Model are not constant, they will change with time. This model can be used to forecast the demand of medication in the future.

In our Production Model, We divide the whole produce process into four stages. On stage Ⅰ~ Ⅲ, the output of the medication is determined by technology and is smaller than the demand. On stage Ⅳ, the output of the medication is determined by the demand.

Our Delivery Model consists of vertical transport（medication distributes from the treatment center to epidemic areas）and lateral transport(medication distributes from one epidemic area to another). There are two objectives in our Delivery Model: To minimize the total cost of the transport and to minimize the distribution time.

Key word: SIR Model, Production Model, Delivery Model, Ebola Virus Disease

Content

I. Introduction

The recent Ebola Virus Disease (EVD), which broke out in West Africa from February 2014, is a sever crisis due to the high mortality rate. Additionally, this disease spreads quickly because the poor medical condition in West Africa and much more frequent international exchange. Up to 1 February 2015, a total of 22,495 have been reported confirmed, probable, and suspected cases of EVD in Guinea, Liberia and Sierra Leone, with 8,981 reported deaths according to the World Health Organization (WHO). As a result, EVD causes widespread concern in the international community.

II. The Description of Problem

The world medical association has announced that their new medication could stop Ebola and cure patients whose disease is not advanced. That is to say, if susceptible people are vaccinated, they will be immune to Ebola and not seriously infected people can be cured with the drugs. But patients whose disease is advanced will die of Ebola even though they take the medication. In this problem, we need to set up a reasonable system to deliver the medication to eradicate Ebola or relieve current intense strain. To optimize the delivery system, we need to take factors like the spread of the disease, the quantity of the medication needed, the locations of delivery, the speed of manufacturing vaccines or drugs and so on into account.

The analytical procedure is as follows:

- Since West Africa has suffered most from Ebola and countries in this region have many similarities, we use Sierra Leone as an example to show the accurate value of the results in our models.
- Set up Disease Spread Model on the basis of SIR Model and simulate the number of infected people in the future. The forecast statistics will affect the demand of medication in each epidemic area.
- According to the administrative division in Sierra Leone, we divide Sierra Leone into 14 epidemic areas and then set up the delivery system.
- Set up Medication Delivery Model to optimize the distribution considering both the cost and time of a complete medication distribution.
- Conclude the final solution to this problem.
- Sum up the whole solution and prepare a non-technical letter for the world medical association to use in their announcement.

III. Models

3.1Spread Model(SIR Model)

3.1.1 Terms, Definitions and Symbols

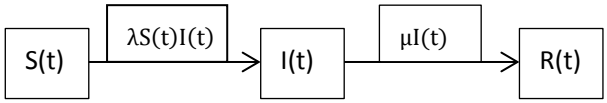
- S(t): The proportion of susceptible people.

- I(t): The proportion of infective people.
- R(t): The proportion of dead and cured people.
- I(0): The proportion of patients in the initial time.
- S(0): The proportion of susceptible people in the initial time.
- N: The total number of people in the epidemic area.
- λ: The number of susceptible people infected by one infective person every week.
- μ: The proportion of dead and cured people in patients every week.

3.1.2 Assumptions

1) We divide the people into three types, including susceptible people, infective people and dead or cured people.
2) When the infected people have recovered, they will never get infected again.
3) The total number of population N is constant, regardless of new birth and death.
4) When patient contact with susceptible people effectively, the susceptible would be infected.
5) Assuming λ is constant.
6) Assuming μ stay constant.
7) When infective people die, the dead people will not infect other people during the circle.

3.1.3 The Process of the Model



3.1.4 The Foundation of Model[1]

According to the assumptions，we set up a SIR model to solve the problem. At the prophase of the outbreak of Ebola (February, 2014 - February, 2015), due to invalid control without medicine, disease spread in the state of nature at this time when λ is assumed to be constant. Every infective person can turn $\lambda S(t)$ susceptible people into the patient. Since the total number of patient is NI(t), there are $\lambda NS(t)I(t)$ susceptible people infected every week. The rate of the dead for each week is μ. The reduction of the susceptible equals to the susceptible infected every week，so：

$$\begin{cases} N\dfrac{dI(t)}{dt} = \lambda NS(t)I(t) - \mu NI(t) \\ N\dfrac{dS(t)}{dt} = -\lambda NS(t)I(t) \\ N\dfrac{dR(t)}{dt} = \mu NI(t) \\ S(t) + I(t) + R(t) = 1 \end{cases}$$

Then

[1] Yang Guan. The Analysis of Infectious Disease Control on the basis of SIR Model. Mathematical Biology Journal.2009, 24 (3): 479-483

$$\begin{cases} \dfrac{dI(t)}{dt} = \lambda S(t)I(t) - \mu I(t), I(0) = I_0 \\ \dfrac{dS(t)}{dt} = -\lambda S(t)I(t), S(0) = S_0 \end{cases}$$

3.1.5 Solution and Result

After the analysis $I_0 = 0.000015,\ S_0 = 0.999985$.

According to the data from the WHO:

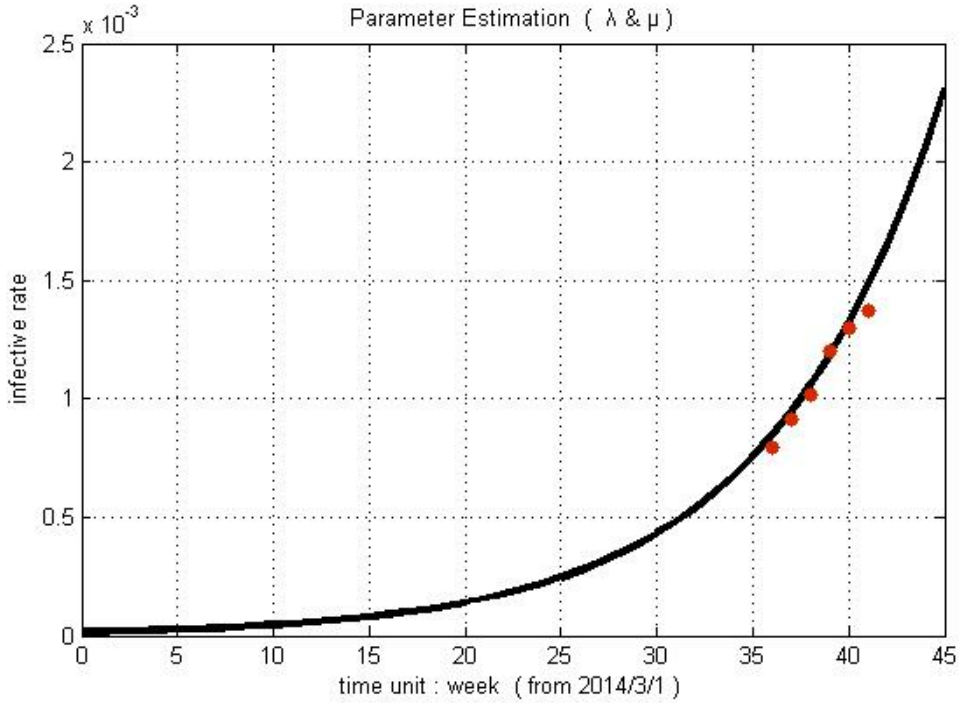
Table 1

Sierra Leone									
date	11.5	11.7	11.12	11.14	11.19	11.21	11.26	12.3	12.8
cases	4759	4862	5368	5586	6073	6190	6599	7312	7897
deaths	1070	1130	1169	1187	1250	1267	1398	1583	1768
date	12.15	12.22	12.29	1.4	1.11	1.18	1.25	2.1	
cases	8356	9004	9446	9780	10124	10340	10518	10740	
deaths	2058	2582	2758	2943	3062	3145	3199	3276	

Through the method of data fitting we can estimate that $\lambda = 0.13$, $\mu = 0.018$.

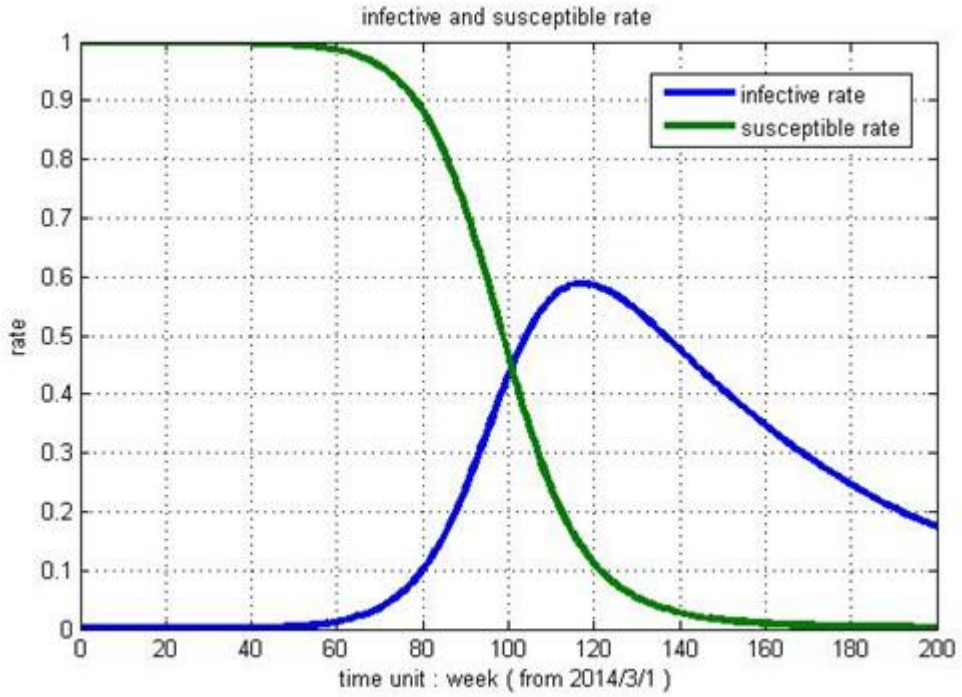
The figure of parameter estimation is as follow:

Figure 1



The figure of I(t) and s(t) is as follows:

Figure 2



3.1.6 Analysis of the Result

According to the result, when Ebola virus was first reported in Sierra Leone on 1st March in 2014, the infective rate was increasing if there were no vaccines or drugs to cure the disease. Then we fitting the curve to the statistics of Sierra Leone's cases from the WHO, which are red dots marked in Figure 2. As the figure shows, our model can well simulate the situation of the spread of Ebola without vaccines or drugs.

Figure 2 shows the change of I(t) and S(t). Under the condition of lack of the medical control, the susceptible rate will continuingly decrease because of the death and cure of infected patient. Finally, the rate of susceptible will maintain a constant, but at a low lever. The infective rate increases at first, but decline then. The reason is that the infected people will infect the susceptible and the number of the infected increases. At the same time, the infected people will die of the disease, which will decline the infective rate. About 120 weeks later, the infective rate rises to the summit and then the rate begins to decrease. During this period, the number of people infected is smaller than the number of people died of the disease or recover.

3.1.7 Strength and Weakness

- **Strength:** This model has proved that the infective rate and the proportion of susceptible people are changing with time. It can predict future trend at a nature spread of disease, including the proportion that the patients will reach. Due to the use of the method of data fitting, the margin of error is small. To sum up, this model is simple, accurate and practical.
- **Weakness:** This model just applies to the nature spread, it is more suitable for natural environment without the external factors. But in fact, we would

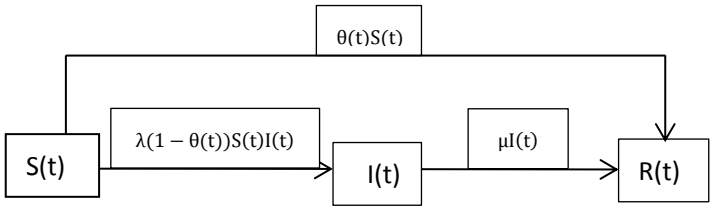
face more complex problems, such as isolation and medical cure. Under these circumstances, we can't use this model, that's what we need to improve in the next models.

3.2Improved Model Ⅰ(Spread Model considering vaccines)

3.2.1 Additional Assumptions

1) We divide people into 3 types, including susceptible people, infective people and dead, cured or vaccinated people.
2) θ will change with t, and the connection between θ and t is linear. (θ(t)=k*t)
3) When people are vaccinated, they will never be infected and then they belong to Type R.
4) If the peak of infective rate is under 0.2, the epidemic situation can be under control.

3.2.2 The Process of the Model



3.2.3 The Foundation of Model

$$\begin{cases} N\dfrac{dI(t)}{dt} = \lambda(1-\theta(t))NS(t)I(t) - \mu NI(t) \\ N\dfrac{dS(t)}{dt} = -\lambda(1-\theta(t))NS(t)I(t) - \theta NS(t) \\ N\dfrac{dR(t)}{dt} = \mu NI(t) + \theta NS(t) \\ \dfrac{d\theta(t)}{dt} = k \\ S(t) + I(t) + R(t) = 1 \end{cases}$$

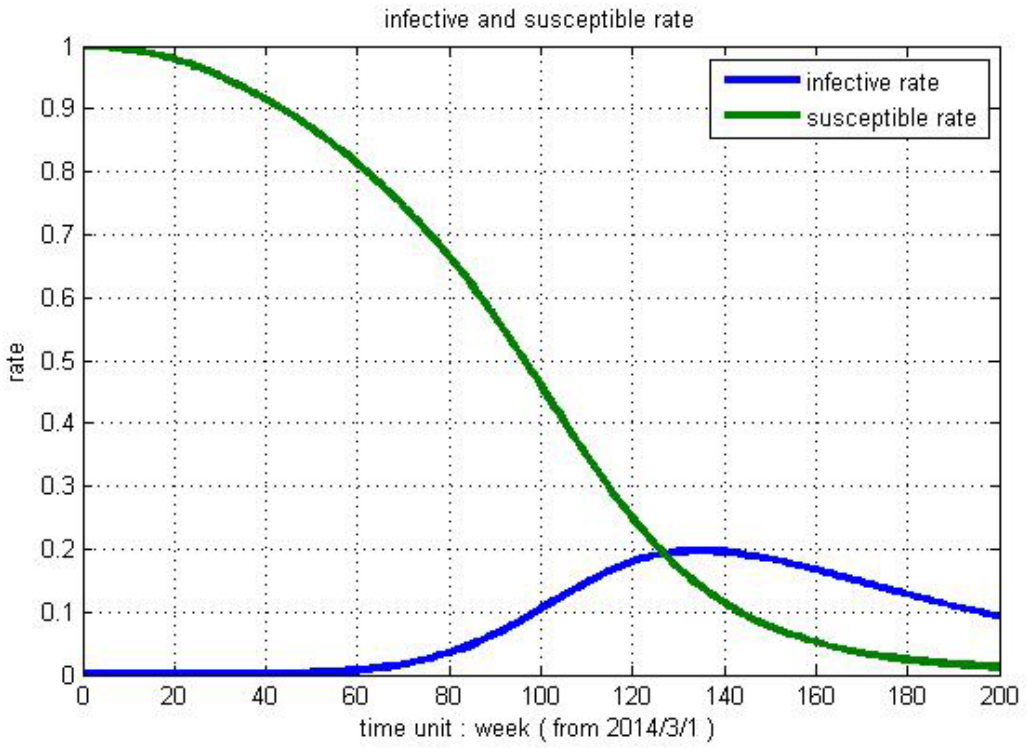
Then :

$$\begin{cases} \dfrac{dI(t)}{dt} = \lambda(1-\theta)S(t)I(t) - \mu I(t), I(0) = I_0 \\ \dfrac{dS(t)}{dt} = -\lambda(1-\theta)S(t)I(t) - \theta S(t), S(0) = S_0 \\ \dfrac{d\theta(t)}{dt} = k \end{cases}$$

Since we assume that if the peak of infective rate is under 0.2, the epidemic situation can be under control, then k=0.00011.

3.2.4 Solution and Result

Figure 3



3.2.5 Analysis of the Result

When we vaccinate people in the epidemic area, the curve of susceptible rate is similar to the Basic Model, but the curve of infective rate changes a lot. Firstly, the maximum of I(t) decrease a lot compared to the Basic Model. The reason is that vaccines can firstly deduct the number of susceptible people remarkably, then, deduct the number of infected people. The trend of infective rate is the same with the previous. It increases at first and then declines, but overall, it is smaller.

3.2.6 Strength and Weakness

- **Strength:** This model takes the vaccines into account, so we can use this model to analyze the effect of vaccines on the epidemic situation.
- **Weakness:** In this model, μ is still a constant, that is to say, we ignore the effect of drugs on the epidemic situation. In fact, when we offer patients with medical treatment, the μ will change and that is what we will consider in the next improved model.

3.3 Improved Model Ⅱ (Spread Model considering vaccines and drugs)

3.3.1 Additional Assumptions

- λ is assumed to be a time-dependent variable and the effective contact people would decrease.

3.3.2 The Foundation of Model

Since the world medical association has announced that their new medication could stop Ebola and cure patients whose disease is not advanced. Medication will be used to cure the infected people and prevent the further spread later, thus λ is assumed to be a time-dependent variable .The effective contact people would decrease. The relationship between λ and t is assumed to be index according to the actual situation.

We analyze the number of infected people during the recent weeks and predict to give the mathematical expression of effective contact people λ and time t:

$$\lambda(t)= 0.13e^{(a*t)}$$

$$\mu(t)= 0.018*e^{(b*t)}$$

Similarly, we consider the proportion of patients in the initial time（t=0）is I_0，the proportion of susceptible is S_0，taking the λ into the following equation，using the MATLAB to solve the differential equation：

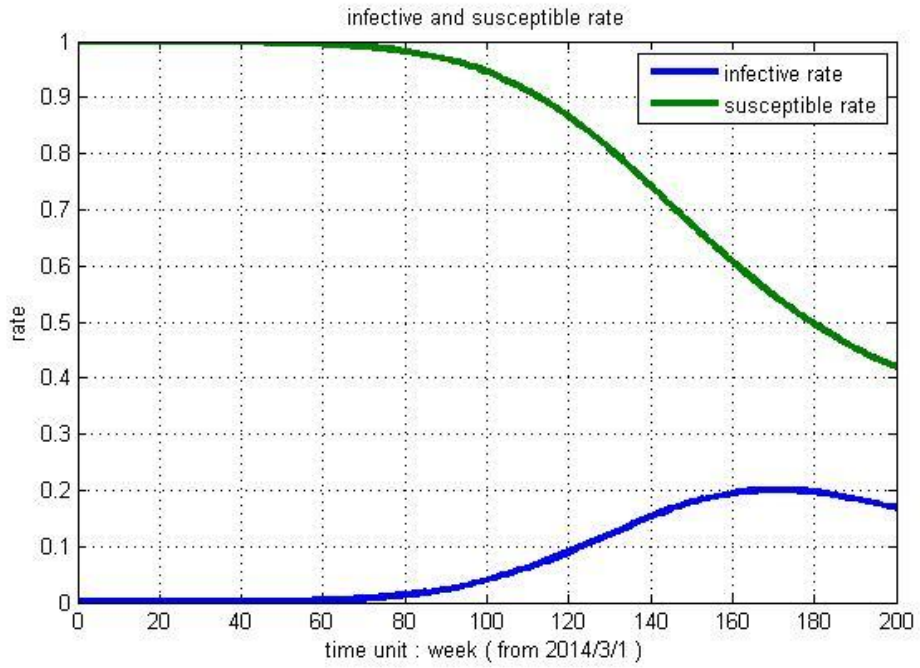
$$\begin{cases} \dfrac{dI(t)}{dt} = \lambda(t)S(t)I(t) - \mu(t)I(t), I(0) = I_0 \\ \dfrac{dS(t)}{dt} = -\lambda(t)S(t)I(t), S(0) = S_0 \\ \dfrac{d\lambda(t)}{dt} = a\lambda(t) \\ \dfrac{d\mu(t)}{dt} = b\mu(t) \end{cases}$$

3.3.3 Solution and Result

Since we assume that if the peak of infective rate is under 0.2, the epidemic situation can be under control, then a=-0.0055, b=0.0025.

From the model analysis we get the numerical result as follows. By the above-mentioned can be obtained, the infection rate of Ebola virus obviously slow down, after using the drugs, we also significantly curb its spread.

Figure 4



3.3.4 Analysis of the Result

From this figure, we can see the infective rate I(t)（the blue line in the figure）and the susceptible rate(the green line in the figure) changes with time. Compare with Improved Model Ⅰ, drugs are taken into consideration. The blue line in the figure shows that infective rate will increase at first and reach the maximum value and then decline. Just like Improved Model Ⅰ, the maximum of I(t) decrease a lot compared to the Basic Model. However, the reason is that drugs can directly deduct the number of infected people，while vaccines can firstly deduct the number of susceptible people remarkably, then, deduct the number of infected people. In comparisons among the three models, we have a conclusion that human intervention has huge impact on EVD.

3.3.5 Strength and Weakness

- **Strength:** Improved Model Ⅱ considers both two factors, including the drugs which have influence on the patient and the vaccines which have influence on the susceptible people. The assumption that the relationships between λ and t, μ and t are index is reasonable. At the same time, the model reflects clearly how the drugs and the vaccine work, which provides the world medical association with data when making the decision. It provide a way of thinking, having certain guiding significance. This model is more reasonable than the two models before and seems to be much more effective and practical.
- **Weakness:** The assumption on index relationships between λ and t , μ and t still has margins of error. Also, we ignore the effect of population migration between the different cities when the disease spread.

3.4 Production Model

3.4.1 Terms, Definitions and Symbols

- u(t): techniques of production at time t.
- L: labor force
- K: capital
- α : the weight of labor force in the production process
- β : the weight of capital in the production process
- t0: the first turning point of the expression of u(t)
- t1: the second turning point of the expression of u(t)
- t2: the second turning point of the expression of u(t)
- Y: the total output of medication in one week
- D(t): the demand of the medication in the epidemic area at time t.

3.4.2 Assumptions

1) L&K are constant, which means that the labor force and capital stay unchanged.
2) α &βare constant, which means the weight of labor force and capital in the production process is always stable.
3) One production process continues for one week.

3.4.3 The Foundation of Model

We divide the whole produce process into four stages. In stage Ⅰ, Y grows with high-increasing speed due to the high development of technology. In stage Ⅱ, Y grows with low-increasing speed due to the speed of technology development slows down as a result of some constraints. Due to the constraints, Y finally becomes stable in StageⅢ(Now we suppose that Y is still less than D at the end of Stage Ⅱ, if Y>D at that time, StageⅢcan be deleted). StageⅣ: Y declines due to the decreasing demand of the medication declines. The demand of the medication in the epidemic area at time t is decided by the improved model Ⅱ .

Stage Ⅰ :Y grows with high-increasing speed

$$\begin{cases} u(t) = e^{\gamma t} \\ Y = u(t) * L^{\alpha} * K^{\beta} \end{cases} \quad (t \in (0, t_0))$$

Stage Ⅱ : Y grows with low-increasing speed

$$\begin{cases} u(t) = u(t_0) + (t - t_0)^{\varepsilon} \\ Y = u(t) * L^{\alpha} * K^{\beta} \end{cases} \quad (t \in (t_0, t_1))$$

StageⅢ: Y becomes stable

$$Y = u(t_1) * L^{\alpha} * K^{\beta} \quad (t \in (t_1, t_2))$$

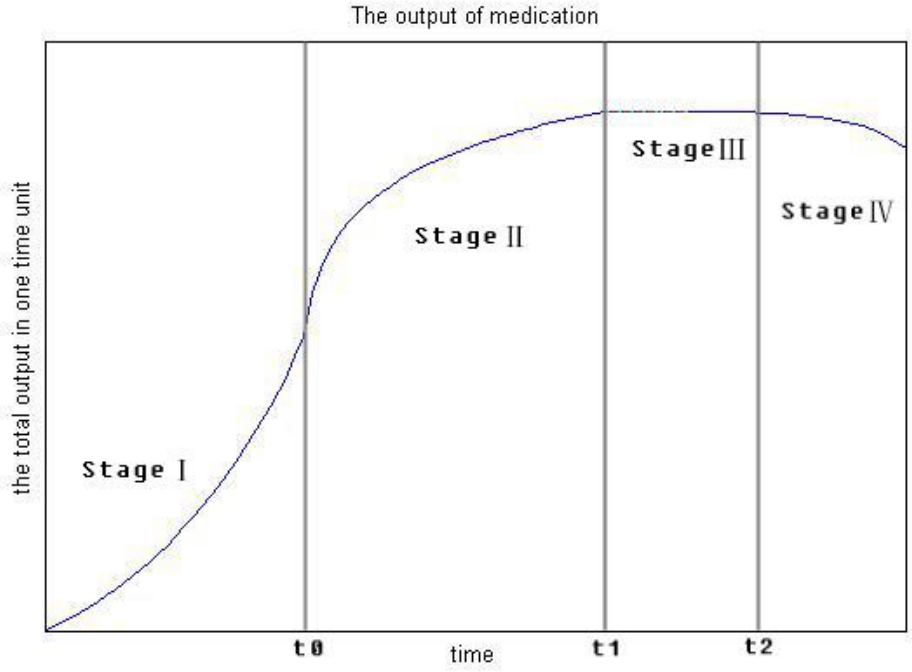
StageⅣ: Y declines due to the decreasing demand of the medication declines

$$Y = D(t) \quad (t > t_2)$$

3.4.4 Solution and Result

Set appropriate values of the parameters (γ and ε), we can draw the yield curve with the help of MATLAB. It is shown in Figure 5.

Figure 5



3.5 Delivery Model

In this model, our team comes up with two models to design the process of medication distribution. The two models both take vertical and lateral transportation into consideration and t, but one assumes that vertical and lateral transportation can take place at the same time while in the other model vertical transportation happens first and lateral transportation happens later in a circle.

Vertical transportation means medication is carried from treatment center to epidemic areas.

Lateral transportation means medication is carried between epidemic areas.

But the two models based on the common assumption that the position of treatment center is determined.

3.5.1 Determination of Treatment Center

3.5.1.1 Terms , Definitions and Symbols

- A_i: The code of epidemic cities (i=1, 2, 3…)
- (x_i, y_i): The location of the epidemic city in the map
- $t1_i$: time of the one-way distribution from treatment center to the epidemic city(A_i)
- S_i: the straight-line distance from treatment center to the epidemic cities(A_i)
- d_i: The amount of the medication the epidemic city(A_i) needed

3.5.1.2 Assumptions

1) d_i is proportion to the cases in epidemic cities during the recent 3 weeks.
2) $t1_i$ is proportion to $S_i * d_i$.

3.5.1.3 Result

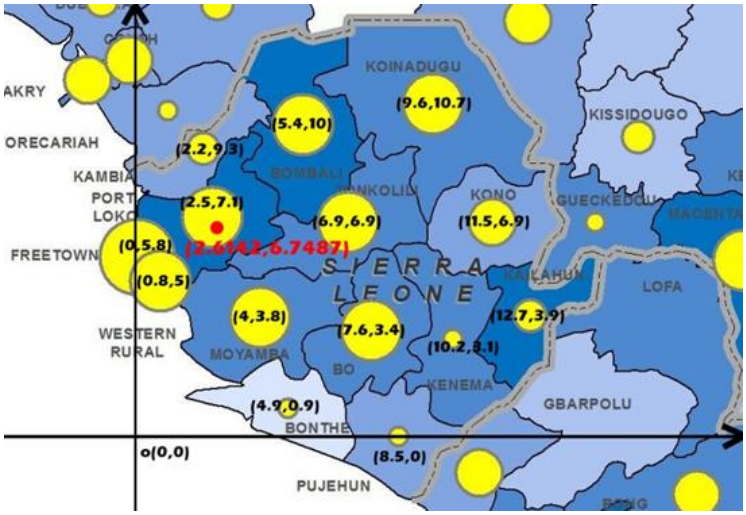
To give a result, we gain the data of Sierra Leone from WHO, which is

shown in Table 2 and Figure 6.

Table 2

Sierra Leone							
City name	WESTERN RURAL	TONKOLILI	PUJEHUN	PORT LOKO	MOYAMBA	KONO	KOINADUGU
Code	A_1	A_2	A_3	A_4	A_5	A_6	A_7
location	(0.8, 5)	(6.9, 6.9)	(8.5, 0)	(2.5, 7.1)	(4, 3.8)	(11.5, 6.9)	(9.6, 10.7)
cases	1130	449	25	1366	268	336	147
City name	BO	BOMBALI	BONTHE	FREETOWN	KAILAHHUN	KAMBIA	KENEMA
Code	A_8	A_9	A_{10}	A_{11}	A_{12}	A_{13}	A_{14}
location	(7.6, 3.4)	(5.4, 10)	(4.9, 0.9)	(8.5, 0)	(12.7,3.9)	(2.2, 9.3)	(10.2, 3.1)
cases	327	971	1	1903	650	156	530

Figure 6



According this model, we can calculate the optimal location of the treatment center with the help of MATLAB, the optimal position is (2.6142 , 6.7287), which is marked in red in Figure 6.

3.5.2 Delivery Model Ⅰ

3.5.2.1 Terms, Definitions and Symbols

- $C1_j$: The cost of each vehicle every time carrying medication from treatment center to epidemic area A_j(j=1, 2, …, 14)
- $C2_{ij}$: The cost of each vehicle every time carrying medication from epidemic area A_i to epidemic area A_j(i, j=1, 2, …, 14)
- $t1_j$: The time of each vehicle every time carrying medication from

treatment center to epidemic area A_j(j=1, 2, ⋯, 14)

- $t2_{ij}$: The time of each vehicle every time carrying medication from epidemic area A_i to epidemic area A_j(i, j=1, 2, ⋯, 14)
- c1 : The capacity of each vehicle from treatment center.
- $c2_i$: The capacity of each vehicle from epidemic area A_i(i=1, 2, ⋯, 14)
- w: The volume of each unit of medication.
- q1: The storage of medication in treatment center.
- $q2_i$: The current storage of medication in epidemic area A_i (i=1, 2, ⋯, 14)
- d_i: The medication demand of epidemic area A_i in this circle (i=1, 2, ⋯, 14)
- u_i: The medication demand of epidemic area A_i in next circle (i=1, 2, ⋯, 14), it a random variable.
- Eu_i: The expected medication demand of epidemic area A_i in next circle (i=1, 2, ⋯, 14).
- $TC1_j$: The total cost of carrying medication from treatment center to epidemic area A_j(j=1, 2, ⋯, 14).
- $TC2_{ij}$: The total cost of carrying medication from epidemic area A_i to epidemic area A_j(i, j=1, 2, ⋯, 14)
- $T1_j$: The actual time of carrying medication from treatment center to epidemic area A_j(j=1, 2, ⋯, 14)
- $T2_{ij}$: The time of each vehicle every time carrying medication from epidemic area A_i to epidemic area A_j(i, j=1, 2, ⋯, 14)
- $y1_j$: The quantity of medication carried from treatment center to epidemic area A_j(j=1, 2, ⋯, 14)
- $y2_{ij}$: The quantity of medication carried from epidemic area A_i to epidemic area A_j(i, j=1, 2, ⋯, 14)

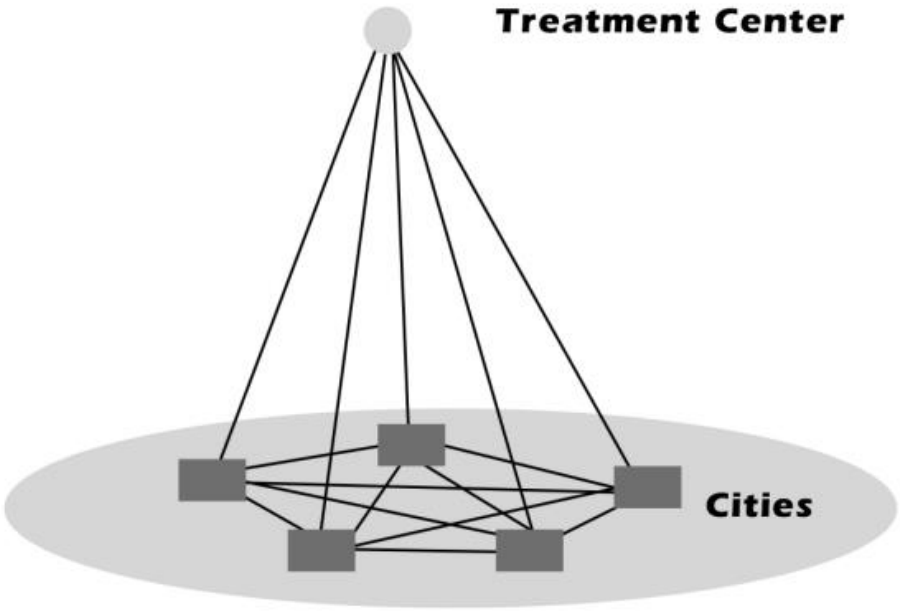
3.5.2.2 Assumptions

1) There is only one treatment center in every country, and vaccines and

drugs are distributed from the center to the epidemic areas.

2) We choose the cities which have been reported confirmed Ebola cases as the epidemic areas. According to the WHO's reports, there are 14 cities have been infected in Sierra Leone until now. We mark these epidemic areas with A1, A2,…, A14. Details refer to Table 2.
3) We simplify the distance between epidemic areas or treatment center and epidemic area as the linear distance between two points on Rectangular Plane Coordinate System.
4) The demand of drugs of each epidemic area is relation to the confirmed cases in this area. Assume they have a linear relation.
5) The increase cases of next circle will obey the SIR model.
6) There are enough vehicles in treatment center and epidemic areas to ensure all of the drugs and vaccines could be delivered once every time.
7) Each infected person needs a unit of drugs, and each susceptible person need a unit of vaccine.
8) The cost of every unit of distance is the same.
9) Only if one epidemic area can meet the demand of itself, that it can deliver vaccines and drugs to other epidemic areas.

3.5.2.3 The Process of Model



3.5.2.4 The Foundation of Model

The treatment center would estimate the different demand for drugs of 14 cities, then, make the distribution of drugs. Considering that there is certain error between the prediction and the actual situation, there would be a case that drugs are sufficient for an epidemic area, but not sufficient for the other. The distance of the two cities is much shorter than the distance from the treatment center to the two cities. In this case,

medication transport between the two cities (lateral transport) is much more reasonable, which saves time and cost. Thus, it is necessary to combine the drug distribution from the treatment center to epidemic areas with drug transport between epidemic areas. Such distribution plan can improve the efficiency.[2]

So we build a model based on both the vertical and lateral transport.

The collaborative optimization scheduling model has two objective functions: Minimizing the total cost of delivery (f_1); Minimizing the total delivery time f_2

Object1: Minimize the total cost of delivery (f_1):

The cost can be divided into 2 parts, one contains the cost of delivering vaccine and drugs from treatment center to each epidemic area, and the other part is the cost of delivering vaccine and drugs between epidemic areas. The two parts of cost are respectively as follows:

$$TC1_j = C1_j * \left\lceil \frac{w * y1_j}{c1} \right\rceil \quad (1)$$

$$TC2_{ij} = C2_{ij} * \left\lceil \frac{w * y2_{ij}}{c2} \right\rceil \quad (2)$$

$\lceil x \rceil$ means to round up the value to an integer, e.g.$\lceil 3.2 \rceil$=4

Function (1) expresses the cost of delivering vaccine and drugs from treatment center to epidemic area A_j (j=1,2, …, 14)

Function (2) means the cost of delivering vaccine and drugs between epidemic areas A_i and A_j (i, j=1,2, …, 14, i≠ j)

Object 2： Minimize the total delivery time f_2

Delivery time refers to the period from the beginning of delivery order to the end of delivering.

The total delivery time depends on the reserve of treatment center and epidemic areas and whether each epidemic area demands vaccine and drugs, so:

$$T1_j = \begin{cases} t1_j , & x1_j = 1 \\ 0 , & x1_j = 0 \end{cases} \quad (3)$$

$$T2_{ij} = \begin{cases} t2_{ij} , & x2_{ij} = 1 \\ 0 , & x2_{ij} = 0 \end{cases} \quad (4)$$

Function (3) means the delivery time of delivering vaccine and drugs from treatment center to epidemic area A_j (j=1,2, …, 14)

Function (4) means the delivery time of delivering vaccine and drugs between epidemic areas A_i and A_j (i, j=1,2, …, 14, i≠ j)

So, the objective function is as follows:

$$\min f_1 = \sum_{j=1}^{n} TC1_j + \sum_{i=1}^{n} \sum_{j=1}^{n} TC2_{ij} \quad (5)$$

[2] Wang Xinping , The Distribution of Emergency Materials under the Law of Infectious Disease Spread. Journal of Southeast University, Volume 36

$$\min f_2 = \max\{T1_j, T2_{ij}\} \quad (6)$$

s.t.

$$\sum_{j=1}^{n} y1_j \ll q1 \quad (7)$$

$$\sum_{j=1}^{n} y2_{ij} \ll q2_i \quad (8)$$

$$q2_j - \sum_{i=1}^{n} y2_{ji} + \sum_{j=1}^{n} y1_j + \sum_{i=1}^{n} y2_{ij} = d_j + Eu_j \quad (9)$$

$$x2_{ij} * x2_{ji} = 0, \quad i \neq j \quad (10)$$

And definition

$$x1_j = \begin{cases} 1, & \text{vaccine center deliver medication to epidemic area } A_j \\ 0, & \text{vaccine center not deliver medication to epidemic area } A_j \end{cases}$$

$$x2_{ij} = \begin{cases} 1, & \text{epidemic area } A_i \text{ deliver medication to } A_j \\ 0, & \text{epidemic area } A_i \text{ not deliver medication to } A_j \end{cases}$$

The constraint function (7) ensures the delivery medication is less than the total reserve in the treatment center. The constraint function (8) ensures the delivery medication from epidemic area A_i to other epidemic area is less than the current reserve of A_i. The constraint function (9) makes sense that only when epidemic area A_i meets the demand of itself, than A_i may deliver surplus medication to other epidemic area. Function (10) prevent cross delivering, and if A_i deliver medication to A_j, than A_j will not deliver medication to A_i in this circle.

3.5.3 Delivery Model Ⅱ (vertical transportation happens first and lateral transportation happens later in a circle)

3.5.3.1 Terms , Definitions and Symbols

- S(t): The proportion of susceptible people.
- I(t): The proportion of infective people.
- R(t): The proportion of dead and cured people.
- I(0): The proportion of patients in the initial time.
- S(0): The proportion of susceptible people in the initial time.
- N: The total number of people in the epidemic area.
- λ: The number of susceptible people infected by one infective person every week.
- μ: The proportion of dead and cured people in patients every week.

3.5.3.2 Assumptions

1) There is only one treatment center in every country, and vaccine and drugs are distributed from the center to the epidemic areas.
2) The demand of drugs of each epidemic area is relation to the confirmed cases in this area. Assume they have a linear relation.
3) The increase cases of next circle will obey to the SIR model.
4) Each infected person need a unit of drugs, and each susceptible person need a unit of vaccine.
5) Only if one epidemic area can meet the demand of itself, than it can deliver vaccine and drugs to other epidemic area

3.5.3.3 The Foundation of Model

- **Distribution Strategy of Drugs**

Step 1: Estimate the total quantity of drugs in demand. On 8th February, 2015, we assume it is the beginning of the 49th week in Improved Model 2, so we only need to forecast the statistics of infected people in Sierra Leone the 50th week and later on. Table 1 shows the forecast through Improved Model 2 with parameters above. Then we can use these data as the total drugs are to distribute.

Step 2: Estimate the demand of each epidemic area. Change the parameters of λ, μ, I_0, S_0 according to the situation of each epidemic area and repeat using Improved Model 2. Then we will get forecast infected people in each area and that is the demand of the area. Our team cannot get enough data of each area, so we cannot calculate further.

Step 3: Drugs distribution strategy. First, treatment center distributes drugs to each epidemic area every week. They distribute drugs according to the number of infected people in each epidemic area. For example, if there are 100 infected people in area A_1 in the next week, 10 infected people in areaA_2, then we distribute 100 units of drugs to A_1 while 10 to A_2 this week. During the period of each week, if area A_i is short of drugs while other areas have surplus drugs, then A_i can choose get drugs from treatment center or other areas. On this occasion, we can refer to Model 3 to get the optimal strategy to get drugs.

- **Distribution Strategy of Vaccine**

Assume P_i as the density of population of A_i (the code of epidemic cities (i=1, 2, 3 …14)) (constant)

According to the Improved Model Ⅱ, we can predict the number of reduced susceptible people (also called the number of people who would be infected) the next week. To prevent the part of people from being infected, the vaccine can be used. Due to the limitation of the level of production in early stage, the supply is supposed to be less than demand, during the time the total number of the vaccine distributed to Sierra Leone equals to the total output of the world medical association.

With the development of the technology and equipment, the supply would increase. With time going by, the supply would meet the demand. From then on, to avoid wasting, the total output should be made according to the demand.

So at the later stage, the number of the vaccine (later stage) is assumed to be linear with the number of reduced susceptible people next week which would

be predicted according to the model. Considering the actual situation, we set the linear relationship between the number N of reduced susceptible people every week and the total number V of the vaccine set to Sierra Leone:

$$V=1.3N$$

So we have the forecast of the total number of the vaccine set to Sierra Leone every week in the future in Table 3.

After the vaccine set to the treatment center in Sierra Leone, the treatment center would estimate the different demand for drugs of 14 cities. Then make the distribution of drugs. Considering that the vaccine distribution not only has contact with the number of reduced susceptible people every week, but also depends on the density of population of 14 cities, we make the assumption that the weight of each city equals to pi*the number of the patient of each city which can be predicted imitating the prediction of the total number of the patient in Sierra Leone (according to every city's specific situation, model 3 would be applied to each city).

3.5.3.4 Solution and Result

When effective drugs are applied into the treatment of Ebola, the parameters of Improved Model Ⅱ will change. Assume the parameters are as follows:

λ =-0.009, μ =0.006, $I_0 = 0.000015$, $S_0 = 0.999985$

According to the calculation of Improved Model Ⅱ, we can get these results:

Table 3

week	50	51	52	53	54	55	56	57	58	59
I(t)	0.000926	0.000982	0.001041	0.001102	0.001166	0.001232	0.001301	0.001373	0.001447	0.001524
drugs	5638.602	5980.871	6338.282	6711.111	7099.612	7504.011	7924.508	8361.276	8814.463	9284.19
vaccine	627.9347	659.8061	692.6611	726.4838	761.2582	796.9683	833.5982	871.1318	909.5531	948.8464
week	60	61	62	63	64	65	66	67	68	69
I(t)	0.001604	0.001687	0.001772	0.00186	0.001951	0.002045	0.002141	0.00224	0.002342	0.002446
drugs	9770.553	10273.62	10793.44	11330.01	11883.35	12453.38	13039.99	13643.04	14262.33	14897.66
vaccine	988.9954	1029.984	1071.797	1114.418	1157.778	1201.721	1246.342	1291.591	1337.403	1383.718
week	70	71	72	73	74	75	76	77	78	79
I(t)	0.002553	0.002663	0.002775	0.002889	0.003006	0.003124	0.003246	0.003369	0.003494	0.003621
drugs	15548.74	16215.26	16896.88	17593.19	18303.75	19028.07	19765.62	20515.83	21278.09	22051.73
vaccine	1430.471	1477.599	1525.041	1572.733	1620.612	1668.616	1716.681	1764.745	1812.745	1860.617
week	80	81	82	83	84	85	86	87	88	89
I(t)	0.00375	0.00388	0.004012	0.004145	0.00428	0.004415	0.004552	0.004689	0.004827	0.004965
drugs	22836.05	23630.3	24433.69	25245.38	26064.51	26890.19	27721.61	28557.72	29397.46	30239.77
vaccine	1908.3	1955.73	2002.845	2049.581	2095.877	2141.615	2186.668	2230.971	2274.464	2317.088
week	90	91	92	93	94	95	96	97	98	99
I(t)	0.005104	0.005243	0.005381	0.005519	0.005657	0.005794	0.00593	0.006065	0.006199	0.006332
drugs	31083.56	31927.75	32771.23	33612.89	34451.6	35286.24	36115.66	36938.71	37754.21	38561.01
vaccine	2358.782	2399.486	2439.14	2477.683	2515.056	2551.198	2586.049	2619.548	2651.636	2682.252
week	100	101	102	103	104	105	106	107	108	109
I(t)	0.006463	0.006592	0.006719	0.006844	0.006966	0.007086	0.007203	0.007317	0.007428	0.007535

drugs	39357.9	40143.69	40917.18	41677.15	42422.37	43151.68	43864.09	44558.45	45233.63	45888.53
vaccine	2711.336	2738.828	2764.667	2788.794	2811.231	2832.096	2851.108	2868.248	2883.518	2896.919
week	110	111	112	113	114	115	116	117	118	119
I(t)	0.007639	0.007739	0.007836	0.007929	0.008017	0.008101	0.008181	0.008256	0.008327	0.008393
drugs	46522.12	47133.39	47721.37	48285.17	48823.9	49336.75	49822.93	50281.7	50712.38	51114.3
vaccine	2908.454	2918.125	2925.933	2931.882	2935.971	2938.205	2938.584	2937.111	2933.788	2928.616
week	120	121	122	123	124	125	126	127	128	129
I(t)	0.008454	0.008511	0.008562	0.008608	0.008649	0.008685	0.008716	0.008741	0.008761	0.008776
drugs	51486.86	51829.51	52141.72	52423.02	52672.97	52891.22	53077.49	53231.62	53353.46	53442.93
vaccine	2921.598	2912.736	2902.032	2889.487	2875.175	2859.308	2841.758	2822.561	2801.77	2779.439
week	130	131	132	133	134	135	136	137	138	139
I(t)	0.008785	0.008789	0.008788	0.008781	0.008769	0.008752	0.00873	0.008703	0.008671	0.008633
drugs	53499.99	53524.67	53517.04	53477.23	53405.41	53301.81	53166.71	53000.46	52803.43	52576.07
vaccine	2755.621	2730.37	2703.739	2675.78	2646.547	2616.094	2584.474	2551.74	2517.945	2483.142
week	140	141	142	143	144	145	146	147	148	149
I(t)	0.008591	0.008544	0.008492	0.008436	0.008375	0.00831	0.00824	0.008167	0.008089	0.008007
drugs	52318.86	52032.36	51717.14	51373.88	51003.25	50605.98	50182.72	49734.32	49261.62	48765.51
vaccine	2447.386	2410.728	2373.223	2334.924	2295.874	2256.157	2215.903	2175.167	2133.998	2092.445

Table 4

This is a table of the vaccine distribution proportion (for the first week):

Sierra Leone							
City name	WESTERN RURAL	TONKOLILI	PUJEHUN	PORT LOKO	MOYAMBA	KONO	KOINADUGU
Code	A1	A2	A3	A4	A5	A6	A7
proportion	1130P1	449P2	25P3	1366P4	268P5	336P6	147P7
City name	BO	BOMBALI	BONTHE	FREETOWN	KAILAHHUN	KAMBIA	KENEMA
Code	A8	A9	A10	A11	A12	A13	A14
proportion	327P1	971P9	P10	1903P11	650P12	156P13	530P14

3.5.3.5 Strength and Weakness

- **Strength**: In the general direction the model has predicted the total country demand for drugs and the vaccine and made a proportion estimation of the demand of each city in the next weeks. The model combines with the improved model 2 effectively and gives a practical distribution way that is based on the demand. On the hand of details, the model gives a more practical way of combining the vertical transport with the lateral transport to save time and cost, having certain guidance meaning to reality. The assumption that one week is a cycle is reasonable. Medical help is timely and it is good for adjusting distribution strategy according to the changing actual situation. The delivery model is practical and reasonable.
- **Weakness**: Due to the lack of data, we can't predict accurately the

demand of each city In the next week. And after drugs used, the situation would change. The model can't predict the concrete effect. That's what we need to improve.

IV. Conclusions

4.1 Spread Model:

To estimate the developments of EVA in the next 200 weeks (from 2014/3/1), we use differential equations to establish the Spread Model. The basic model is model 1, in this model we suppose that EVA spreads in the absence of any additional medication, which means λ (The number of susceptible people infected by one infective person every week) and μ (the proportion of dead and cured people in patients every week) are constant. The two improved models take the function of the drugs and the vaccine into consideration. In the improved models, the amounts of vaccines used on the susceptible people are related to λ, the amounts of drugs used on the infective people have relationship with μ, both λ and μ change with time. In improved mode Ⅰ, we only consider the influence of the vaccines and in improved model Ⅱ, we take the influence of both the drugs and the vaccines into consideration). As a result, the two improved models are more realistic.

4.2 Production model:

Due to the restrictions with technology, supply is supposed to be less than demand at the beginning. In such situation, the number of the vaccines sent to the epidemic areas in one week equals to the number of one-week output of the medication. With the advance of technology, supply will first grow with high-increasing speed and then grows with low-increasing speed. Finally, the speed will become stable as a result of more and more constraints. When supply meets demand, though more medication is able to be produced, supply will change with the demand in the future in order to avoid waste.

4.3 Delivery model:

Our delivery model consists of vertical transport（drugs distribute from the treatment center to epidemic areas）and lateral transport(drugs distribute from one epidemic area to another). We take Sierra Leone as an example, making assumptions that one week is a vertical transport cycle and every country has one treatment center. During every vertical transport cycle, we use lateral transport. Firstly, we estimate the weight of each city in country according to the infective and susceptible people in these cities at present so that we can

determine the suitable position of the treatment center. Secondly, we make a table based on the improved EVA spread model 2 to predict the total number of the drugs and the vaccines needed to be distributed from the treatment center in the next week. Thirdly, we predict the proportion of medication needed to be distributed to each city based on the improved EVA spread model 2. There are two objectives in our delivery model: To minimize the total cost of the transport and to minimize the distribution time.

V. Future work

- The Spread Model using actual data to simulate future trend provides a practical way of thinking which could be applied to many fields such as finance, administration etc. The Spread Model needs to be improved because the relationships between λ and t, μ and t are so sophisticated that they cannot always remain unchanged. In fact, the relationships between λ and t, μ and t will change with time goes by. Consequently, we will spare much more effort to determine the concrete relationships between λ and t, μ and t in the future.
- The Delivery Model has only considered the situation that there is one treatment center in one country. But in fact, there can be no or more than one treatment center in one country. In the future we would improve our model to make it be able to apply to more situations.

VI. The Non-technical Letter

WMA Announcement on Ebola Treatment

9 February 2015

Ladies and gentlemen:

The outbreak of Ebola Virus Disease (EVD) in West Africa is a crisis, which is unprecedented in its size, severity, and complexity. Up to 1 February 2015, a total of 22,495 people have been reported confirmed, probable, and suspected cases of EVD in Guinea, Liberia and Sierra Leone, with 8,981 reported deaths.

Ebola spreads through contact with the blood or other fluids of infected people. This virus causes high body temperature and bleeding. Victims may experience vomiting -- the involuntary emptying of the stomach. They may also have diarrhea, the uncontrolled expulsion of body wastes.

According to the research of WMA, the spread of EVA is still in the primary stage, the number of cases in one day may still increase for some time. But fortunately, after almost one-year hard work of WMA, we have developed new medication which could

stop Ebola and cure patients whose disease is not advanced. It marks the turn of this crisis.

Though such medication cannot be mass produced at present, large scale of production can be realized in the near future with the high-increasing speed of development in our production technology. According to the epidemic situation, WMA has decided to take most serious epidemic countries for priority in the distribution of vaccine and drug, including Guinea, Sierra Leone, Liberia, etc. WMA will build new Ebola treatment centers in these countries and formulate a reasonable plan of medication distribution to have the epidemic under control. What's more, the situation of Ebola in all epidemic areas will be closely monitored by WMA to ensure the epidemic would not break out in large scale again.

In particular, what we would like to remind everyone are as follows:

- Do not contact with people who are sick or dead due to EVD.
- Do not touch or eat the meat of primates or bats because EVD can spread through them too.
- Keep your hands clean with soap and water and wash more often.
- If you get sick and have symptoms like fever and vomit, call the treatment center tell them about your symptoms and obey the arrangement of it.

Now, WMA has set the goal of having 90 percent of Ebola patients under treatment, and 95 percent of victims safely buried half year later. We are confident of realizing this goal if all of us can pull together and take our responsibilities.

Thank you!

VII. References

[1] Yang Guan. The Analysis of Infectious Disease Control on the basis of SIR Model.Mathematical Biology Journal.2009, 24 (3): 479-483

[2] Wang Xinping , The Distribution of Emergency Materials under the Law of Infectious Disease Spread. Journal of Southeast University, Volume 36