Research and Implementation of Grammar Transformation and Parser Generators

Yihan Zhang

Northeastern University

zhng1573@hotmail.com

May 21, 2016

Overview

Notations and preliminaries

Definition

A context-free grammar[Cho56] G is a quadruple $G = (V, T, P, S)$, where V is nonterminal(variable) set, T is terminal set, P is production set with productions shaped like $A \to \alpha, A \in V, \alpha \in (V \cup T)^*$, S is start variable.

Iteration to a fixed point

In this frame, we will introduce a technique called "iteration to a fixed point". Take computing the generating symbols as an example. A symbol is said to be generating if $A \stackrel{*}{\Rightarrow} \omega, \omega \in T^*$. In most textbooks, generating symbols are generally computed by the following rules[HMU79]:

- All nonterminals are generating;
- If $A \to \alpha \in P$, $\alpha = \varepsilon$ or every symbol in α is generating. Then A is generating.

Given any system of set equations, it's easy to get corresponding algorithm using the technique iteration to a fixed point. Algorithm for the last example is:

Algorithm 1 Compute generating variables

Require: $G = (V, T, P, S)$

Ensure: $V_{old} = \emptyset, V_{new} = \{A : A \to \omega \in P, \omega \in T^*\}$

```
while V_old ≠ V_new do
    V_old = V_new
    V_new = V_old ∪ {A : A → α ∈ P, α ∈ (T ∪ V_old)*}
end while
V = V_new
```

Original CYK algorithm

Algorithm 2 Original CYK

Require: $G = (V, T, P, S)$ in CNF, $a_1 a_2 \cdots a_n \in T^n$
Ensure:

```
for i = 0 to n − 1 do
    T_{i0} = {A : A → a_i ∈ P}
end for
for j = 1 to n − 1 do
    for i = 0 to n − j − 1 do
        T_{ij} = ∅
        for k = 0 to j − 1 do
            T_{ij} = T_{ij} ∪ {A : A → BC ∈ P, B ∈ T_{ik}, C ∈ T_{(i+k)(j−k)}}
        end for
    end for
end for
```

Remark

CYK algorithm[You76][Kas65] requires the input grammar to be in CNF, whose productions are all in the form $A \to BC, B, C \in V$. *Definition of* T_{ij} *in the algorithm is* $T_{ij} = \{A : A \overset{*}{\Rightarrow} a_i a_{i+1} \cdots a_{i+j-2} a_{i+j-1}\}$. *After running the algorithm, if* $S \in T_{1n}$, *then* $a_1 a_2 \cdots a_n \in L(G)$.

Improved CYK algorithm

Algorithm 3 Improved CYK

Require: $G = (V, T, P, S)$ in CNF, $a_1 a_2 \cdots a_n \in T^n$

Ensure: $M =$ all **false**, $M[0, i, j] =$**true**, $i = 0, 1, \cdots, n$, $V_j \rightarrow a_i \in P$

for $i = 1$ **to** $n - 1$ **do**
    for $j = 0$ **to** $n - i - 1$ **do**
        for $k = 0$ **to** $i - 1$ **do**
            for all V_A**to**$V_B V_C$ **do**
                if $M[k, j, B], M[i - k - 1, j + k + 1, C]$ **then**
                    $M[i, j, A] =$**true**
                end if
            end for
        end for
    end for
end for

Remark

After running the improved version, if $M[n-1, 0, S]$*, then* $V_S \in L(G)$*. In fact,* $V_k \in T[i,j] \iff M[i,j,k]$*. Considering* V *is a finite set, we use a 3-dimensional boolean matrix* M *to represent whether a variable is in an entry of the original 2-dimensional variable-set matrix* T*. This improvement avoids union-find operation of sets which may have a high-complexity implementation and makes the algorithm more practical.*

Original Earley algorithm

Given any augmented CFG(with additional variable S' and production $S' \to S$) G and $a_1 a_2 \cdots a_n \in T^n$, Earley algorithm[Ear70] maintains a list of $n+1$ sets and can be divided into two stages:

1. Initialization: $S_0 = \{(S' \to \cdot S, 0)\}$;
2. Assuming the current position is in the k-th slot of input string, perform the following operation until a fixed point:
   - Predict S_k;
   - Scan S_k;
   - Complete S_k.

Improved Earley algorithm

Algorithm 4 Earley

Ensure: $S_0 = \{(S' \to \cdot S, 0)\}$

for $k = 0$ **to** n **do**
    for all $s \in S_k$ **do**
        if $\cdot$ is at the end **then**
            if nonterminal after $\cdot$ **then**
                Predict(G, k, s)
            else
                Scan$(a_1 a_2 \cdots a_n, k, s)$
            end if
        else
            Complete(k, s)
        end if
    end for
end for

Remark

The improved version modifies three operations to make them act on a single state rather than a state set and puts them into a wider loop, reducing repeated search.

System Design

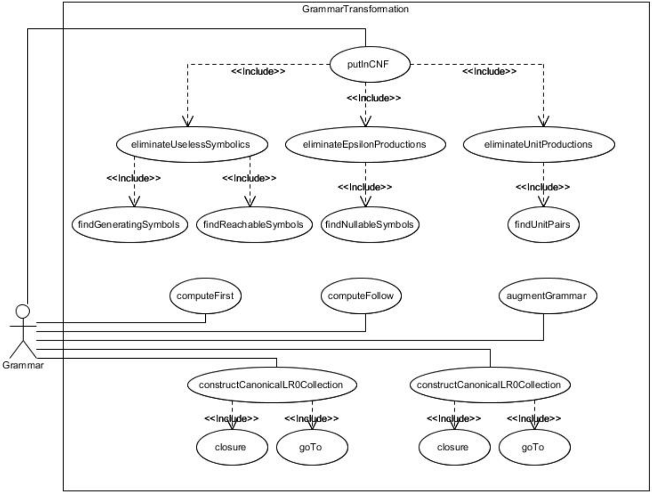


Figure: Use case diagram for grammar tramsformer

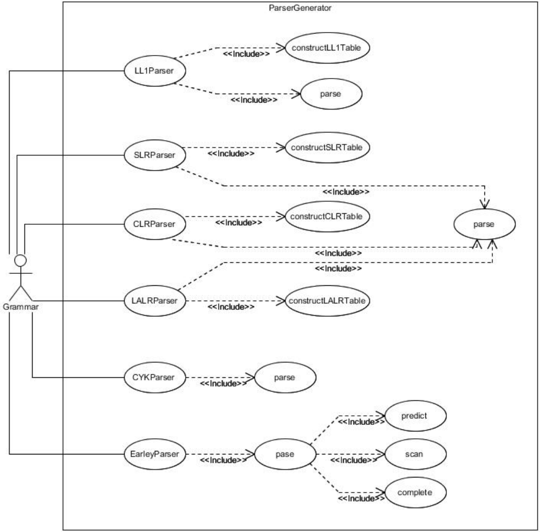


Figure: Use case diagram for parser generators

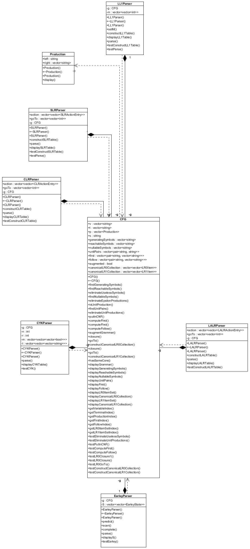


Figure: Class diagram

Grammar transformer(simplify any given CFG and compute relevant sets automatically):

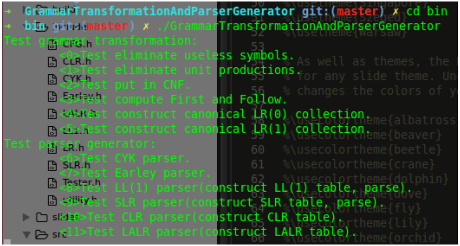
- Eliminate useless symbols;
- Eliminate ε-productions;
- Eliminate unit production;
- Put in CNF.

Parser generators(generate parse table for any given CFG automatically):

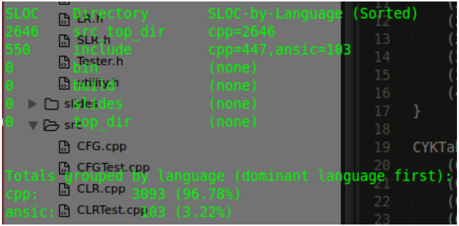
- Earley
- LL(1)
- SLR
- CLR
- LALR

System Implementation

- Hardware:ThinkPad-Edge-E530, 3.5GiB memory, Intel Core i5-3210M CPU, 2.50GHzx4, Intel Ivybridge Mobile;
- Operating system: Ubuntu 14.04 LTS, 64-bit;
- Tools: C++1y, Sublime Text 3, makefile;
- Environment requirement: g++ (Ubuntu 4.8.4-2ubuntu1 14.04.1), Copyright (C) 2013 Free Software Foundation, Inc.



(a) User interface



(b) 3000+ lines codes

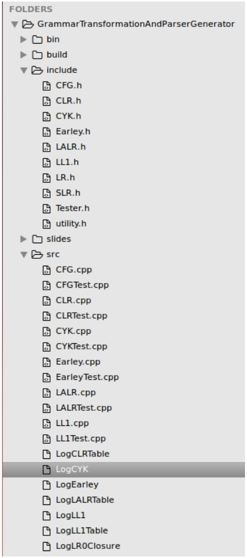


Figure: Directory structure

System Testing

```
CYKTable = {
      (0, 0) { B, },
      (0, 1) { A, C, },
      (0, 2) { A, C, },
      (0, 3) { B, },
      (0, 4) { A, C, },
      (1, 0) { S, A, },
      (1, 1) { B, },
      (1, 2) { S, C, },
      (1, 3) { S, A, },
      (2, 0) { },
      (2, 1) { B, },
      (2, 2) { B, },
      (3, 0) { },
      (3, 1) { S, A, C, },
      (4, 0) { S, A, C, },
}

CYKTable = {
      (0, 0) { NP, },
      (0, 1) { VP, V, },
      (0, 2) { Det, },
      (0, 3) { N, },
      (0, 4) { P, },
      (0, 5) { Det, },
      (0, 6) { N, },
      (1, 0) { S, },
      (1, 1) { },
      (1, 2) { NP, },
      (1, 3) { },
      (1, 4) { },
      (1, 5) { NP, },
      (2, 0) { },
      (2, 1) { VP, },
      (2, 2) { },
      (2, 3) { },
      (2, 4) { PP, },
      (3, 0) { S, },
      (3, 1) { },
      (3, 2) { },
```

(a) CYK log

```
S = {
      (0, 0) (_S -> . S , 0)
      (0, 1) (S -> . S + M , 0)
      (0, 2) (S -> . M , 0)
      (0, 3) (M -> . M * T , 0)
      (0, 4) (M -> . T , 0)
      (0, 5) (T -> . 1 , 0)
      (0, 6) (T -> . 2 , 0)
      (0, 7) (T -> . 3 , 0)
      (0, 8) (T -> . 4 , 0)
      (1, 0) (T -> 2 . , 0)
      (1, 1) (M -> T . , 0)
      (1, 2) (S -> M . , 0)
      (1, 3) (M -> M . * T , 0)
      (1, 4) (_S -> S . , 0)
      (1, 5) (S -> S . + M , 0)
      (2, 0) (S -> S + . M , 0)
      (2, 1) (M -> . M * T , 2)
      (2, 2) (M -> . T , 2)
      (2, 3) (T -> . 1 , 2)
      (2, 4) (T -> . 2 , 2)
      (2, 5) (T -> . 3 , 2)
      (2, 6) (T -> . 4 , 2)
      (3, 0) (T -> 3 . , 2)
      (3, 1) (M -> T . , 2)
      (3, 2) (S -> S + M . , 0)
      (3, 3) (M -> M . * T , 2)
      (3, 4) (_S -> S . , 0)
      (3, 5) (S -> S . + M , 0)
      (4, 0) (M -> M * . T , 2)
      (4, 1) (T -> . 1 , 4)
      (4, 2) (T -> . 2 , 4)
      (4, 3) (T -> . 3 , 4)
      (4, 4) (T -> . 4 , 4)
      (5, 0) (T -> 4 . , 4)
      (5, 1) (M -> M * T . , 2)
      (5, 2) (S -> S + M . , 0)
      (5, 3) (M -> M . * T , 2)
      (5, 4) (_S -> S . , 0)
      (5, 5) (S -> S . + M , 0)
```

(b) Earley log

```
LL1Table = {
    (E, +) error,
    (E, *) error,
    (E, () E -> T E_ ,
    (E, )) error,
    (E, 0) E -> T E_ ,
    (E, $) error,
    (T, +) error,
    (T, *) error,
    (T, () T -> F T_ ,
    (T, )) error,
    (T, 0) T -> F T_ ,
    (T, $) error,
    (E_, +) E_ -> + T E_ ,
    (E_, *) error,
    (E_, () error,
    (E_, )) E_ ->  ,
    (E_, 0) error,
    (E_, $) E_ ->  ,
    (F, +) error,
    (F, *) error,
    (F, () F -> ( E ) ,
    (F, )) error,
    (F, 0) F -> 0 ,
    (F, $) error,
    (T_, +) T_ ->  ,
    (T_, *) T_ -> * F T_ ,
    (T_, () error,
    (T_, )) T_ ->  ,
    (T_, 0) error,
    (T_, $) T_ ->  ,
}
```

(a) LL(1) table log

```
stk: { E, $, }
x: E
a: 0
E -> T E_

stk: { T, E_, $, }
x: T
a: 0
T -> F T_

stk: { F, T_, E_, $, }
x: F
a: 0
F -> 0

stk: { 0, T_, E_, $, }
x: 0
a: 0
match 0

stk: { T_, E_, $, }
x: T_
a: +
T_ ->

stk: { E_, $, }
x: E_
a: +
E_ -> + T E_

stk: { +, T, E_, $, }
x: +
a: +
match +

stk: { T, E_, $, }
x: T
a: 0
T -> F T_
```

(b) LL(1) parsing log

```
canonicalLR0Collection = {
    I0 = {
        _E -> . E ,
        E -> . E + T ,
        E -> . T ,
        T -> . T * F ,
        T -> . F ,
        F -> . ( E ) ,
        F -> . 0 ,
    }

    I1 = {
        _E -> E . ,
        E -> E . + T ,
    }

    I2 = {
        E -> T . ,
        T -> T . * F ,
    }

    I3 = {
        T -> F . ,
    }

    I4 = {
        F -> ( . E ) ,
        E -> . E + T ,
        E -> . T ,
        T -> . T * F ,
        T -> . F ,
        F -> . ( E ) ,
        F -> . 0 ,
    }

    I5 = {
        F -> 0 . ,
    }

    I6 = {
```

(a) LR(0) collection log

```
action = {
    (0, +) error,
    (0, *) error,
    (0, () shift 4,
    (0, )) error,
    (0, 0) shift 5,
    (0, $) error,
    (1, +) shift 6,
    (1, *) error,
    (1, () error,
    (1, )) error,
    (1, 0) error,
    (1, $) accept,
    (2, +) reduce E -> T ,
    (2, *) shift 7,
    (2, () error,
    (2, )) reduce E -> T ,
    (2, 0) error,
    (2, $) reduce E -> T ,
    (3, +) reduce T -> F ,
    (3, *) reduce T -> F ,
    (3, () error,
    (3, )) reduce T -> F ,
    (3, 0) error,
    (3, $) reduce T -> F ,
    (4, +) error,
    (4, *) error,
    (4, () shift 4,
    (4, )) error,
    (4, 0) shift 5,
    (4, $) error,
    (5, +) reduce F -> 0 ,
    (5, *) reduce F -> 0 ,
    (5, () error,
    (5, )) reduce F -> 0 ,
    (5, 0) error,
    (5, $) reduce F -> 0 ,
    (6, +) error,
    (6, *) error,
    (6, () shift 4,
```

(b) SLR table log

```
canonicalLR1Collection = {
    I0 = {
        _S -> . S , $,
        S -> . C C , $,
        C -> . c C , c,
        C -> . c C , d,
        C -> . d , c,
        C -> . d , d,
    }

    I1 = {
        _S -> S . , $,
    }

    I2 = {
        S -> C . C , $,
        C -> . c C , $,
        C -> . d , $,
    }

    I3 = {
        C -> c . C , c,
        C -> c . C , d,
        C -> . c C , c,
        C -> . d , c,
        C -> . c C , d,
        C -> . d , d,
    }

    I4 = {
        C -> d . , c,
        C -> d . , d,
    }

    I5 = {
        S -> C C . , $,
    }

    I6 = {
        C -> c . C , $,
```

(a) LR(1) collection log

```
action = {
    (0, c) shift 3,
    (0, d) shift 4,
    (0, $) error,
    (1, c) error,
    (1, d) error,
    (1, $) accept,
    (2, c) shift 6,
    (2, d) shift 7,
    (2, $) error,
    (3, c) shift 3,
    (3, d) shift 4,
    (3, $) error,
    (4, c) reduce C -> d ,
    (4, d) reduce C -> d ,
    (4, $) error,
    (5, c) error,
    (5, d) error,
    (5, $) reduce S -> C C ,
    (6, c) shift 6,
    (6, d) shift 7,
    (6, $) error,
    (7, c) error,
    (7, d) error,
    (7, $) reduce C -> d ,
    (8, c) reduce C -> c C ,
    (8, d) reduce C -> c C ,
    (8, $) error,
    (9, c) error,
    (9, d) error,
    (9, $) reduce C -> c C ,
}

goTo = {
    (0, S) 1,
    (0, C) 2,
    (1, S) error,
    (1, C) error,
    (2, S) error,
    (2, C) 5,
```

(b) CLR table log

```
action = {
    (0, c) shift 3,
    (0, d) shift 4,
    (0, $) error,
    (1, c) error,
    (1, d) error,
    (1, $) accept,
    (2, c) shift 3,
    (2, d) shift 4,
    (2, $) error,
    (3, c) shift 3,
    (3, d) shift 4,
    (3, $) error,
    (4, c) reduce C -> d ,
    (4, d) reduce C -> d ,
    (4, $) reduce C -> d ,
    (5, c) error,
    (5, d) error,
    (5, $) reduce S -> C C ,
    (6, c) reduce C -> c C ,
    (6, d) reduce C -> c C ,
    (6, $) reduce C -> c C ,
}

goTo = {
    (0, S) 1,
    (0, C) 2,
    (1, S) error,
    (1, C) error,
    (2, S) error,
    (2, C) 5,
    (3, S) error,
    (3, C) 6,
    (4, S) error,
    (4, C) error,
    (5, S) error,
    (5, C) error,
    (6, S) error,
    (6, C) error,
}
```

(c) LALR table log

Conclusions

We design a C-like grammar which supports most basic syntax of C and use (part of) it to test the system. Conclusions are shown below:

- Any grammar which $L(G)\backslash\{\varepsilon\} \neq \emptyset$ can be put in CNF;
- Complexity of CYK and Earley are both $O(n^3)$, while $O(n^2)$ for Earley when handling unambiguous grammars;
- Construction of parser generators is equivalent to construction of parse tables;
- LL(1) parser is equivalent to recursive descent parser without backtrack;
- Range of applicable grammars: $\mathrm{SLR} \subset \mathrm{LALR} \subset \mathrm{CLR}$, scale of generated parse tables: $\mathrm{SLR} = \mathrm{LALR} < \mathrm{CLR}$.

References

John E. Hopcroft, Rajeev Motwani, Jeffery D. Ullman. (1979)
Introduction to Automata Theory, Languages, and Computation

Daniel H. Younger. (1976)
Recognition and Parsing of Context-Free Languages in Time n^3
Information and Control 10(2): 189-208.

T. Kasami. (1965)
An Efficient Recognition and Syntax-Analysis Algorithm for Context-Free Languages

Jay Earley. (1970)
An Efficient Context-Free Parsing Algorithm
Communications of ACM 13(2): 94-102.

Chomsky, N. (1956)
Three models for the description of language
Information Theory 2(3): 113-124.

Q & A