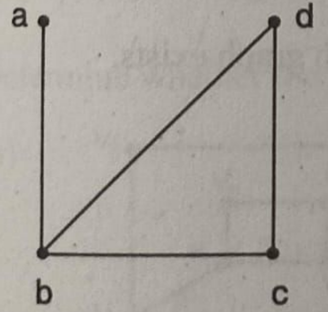
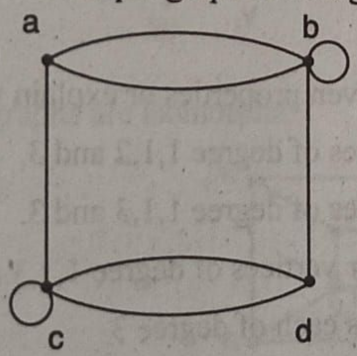
1. Determine whether the graphs shown is a simple graph a multigraph, a pseudograph.

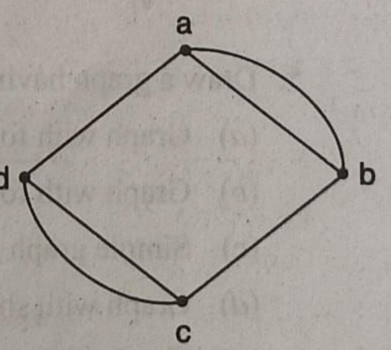
(*a*)



(*b*)

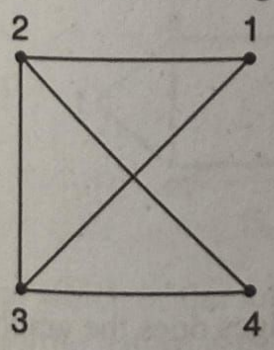


(*c*)

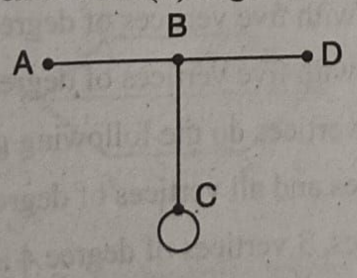


2. Consider the following graphs, determine the (*i*) vertex set and (*ii*) edge set.

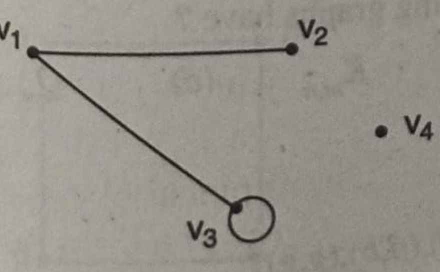
(*a*)



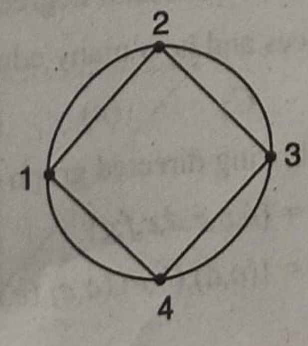
(*b*)



(c)

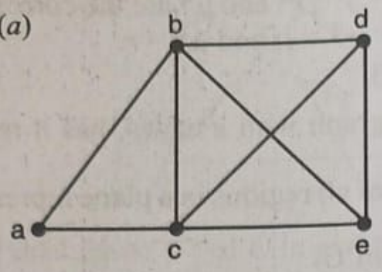


(*d*)

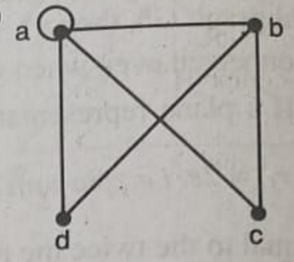


3. Consider the following graphs and determine the degree of each vertex.

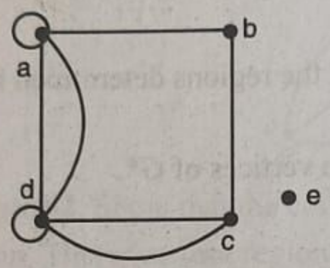
(a)



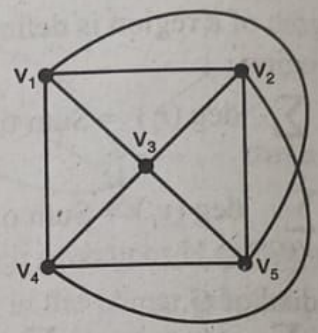
(b)



(c)

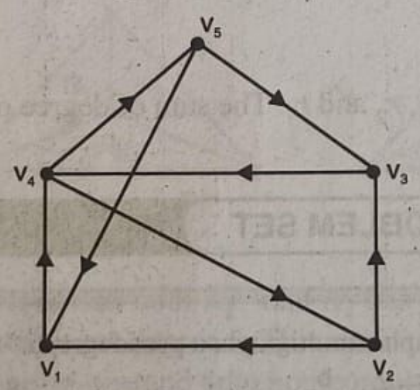


(d)

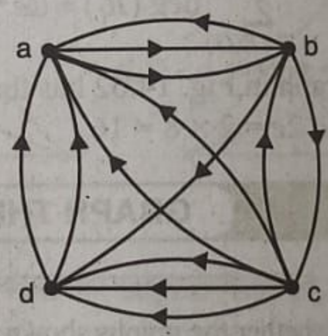


4. Find the in-degree and out-degree of each vertex of the following directed graphs

(a)



(b)



5. Draw a graph having the given properties or explain why no such graph exists.
   (a) Graph with four vertices of degree 1,1,2 and 3.
   (b) Graph with four vertices of degree 1,1,3 and 3.
   (c) Simple graph with four vertices of degree 1,1,3 and 3.
   (d) Graph with six vertices each of degree 3
   (e) Graph with six vertices and four edges.
   (f) Graph with five vertices of degree 3, 3, 3, 3, 2
   (g) Graph with five vertices of degree 0, 1, 2, 2, 3
6. How many vertices do the following graphs have if they contain
   (a) 16 edges and all vertices of degree 2.
   (b) 21 edges, 3 vertices of degree 4 and others each of degree 3.
7. Suppose a graph has vertices of degree 0,2,2,3 and 9. How many edges does the graph have?
8. How many vertices and how many edges do the following graphs have?
   (a) K_n (b) C_n (c) W_n (d) $K_{m,n}$ (d) Q_n
9. Consider the following directed graph G:

$$V(G) = \{a,b,c,d,e,f,g\}$$

$$E(G) = \{(a,a),(b,e),(a,e),(e,b),(g,c),(a,e),(d,f),(d,b),(g,g)\}$$

(i) Indentify any loops or parallel edges

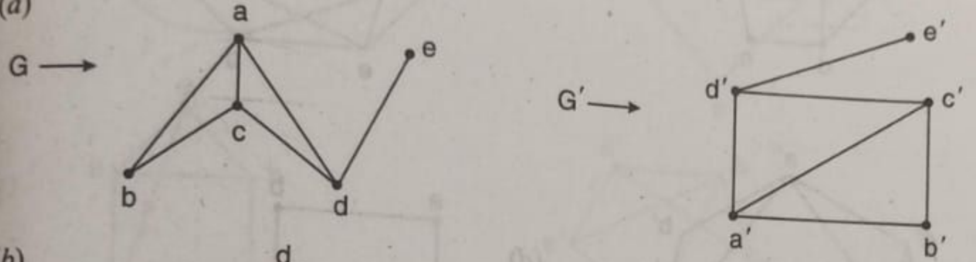
(ii) Are there any sources in G ?

(iii) Are there any sinks in G ?

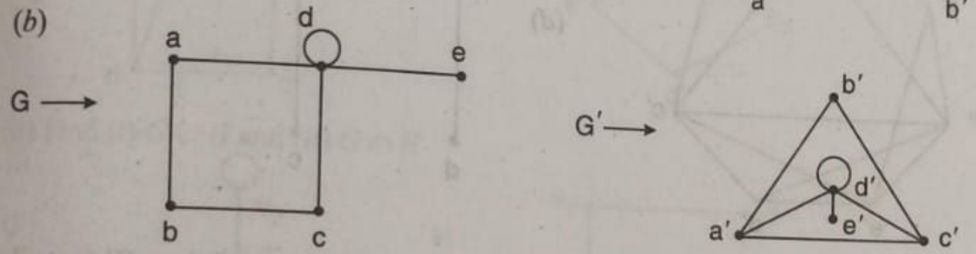
(iv) Find the subgraph H of G determined by the vertex set $V' = \{a, b, c, d\}$

10. Show that the following graphs are isomorphic.

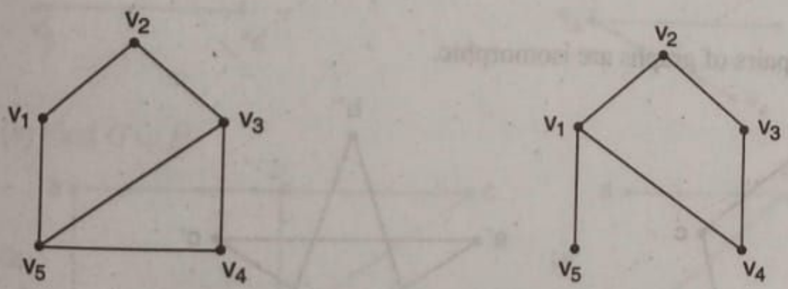
(a)



(b)

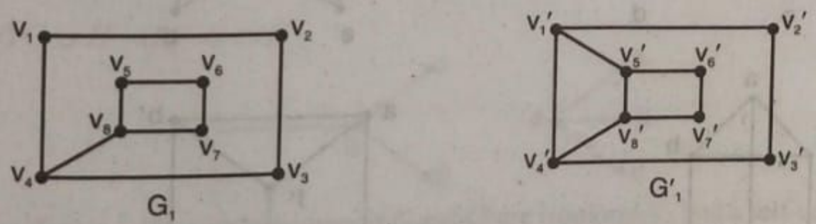


11. Show that graphs are not isomorphic.

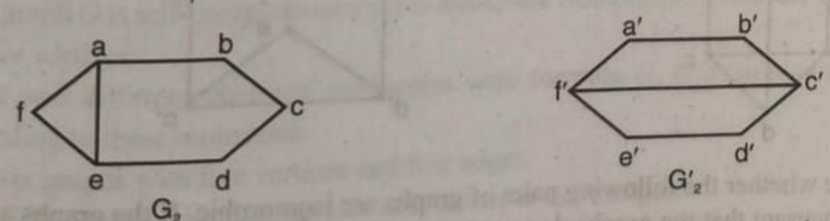


12. Determine whether the following graphs are isomorphic.

(a)

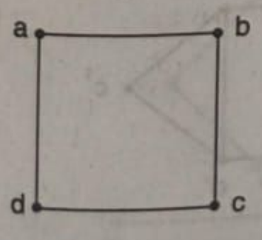


(b)

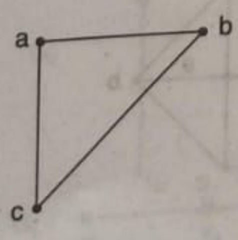


13. Find which of the following graphs are bipartite, redraw the bipartite graphs so that their bipartite nature is evident.

(a)

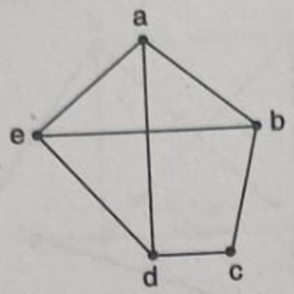


(b)

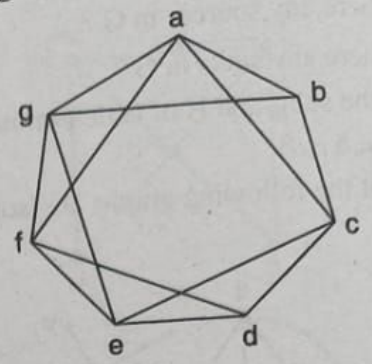


14. Find the complements of each of the following graphs.

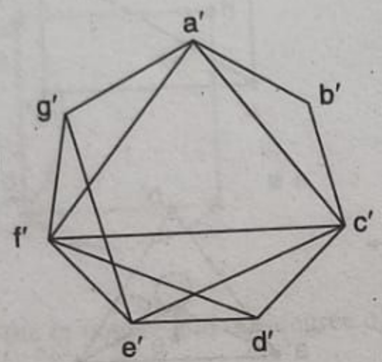
(a)



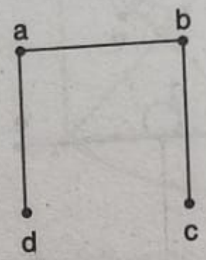
(b)



(c)



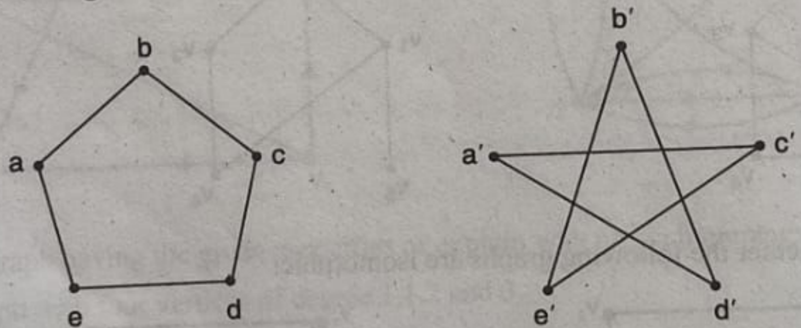
(d)



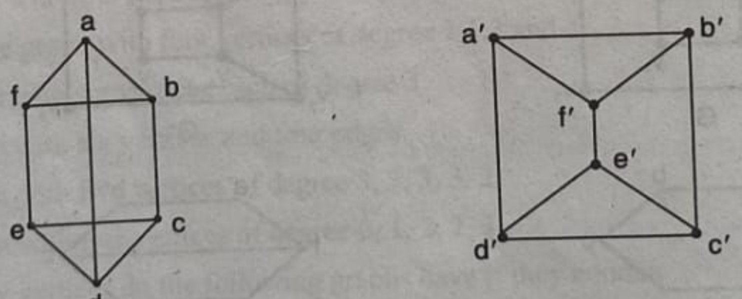
15. If the simple graph G has V vertices and e edges, how many edges does G′ (complement of G) have?

16. Show that the given pairs of graphs are isomorphic.

(a)

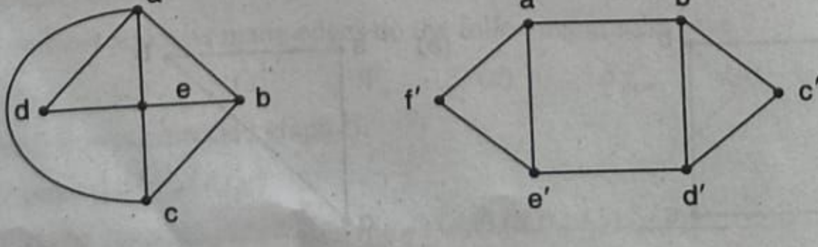


(b)

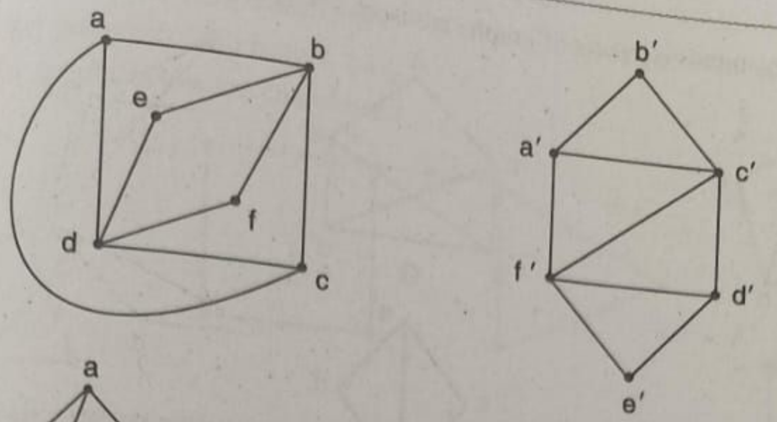


17. Determine whether the following pairs of graphs are isomorphic. If the graphs are not isomorp give an invariant that the graphs do not share.

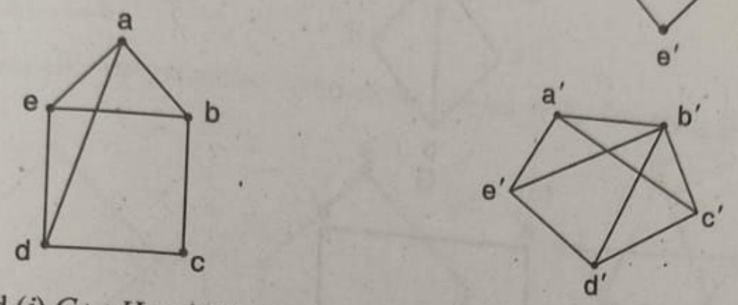
(a)



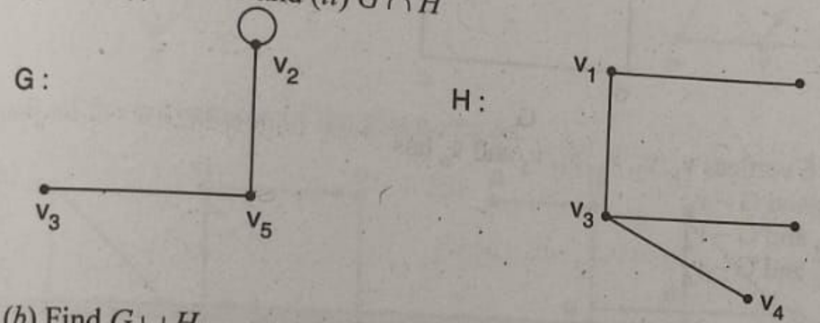
(b)



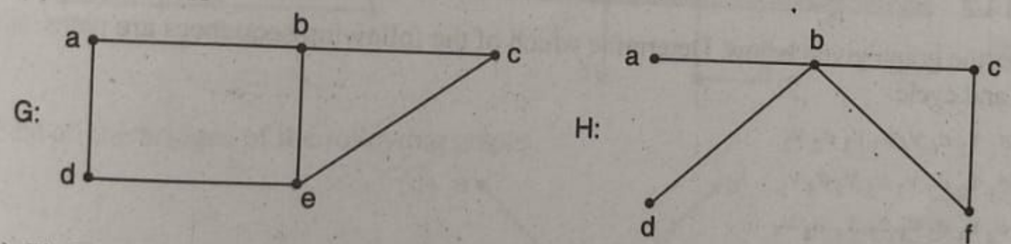
(c)



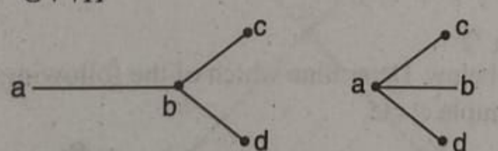
18. (a) Find (i) $G \cup H$ and (ii) $G \cap H$



(b) Find $G \cup H$

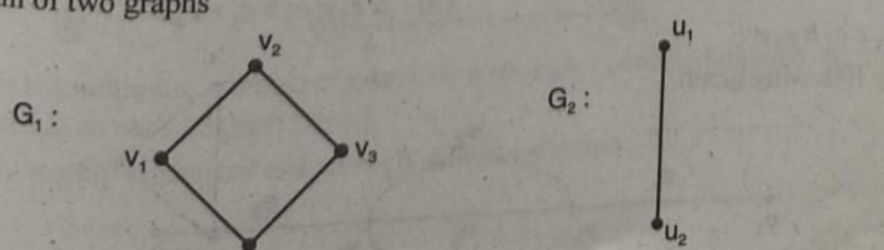


(c) Find (i) $G \cup H$ (ii) $G \cap H$

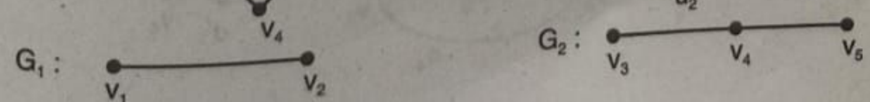


19. A simple graph G is self-complementary if G and G' are isomorphic, find a self-complementary graphs having five vertices.
20. There are two different chemical molecules with formula C_4H_{10} (isobutane). Draw the graphs corresponding to these molecules.
21. Draw all six graphs with five vertices and five edges.
22. Draw all eight graphs with five vertices and seven or more edges.
23. Find sum of two graphs

(a)

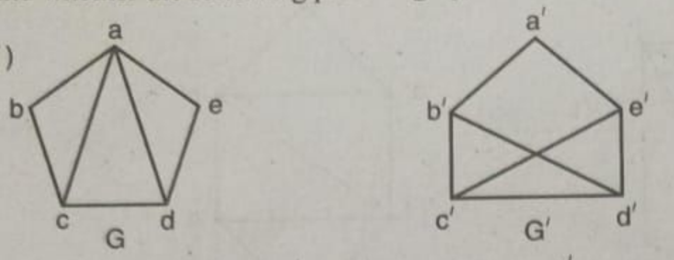


(b)

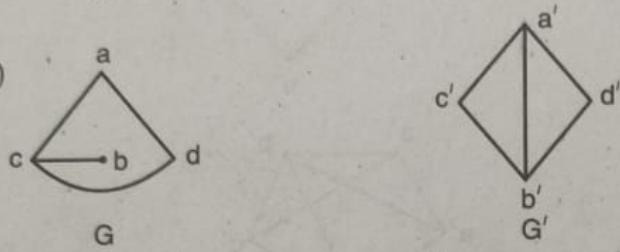


24. Find whether the following pairs of graphs are isomorphic or not.

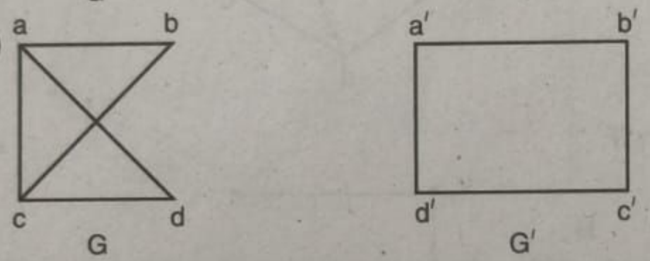
(*i*)



(*ii*)



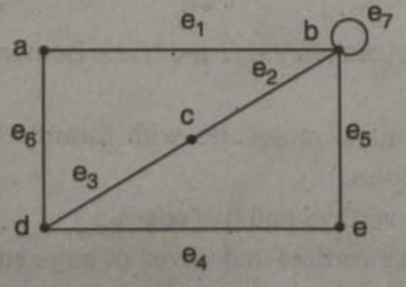
(*iii*)



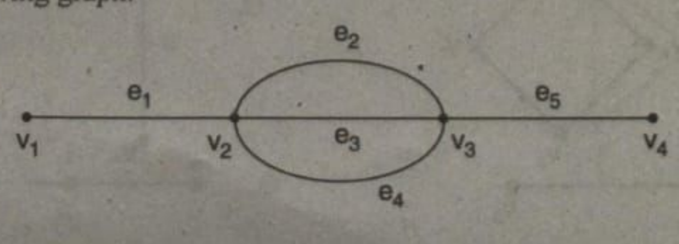
25. A simple graph G on 6 vertices v_1, v_2, v_3, v_4, v_5 and v_6 has
(*i*) 7 edges in $G - v_1$ and $G - v_2$
(*ii*) 6 edges in $G - v_3$ and $G - v_4$
(*iii*) 5 edges in $G - v_5$ and $G - v_6$

Problem Set 14.2

1. Consider the graph given below. Determine which of the following sequences are paths, simple paths, circuit, and cycle.
   (*a*) $v_1 e_1 v_2 e_6 v_4 e_3 v_3 e_2 v_2$
   (*b*) $v_1 e_1 v_2 e_2 v_3 e_3 v_4 e_4 v_5$
   (*c*) $v_1 e_8 v_4 e_3 v_3 e_7 v_1 e_8 v_4$
   (*d*) $v_5 e_5 v_1 e_8 v_4 e_3 v_3 e_2 v_2 e_6 v_4 e_4 v_5$
   (*e*) $v_2 e_2 v_3 e_3 v_4 e_4 e_5 v_1 e_1 v_2$
2. Consider the graph given below. Determine which of the following sequences are paths, simple paths, cycle and simple cycle.



   (*a*) $b\, e_7\, b$ (*b*) $d\, e_3\, c\, e_2\, b\, e_5\, e\, e_4\, d$
   (*c*) $a\, e_6\, d\, e_3\, c\, e_2\, b\, e_5\, e$
3. Consider the following graph.



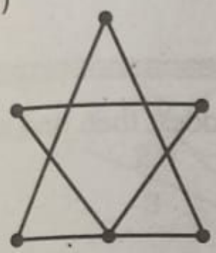
(*a*) How many simple paths are there from v_1 to v_4 ?

(*b*) How many trails are there from v_1 to v_4 ?

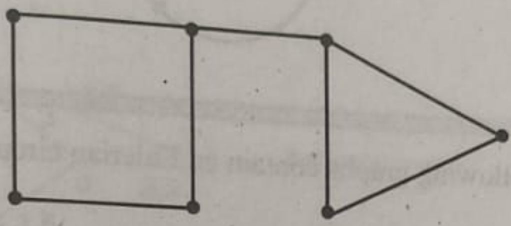
(*c*) How many paths are there from v_1 to v_4 ?

4. Which of the graphs below are connected ?

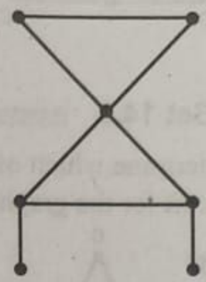
(*a*)



(*b*)

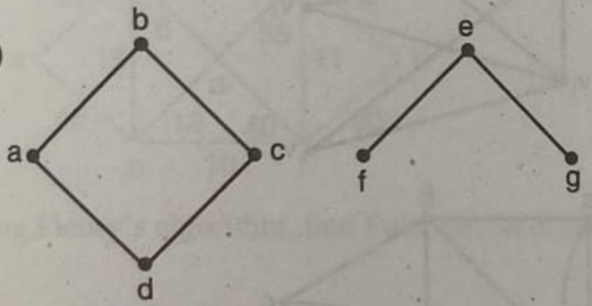


(*c*)

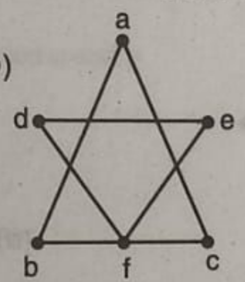


5. Find the number of connected components of each of the following graphs.

(*a*)

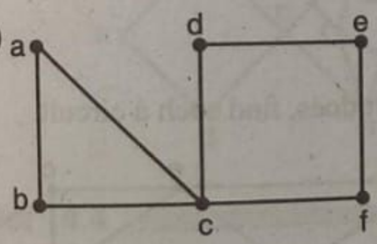


(*b*)

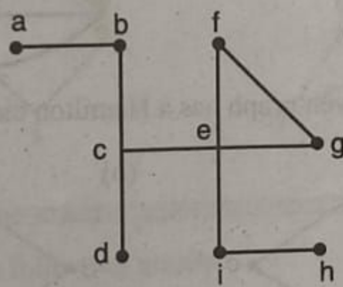


6. Find all the cutvertices of the given graphs.

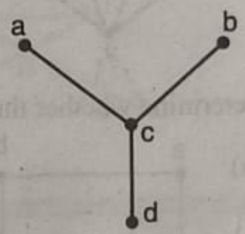
(*a*)



(*b*)

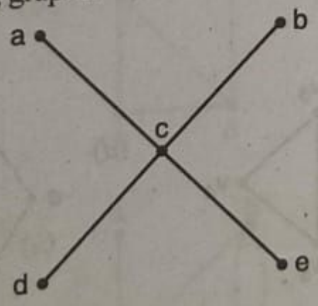


(*c*)



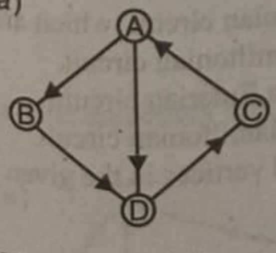
7. Find all the bridges of the following graphs.

(*a*)

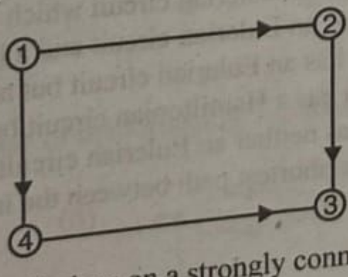


8. Is the following graph strongly connected ?

(*a*)



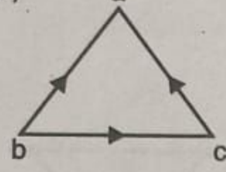
(*b*)



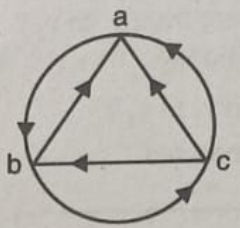
9. What is the minimum number of edges on a strongly connected digraph on *n* vertices? What shape do such adgraph have ?

10. Identify weekly connected and strongly connected graph.

(a)



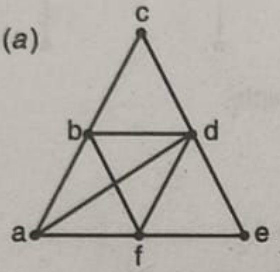
(b)



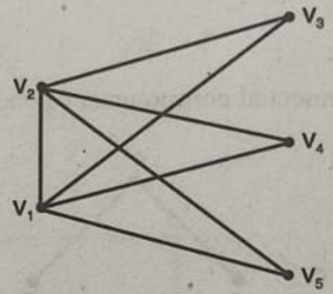
Problem Set 14.3

1. Determine which of the following graphs contain an Eulerian circuit. If it does, then find an Eulerian circuit for the graph.

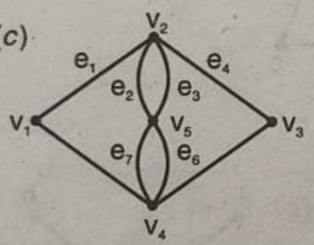
(a)



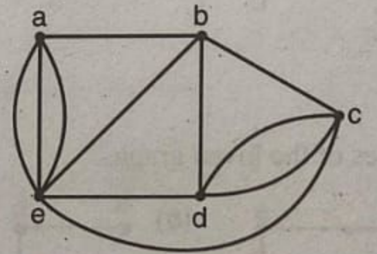
(b)



(c)

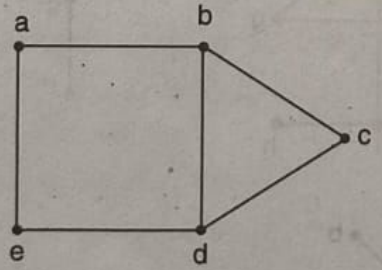


(d)

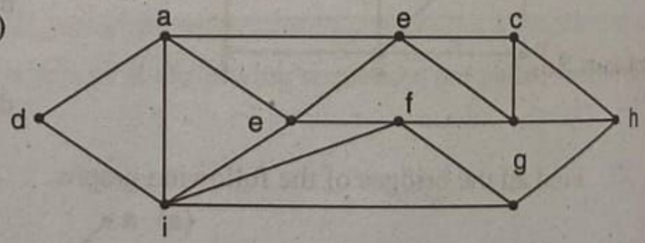


2. Determine whether the given graph has a Hamilton circuit. If it does, find such a circuit.

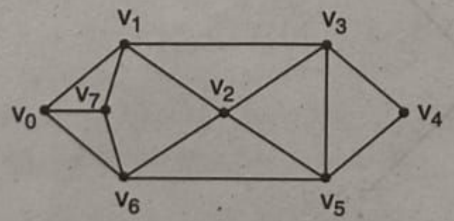
(a)



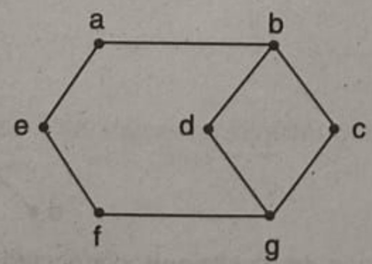
(b)



(c)

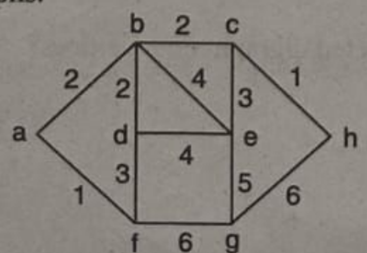


(d)

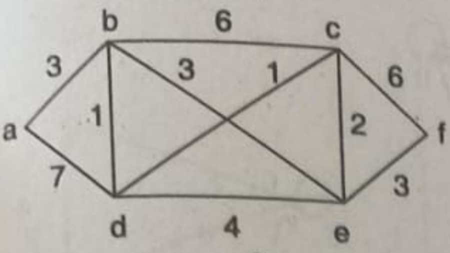


3. Give an example of a graph that has an Eulerian circuit which is also a Hamiltonian circuit.
4. Give an example of a graph that has an Eulerian circuit and Hamiltonian circuit, which are distinct.
5. Give an example of a graph which has an Eulerian circuit but not a Hamiltonian circuit.
6. Give are example of a graph which has a Hamiltonian circuit but not an Eulerian circuit.
7. Give an example of a graph that has neither an Eulerian circuit nor a Hamiltonian circuit.
8. Use Dijkstra's algorithm to find the shortest path between the indicated vertices in the given weighted graphs.

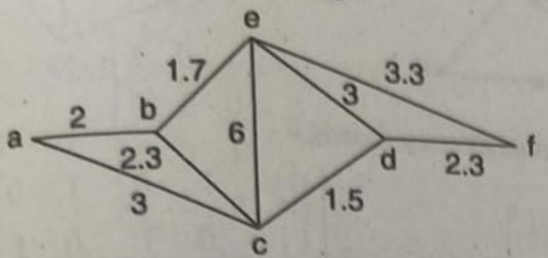
(a)



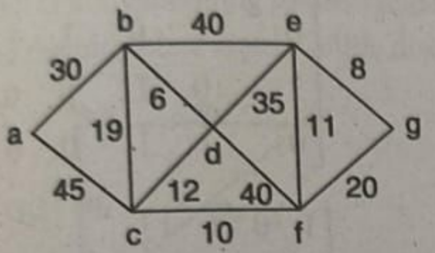
between a and h

(*b*) 

between *a* and *f*

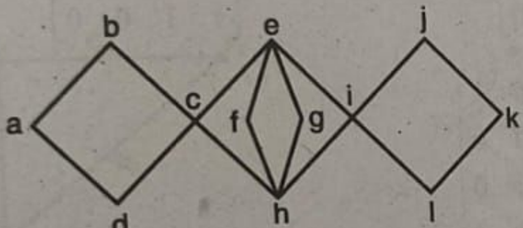
(*c*) 

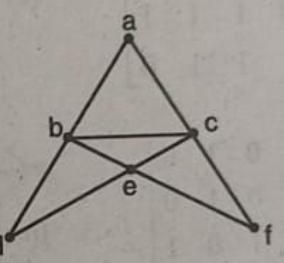
between *a* and *f*

(*d*) 

between *a* and *g*

9. Using Fleury's algorithm, find Euler circuit of the graphs.

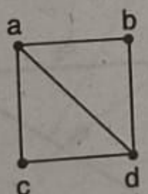
(a) 

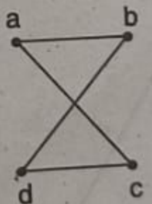
(b) 

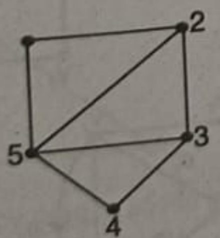
Problem Set 14.4

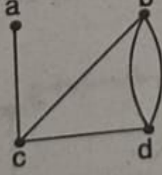
1. Find the adjacency matrix of each of the following graphs:

(*a*) 

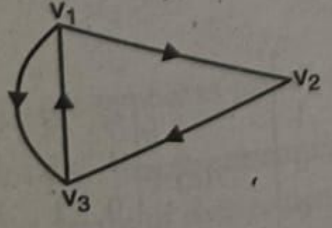
(*b*) 

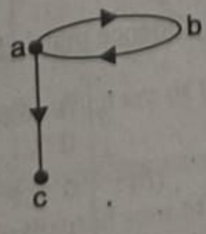
(*c*) 

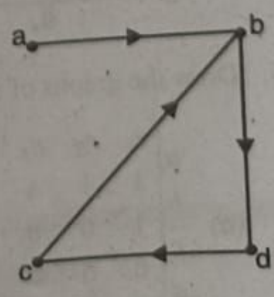
(*d*) 

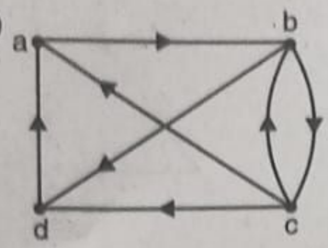
(*e*) 

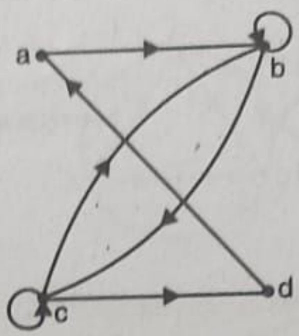
2. Find the adjacency matrix of each of the following digraphs:

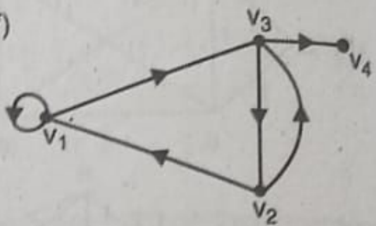
(a) 

(b) 

(c) 

(d) 

(e) 

(f) 

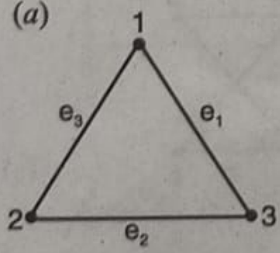
3. Draw the graph/digraph represented by the given adjacency matrix.

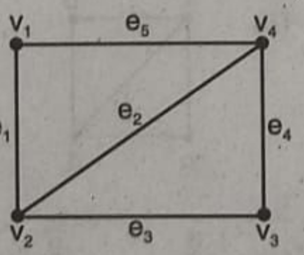
(a) $\begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$ (b) $\begin{bmatrix} 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \end{bmatrix}$ (c) $\begin{bmatrix} 0 & 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 & 1 \\ 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 \end{bmatrix}$

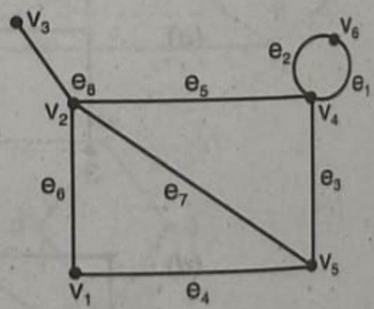
(d) $\begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 1 & 1 & 1 \\ 1 & 1 & 1 & 0 \end{bmatrix}$ (e) $\begin{bmatrix} 0 & 1 & 1 & 1 \\ 1 & 0 & 1 & 0 \\ 1 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \end{bmatrix}$ (f) $\begin{bmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 1 & 0 & 0 \end{bmatrix}$

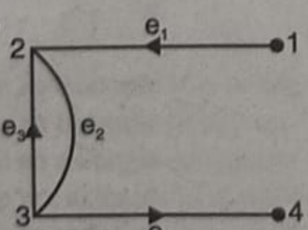
(g) $\begin{bmatrix} 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$ (h) $\begin{bmatrix} 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \end{bmatrix}$

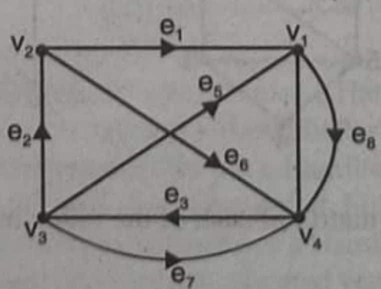
4. Find the incidence matrix of each of the following graphs or diagraphs.

(a) 

(b) 

(c) 

(d) 

(e) 

5. Draw the graphs of the following incidence matrices.

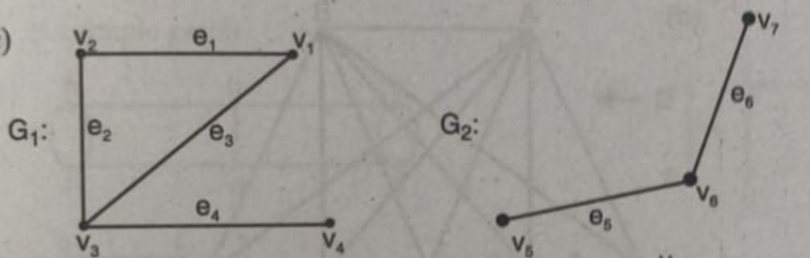
(a) $\begin{array}{c} \\ a \\ b \\ c \\ d \end{array} \begin{array}{c} \begin{array}{ccccc} e_1 & e_2 & e_3 & e_4 & e_5 \end{array} \\ \begin{bmatrix} 1 & 1 & 1 & 0 & 0 \\ 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 & 1 \end{bmatrix} \end{array}$ (b) $\begin{array}{c} \\ a \\ b \\ c \\ d \end{array} \begin{array}{c} \begin{array}{ccccc} e_1 & e_2 & e_3 & e_4 & e_5 \end{array} \\ \begin{bmatrix} 0 & 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 1 & 0 \\ 1 & 0 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 0 \end{bmatrix} \end{array}$ (c) $\begin{array}{c} \\ a \\ b \\ c \end{array} \begin{array}{c} \begin{array}{ccccc} e_1 & e_2 & e_3 & e_4 & e_5 \end{array} \\ \begin{bmatrix} 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 \end{bmatrix} \end{array}$

(d) $\begin{array}{c} \\ a \\ b \\ c \\ d \end{array}\begin{array}{c} \begin{array}{cccccc} e_1 & e_2 & e_3 & e_4 & e_5 & e_6 \end{array} \\ \begin{bmatrix} 1 & 0 & 0 & 1 & -1 & 0 \\ 0 & 0 & 1 & -1 & 1 & 0 \\ 0 & 1 & -1 & 0 & 0 & -1 \\ -1 & -1 & 0 & 0 & 0 & 1 \end{bmatrix} \end{array}$ (e) $\begin{bmatrix} 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 \end{bmatrix}$ (f) $\begin{array}{c} \\ a \\ b \\ c \\ d \end{array}\begin{array}{c} \begin{array}{cccc} e_1 & e_2 & e_3 & e_4 \end{array} \\ \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & 0 \end{bmatrix} \end{array}$

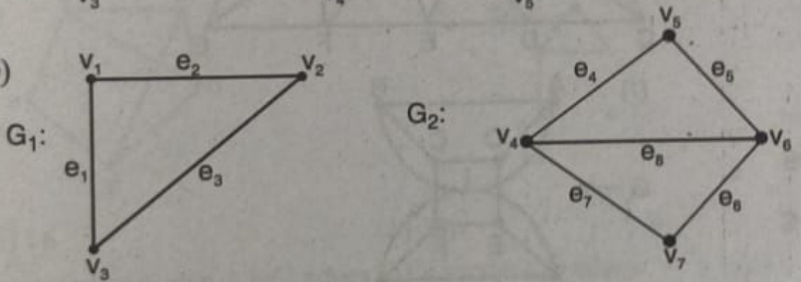
6. Let $I(G) = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$. Without actually constructing the gralp G, show that there exists no connected graph G corresponding to this incidence disconnected graph.

7. Find the incidence matrix of the disconnected graphs :

(a)

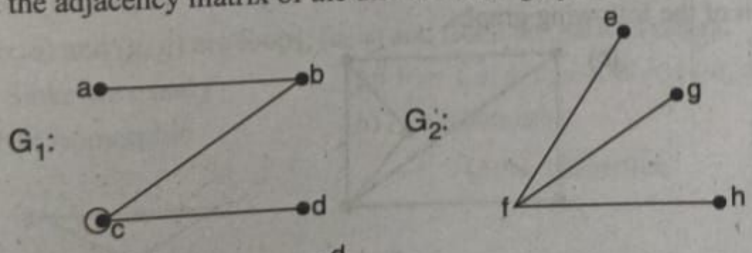


(b)

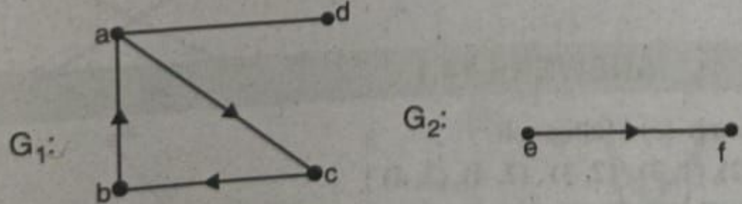


8. Find the adjacency matrix of the disconnected graphs :

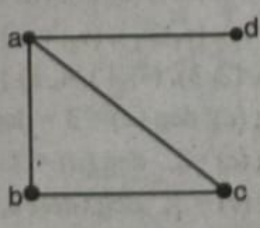
(a)



(b)



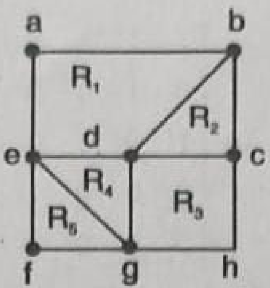
9. Find the adjacency matrix of the graph



and find the number of paths between b and which are of length 4.

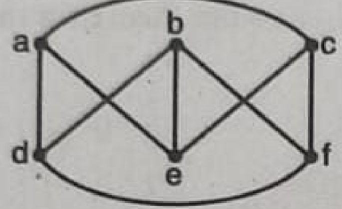
Problem Set 14.5

1. Find the degree of each region and verify that the sum of degrees of the regions is equal to twice the number of edges.

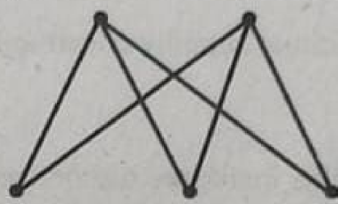


2. Show that each is planar graph by redrawing it so that no edges cross.

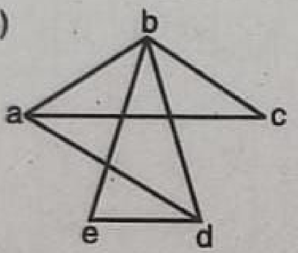
(*a*)



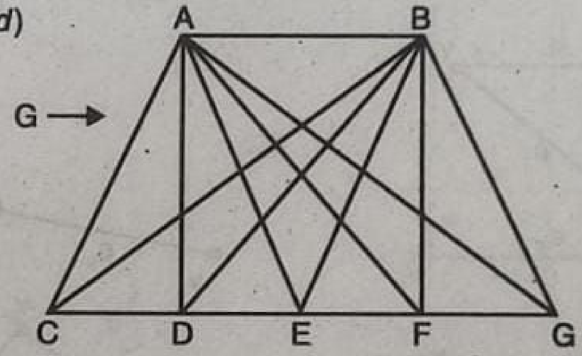
(*b*)



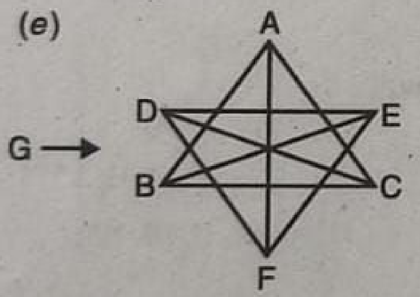
(*c*)



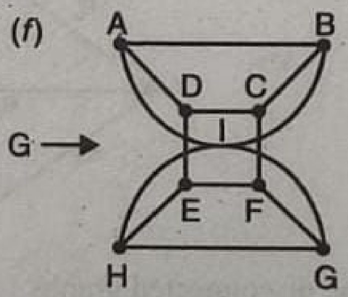
(*d*)



(*e*)

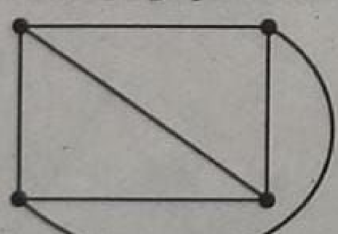


(*f*)



3. Draw the dual graph for each of the following graphs,

(a)



(b)