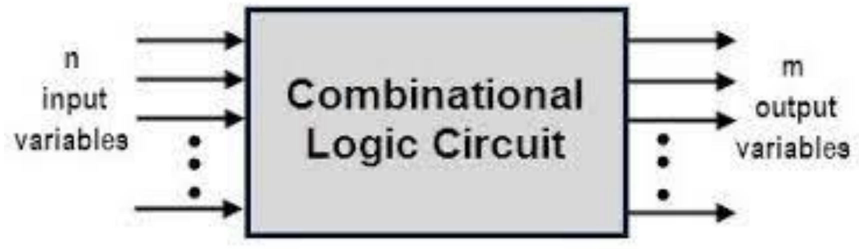
3	**Logic Design Minimization Techniques:** Logic minimization representation of truth-table SOP form POS form simplification of logical functions minimization of SOP and POS forms don't care conditions reduction techniques: k-maps up to 4 variables	**8**	**8**

Combinational circuits :

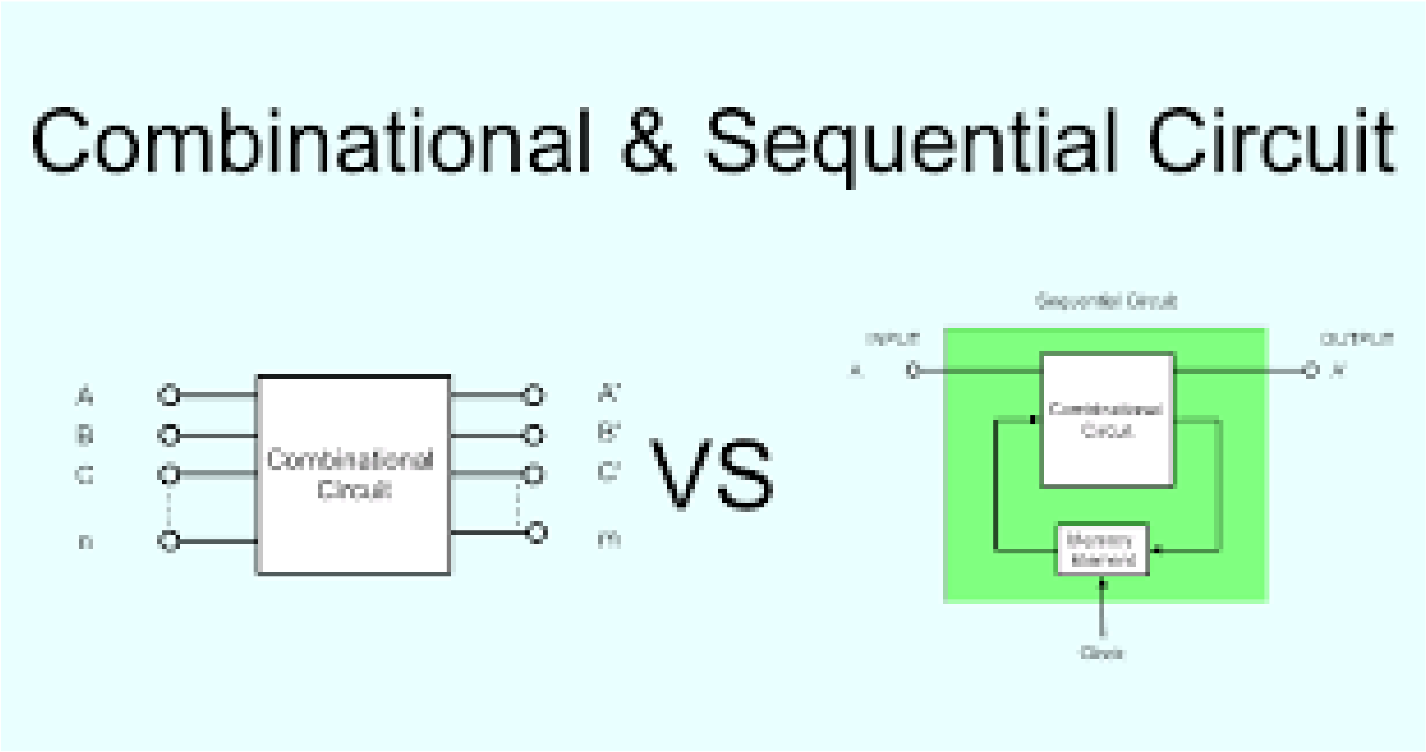
1. A combinational circuit is a **logic** circuit, the output of which depends only on the combination of the inputs.
2. The output does not depend on the past value of inputs or outputs.
3. Hence combinational circuits do not require **any memory.**
4. The block diagram of a combinational circuit is shown in Fig.



Classification of Digital Circuits :

- The digital systems in general are classified into two categories namely:

1. Combinational logic circuits
2. Sequential logic circuits.



Examples of combinational circuits :

Following are the examples of some combinational circuits:

1. Adders, subtractors
2. Comparator
3. Code converters
4. Encoders, decoders
5. Multiplexers, demultiplexers

SOP and POS Representations for Logic Expressions :

- Consider that the logic expression given to us is as follows:

$$Y = AC + BC$$

Then it can be realized using basic gates as shown in Fig

In this Boolean expression, Y is the result or output and A, B,C are called as **literals.**

Any logic expression can be expressed in the following two standard forms:

1. **Sum-of-products form (SOP) and**

2 Product-of-sums form (POS)

These two forms are suitable for **reducing the given logic expression to its simplest form.**

1. **Sum-of-products form (SOP)**

Y =ABC+BCD+ABD

A = XY+XY+XỸ

Y = PO+POR + POR

2 Product-of-Sums form (POS)

Y = (A+B+C).(A + B).(A + C)

A = (X+Y). (X+Y+Z)

Y = (P+R).(P+Q). (P+R)

Z=(A+B).(C+D)

- **Standard Form-In** this form each term may contain one, two or any number of literals. It is not necessary that each term should contain all the literals
- **Canonical Form-This** rule states that each term used in a equation **must contain all the available input variables.**

Sr.No	Expression	Type
1	Y=AB+ ABC + ABC	Standard SOP
2	$Y=AB+\overline{A}B+AB$	Canonical SOP
3	Y=(A+B).(A+B).(A+B)	Canonical POS
4	Y=(A+B).(A+B+C)	Standard POS

Q. Design a combinational logic circuit whose output is high (1)only when majority of inputs (A,B,C,D) are low(0).

- Solution:

  Write the truth table

AB \ CD	00	01	11	10
00				
01				
11				
10				

A	B	C	D	Output
0	0	0	0	**1**
0	0	0	1	**1**
0	0	1	0	**1**
0	0	1	1	0
0	1	0	**0**	**1**
0	1	0	1	0
0	1	1	0	0
0	1	1	1	0
1	0	0	0	**1**
1	0	0	1	0
1	0	1	0	0
1	0	1	1	0
1	1	0	0	0
1	1	0	1	0
1	1	1	0	0
1	1	1	1	0

Concept of Minterm and Maxterm

- **Minterm-** Each individual term in the canonical SOP Form is called as Minterm
- **Maxterm-** Each individual term in the canonical POS form is called as maxterm.
- Canonical SOP –$Y=AB\bar{C}+AB\bar{C}+\bar{A}B\bar{C}$ each individual term **is minterm**
- mo m1 m2
- Canonical POS-$Y=(A+B).(A+\bar{B})$
- M0 M1

Q. Write the Boolean equation and draw logic diagram for the logic that will have output as Y and inputs A,B,C.

The logic performs the following operation:

1.Y=1

when A=B=C=1

and when A=B=0 and C=1

3.Y=0 for all other cases.

Conversion from Standard SOP to Canonical SOP form :

- Steps to be followed:

Step 1: For each term in the given standard SOP expression find the missing literal.

Step 2: Then AND this term with the term formed by ORing the missing literal and its complement.

Step 3: Simplify the expression to get the canonical SOP expression

Convert the expression $Y = AB + A\bar{C} + BC$ into the canonical SOP form :

Soln. :

Step 1: Find the missing literal for each term :

$$Y = AB + A\bar{C} + BC$$

Missing literal → C B A

Step 2: AND each term with (Missing literal + Its C(complement)) :

$$Y = AB.(C+\bar{C}) + A\bar{C}.(B+\bar{B}) + BC.(A+\bar{A})$$

(Missing literal + Its complement)

Original product term

Step 3: Simplify the expression to get the canonical SOP :

$Y = AB\ (C+C) + A\bar{C}\ (B+B) + BC\ (A+\bar{A})$

$= ABC + ABC + ABC + ABC + ABC + ABC$

$= (ABC + ABC) + (ABC + AB\ C) + ABC + AB\ C$

But $A + A = A$ & $(ABC + ABC) = ABC$ and $(A\bar{B}C + AB\bar{C}) = AB\bar{C}$

$\therefore Y = ABC + AB\bar{C} + A\bar{B}\bar{C} + \bar{A}BC$ Canonical SOP form

Each term contains all the literals

Soln. :

1. Given expression is,

$$Y = \bar{A} + B\bar{C}\,\bar{D}$$

$$\therefore \quad Y = \bar{A}\,(B + \bar{B})(C + \bar{C})(D + \bar{D}) + B\bar{C}\,\bar{D}\,(A + \bar{A})$$

$$= \bar{A}BCD + \bar{A}BC\bar{D} + \bar{A}B\bar{C}D + \bar{A}B\bar{C}\,\bar{D}$$

$$+ \bar{A}\,\bar{B}CD + \bar{A}\,\bar{B}C\bar{D} + \bar{A}\,\bar{B}\,\bar{C}D + \bar{A}\,\bar{B}\,\bar{C}\,\bar{D}$$

$$+ AB\bar{C}\,\bar{D} + \bar{A}B\bar{C}\,\bar{D}$$

$$\therefore \quad Y = \bar{A}BCD + \bar{A}BC\bar{D} + \bar{A}B\bar{C}D + \bar{A}B\bar{C}\,\bar{D} + \bar{A}\,\bar{B}CD$$

$$+ \bar{A}\,\bar{B}C\bar{D} + \bar{A}\,\bar{B}\,\bar{C}D + \bar{A}\,\bar{B}\,\bar{C}\,\bar{D} + AB\bar{C}\,\bar{D}$$

$$\ldots (\because A + A = A)$$

This is the required expression in the standard SOP form.

1. Convert following equation into standard SOP form (canonical) :
$Y = \bar{A} + BC\bar{D}$

2. Convert following equation into standard POS form (canonical) :
$Y = (\bar{A} + B)(B + \bar{C})(\bar{A} + \bar{C})$

Soln. :

1. Given expression is,

$$Y = \bar{A} + B\bar{C}\,\bar{D}$$

$$\therefore \quad Y = \bar{A}\,(B+\bar{B})(C+\bar{C})(D+\bar{D}) + B\bar{C}\,\bar{D}\,(A+\bar{A})$$

$$= \bar{A}BCD + \bar{A}BC\bar{D} + \bar{A}B\bar{C}D + \bar{A}B\bar{C}\,\bar{D} + \bar{A}\,\bar{B}CD$$

$$+ \bar{A}\,\bar{B}C\bar{D} + \bar{A}\,\bar{B}\,\bar{C}D + \bar{A}\,\bar{B}\,\bar{C}\,\bar{D}$$

$$+ AB\bar{C}\,\bar{D} + \bar{A}B\bar{C}\,\bar{D}$$

$$\therefore \quad Y = \bar{A}BCD + \bar{A}BC\bar{D} + \bar{A}B\bar{C}D$$

$$+ \bar{A}B\bar{C}\,\bar{D} + \bar{A}\,\bar{B}CD + \bar{A}\,\bar{B}C\bar{D} + \bar{A}\,\bar{B}\,\bar{C}D$$

$$+ \bar{A}\,\bar{B}\,\bar{C}\,\bar{D} + AB\bar{C}\,\bar{D} \qquad \ldots (\because A + A = A)$$

This is the required expression in the canonical SOP form.

2. $$Y = (\bar{A} + B)(B + \bar{C})(\bar{A} + \bar{C})$$

$$= (\bar{A} + B + C\bar{C}) \cdot (B + \bar{C} + A\bar{A}) \cdot (\bar{A} + \bar{C} + B\bar{B})$$

But $A + BC = (A + B)(A + C)$

$$\therefore \quad Y = (\bar{A} + B + C)(\bar{A} + B + \bar{C}) \cdot (B + \bar{C} + A)(B + \bar{C} + \bar{A})$$

$$\cdot (\bar{A} + \bar{C} + B)(\bar{A} + \bar{C} + \bar{B})$$

$$\therefore \quad Y = (\bar{A} + B + C)(\bar{A} + B + \bar{C}) \cdot (A + B + \bar{C})(\bar{A} + B + \bar{C})$$

$$\cdot (\bar{A} + B + \bar{C})(\bar{A} + \bar{B} + \bar{C}) \qquad \ldots \text{Ans.}$$

Ex. 4.2.6 : Convert following equation to canonical SOP form : $Y = (A + B\bar{C})(B + AC)$

S-09, 2 Marks

Soln. :

$$Y = (A + B\bar{C})(B + AC)$$

$$= AB + AAC + BB\bar{C} + ABC\bar{C}$$

$$= AB + AC + B\bar{C}$$

$$[\because A \cdot A = A,\ B \cdot B = B \text{ and } C \cdot \bar{C} = 0]$$

$$= AB(C + \bar{C}) + AC(B + \bar{B}) + B\bar{C}(A + \bar{A})$$

$$= ABC + AB\bar{C} + ABC + A\bar{B}C + AB\bar{C} + \bar{A}B\bar{C}$$

$$Y = ABC + AB\bar{C} + A\bar{B}C + \bar{A}B\bar{C}$$

This is required canonical SOP form.

Ex. 4.2.7 : Convert following expression into canonical SOP form : $Y = A + B$

W-09, 2 Marks

Soln. :

$$Y = A + B$$

$$Y = A(B + \bar{B}) + B(A + \bar{A}) = AB + A\bar{B} + AB + \bar{A}B$$

$$Y = AB + A\bar{B} + \bar{A}B$$

This is canonical SOP form.

Conversion from standard POS to canonical POS form :

Steps to be followed:

Step 1: For each term find the missing literal

Step 2: Then OR each term with the term formed by ANDing the missing literal in that term with its complement.

Step 3: Simplify the expression to get the canonical POS

Ex. 4.2.8 : Convert the following into canonical SOP :

1. $AC + CD + BC$ 2. $\bar{A}(B + \bar{C})$

W-12, 4 Marks

Soln. :

1. $AC + CD + BC$:

$$\therefore Y = AC(B + \bar{B})(D + \bar{D}) + CD(A + \bar{A})(B + \bar{B})$$
$$+ BC(A + \bar{A})(D + \bar{D})$$
$$= ABCD + ABC\bar{D} + A\bar{B}C\bar{D} + A\bar{B}CD + ABCD$$
$$+ \bar{A}BCD + A\bar{B}CD + \bar{A}\bar{B}CD + ABCD + \bar{A}BCD$$
$$+ ABC\bar{D} + \bar{A}BC\bar{D}$$
$$= ABCD + ABC\bar{D} + A\bar{B}C\bar{D} + A\bar{B}CD + \bar{A}BCD$$
$$+ \bar{A}\bar{B}CD + \bar{A}BC\bar{D}$$

2. $\bar{A}(B + \bar{C})$:

$$\therefore Y = \bar{A}B + \bar{A}\bar{C}$$
$$= \bar{A}B(C + \bar{C}) + \bar{A}\bar{C}(B + \bar{B})$$
$$= \bar{A}BC + \bar{A}B\bar{C} + \bar{A}B\bar{C} + \bar{A}\bar{B}\bar{C}$$

Convert the following SOP equation into standard SOP equation.

$Y = AB + \bar{A}B + A\bar{B}\bar{C}$

S-15, 2 Marks

$$Y = AB + \bar{A}B + A\bar{B}\bar{C}$$
$$= AB(C + \bar{C}) + \bar{A}B(C + \bar{C}) + A\bar{B}\bar{C}$$

$$Y = ABC + AB\bar{C} + \bar{A}BC + \bar{A}B\bar{C} + A\bar{B}\bar{C}$$

$$Y = ABC + AB\bar{C} + \bar{A}BC + \bar{A}B\bar{C} + A\bar{B}\bar{C} \quad \text{...Ans}$$

Ex. 4.2.11 : Convert the following expression into its standard forms :

1. $Y = \bar{A}BC + AC + \bar{B}$
2. $Y = (B + \bar{C}) \times (A + D) \times (\bar{B} + \bar{D})$

W-16, 4 Marks

Soln. :

1. $Y = \bar{A}BC + AC + \bar{B}$

$$= \bar{A}BC + AC(B + \bar{B}) + \bar{B}(A + \bar{A})(C + \bar{C})$$

$$= \bar{A}BC + ABC + A\bar{B}C + A\bar{B}C + A\bar{B}\bar{C} + \bar{A}\bar{B}C + \bar{A}\bar{B}\bar{C}$$

$$Y = \bar{A}BC + ABC + A\bar{B}C + A\bar{B}\bar{C} + \bar{A}\bar{B}C + \bar{A}\bar{B}\bar{C}$$

$$Y = (B + \bar{C}) \cdot (A + D) \cdot (\bar{B} + \bar{D})$$

$$= (B + \bar{C} + A\bar{A} + D\bar{D}) \cdot (A + D + B\bar{B} + C\bar{C})$$

$$(\bar{B} + \bar{D} + A\bar{A} + C\bar{C})$$

$$= (B + \bar{C} + A + D)(B + \bar{C} + A + \bar{D})(B + \bar{C} + \bar{A} + D)$$

$$(B + \bar{C} + \bar{A} + \bar{D})(A + D + B + C)(A + D + B + \bar{C})$$

$$(A + D + \bar{B} + C)(A + D + \bar{B} + \bar{C})(\bar{B} + \bar{D} + A + C)$$

$$(\bar{B} + \bar{D} + A + \bar{C})(\bar{B} + \bar{D} + \bar{A} + C)(\bar{B} + \bar{D} + \bar{A} + \bar{C})$$

$$= (A + B + \bar{C} + D)(A + B + \bar{C} + \bar{D})(\bar{A} + \bar{B} + \bar{C} + D)$$

$$(\bar{A} + B + \bar{C} + \bar{D})(A + B + C + D)(A + \bar{B} + C + D)$$

$$(A + \bar{B} + \bar{C} + D)(A + \bar{B} + C + \bar{D})(A + \bar{B} + \bar{C} + \bar{D})$$

$$(\bar{A} + \bar{B} + C + \bar{D})(\bar{A} + \bar{B} + \bar{C} + \bar{D})$$

Ex. 4.4.3 : Convert the following boolean expression into its standard forms :

1. $Y = A\bar{B} + AC + \bar{B}C$

2. $Y = (A + \bar{B}) \cdot (A + C) \cdot (B + \bar{C})$

S-14, 4 Marks

Soln. :

1.
$$Y = A\bar{B} + AC + \bar{B}C$$
$$= AB\,(C + \bar{C}) + AC\,(B + \bar{B}) + \bar{B}C\,(A + \bar{A})$$
$$= ABC + AB\bar{C} + ABC + A\bar{B}C + A\bar{B}C + \bar{A}\,\bar{B}C$$
$$Y = ABC + AB\bar{C} + A\bar{B}C + \bar{A}\,\bar{B}C$$
$$\ldots(\because ABC + ABC = ABC,\ A\bar{B}C + A\bar{B}C = A\bar{B}C)$$

This is required standard SOP form.

2.
$$Y = (A + \bar{B})\,(A + C)\,(B + \bar{C})$$
$$= (A + \bar{B} + C\bar{C})\,(A + C + B\bar{B})\,(B + \bar{C} + A\bar{A})$$
$$= (A + \bar{B} + C)\,(A + \bar{B} + \bar{C})\,(A + B + C)\,(A + \bar{B} + C)$$
$$(A + B + \bar{C})\,(\bar{A} + B + \bar{C})$$
$$= (A + \bar{B} + C)\,(A + \bar{B} + \bar{C})\,(A + B + C)$$
$$(A + B + \bar{C})\,(\bar{A} + B + \bar{C})$$
$$\ldots(\because (A + \bar{B} + C)\,(A + \bar{B} + C) = A + \bar{B} + C)$$

This is required standard POS form.

Karnaugh-Map Simplification :

What is K-map ? Explain the rules to simplify

1. This is another simplification technique to reduce the Boolean equation.
2. It overcomes all the disadvantages of the algebraic simplification technique.
3. K-map (short form of Karnaugh map) is a graphical method of simplifying a Boolean equation.
4. K-map is a graphical chart made up of rectangular boxes.
5. The information contained in a truth table or available in the
6. SOP or POS form can be represented on a K-map. The K-map can be used for systematic simplification of Boolean expression.
7. K-maps can be written for 2, 3, 4 ... upto 6 variables. Beyond that the K-map technique becomes very cumbersome.
8. K-map is ideally suitable for designing the combinational logic circuits using either a SOP method or a POS method.

K-map Structure

- 2 variable k map

A. SOP: -

A \ B	$\overline{B}$ 0	B 1
$\overline{A}$ 0	$\overline{A}.\overline{B}$	$\overline{A}.B$
A 1	$A.\overline{B}$	$A.B$

4 variable k map

AB \ CD	00	01	11	10
00				
01				
11				
10				

	C'D'	C'D	CD	CD'
A'B'	0	1	3	2
A'B	4	5	7	6
AB	12	13	15	14
AB'	8	9	11	10

- 3 variable k map

2 elements in one group

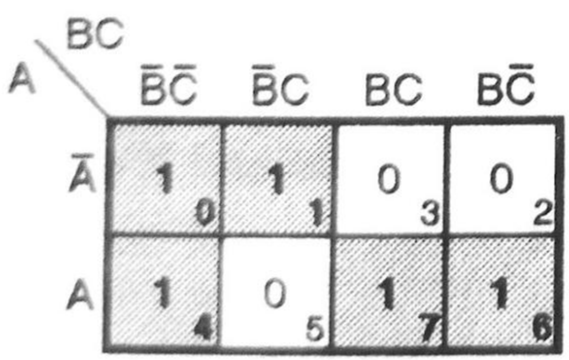
A \ BC	00	01	11	10
0	0 (0)	1 (1)	0 (3)	1 (2)
1	1 (4)	1 (5)	0 (7)	0 (6)

Ex. 4.5.1 : Represent the equation given below on Karnaugh map :

$$Y = \bar{A}\,\bar{B}\bar{C} + \bar{A}\,\bar{B}\,C + A\bar{B}\bar{C} + AB\bar{C} + ABC.$$

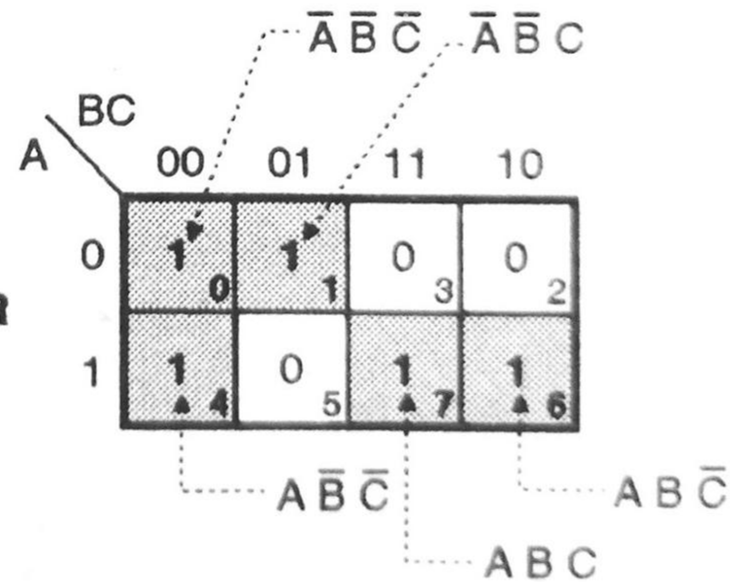
Soln. : The given expression is in the standard SOP form. Each term represents a minterm.

We have to enter 1's in the boxes corresponding to each minterm, as shown in Fig. P. 4.5.1.



"1"s correspond to the minterms in the given Boolean expression

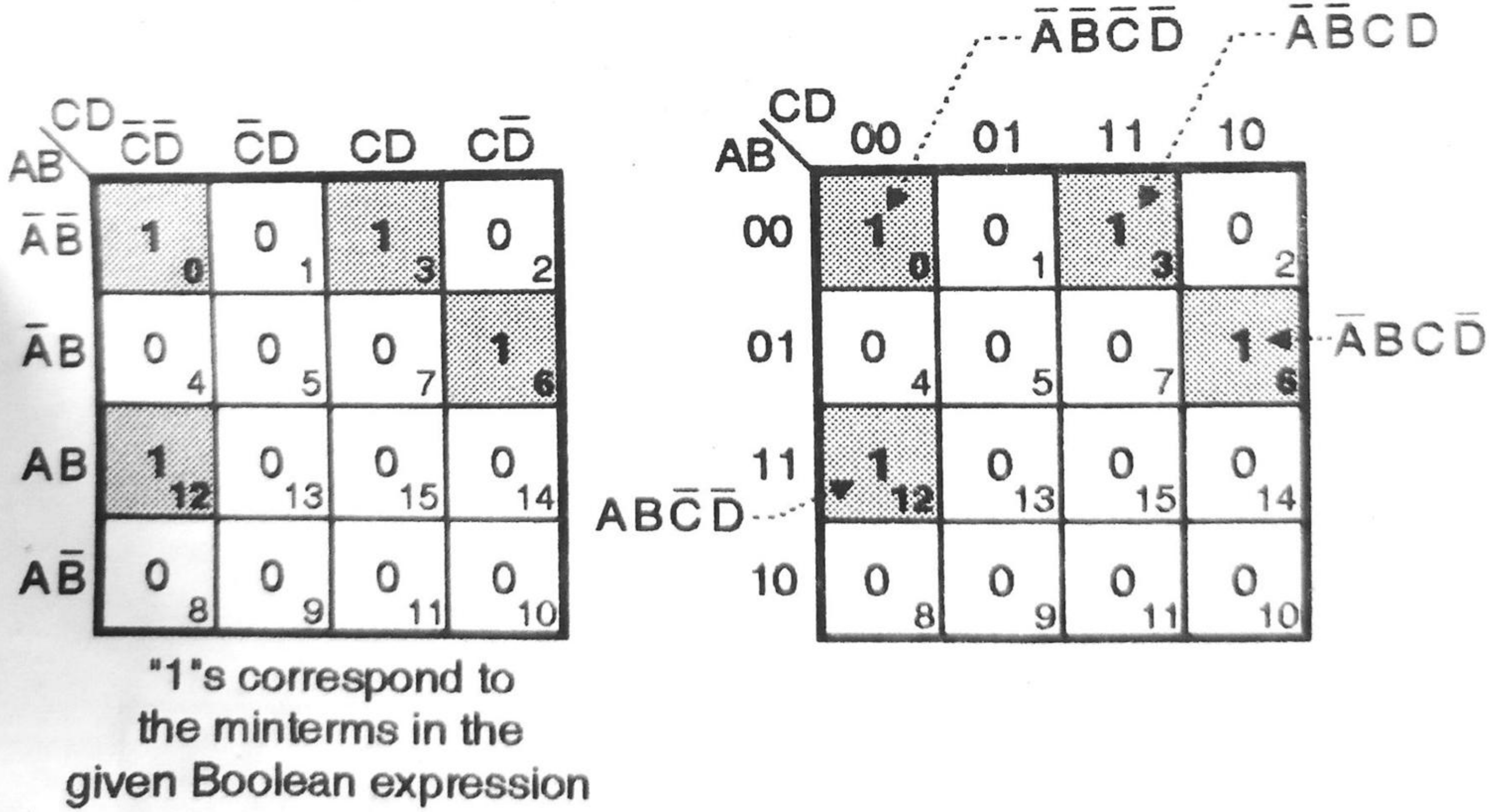
OR



Ex. 4.5.2 : Plot the following Boolean expression on K-map :

$$Y = \bar{A}\,\bar{B}\,\bar{C}\,\bar{D} + \bar{A}\,\bar{B}\,C\,D + \bar{A}\,B\,C\,\bar{D} + A\,B\,\bar{C}\,\bar{D}$$

Soln. : Refer Fig. P. 4.5.2.



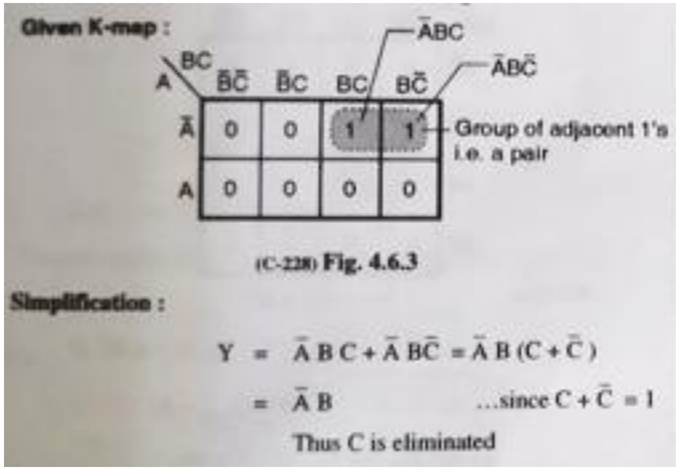
Way of Grouping (Pairs, Quads and Octets):

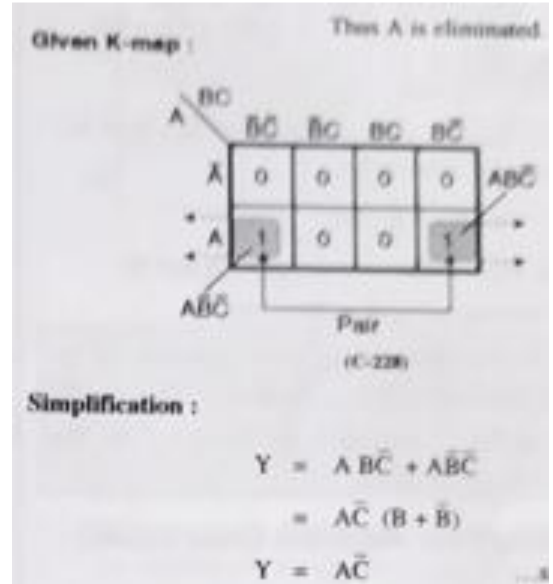
While grouping, we **should group most number of 1's** (or 0's).

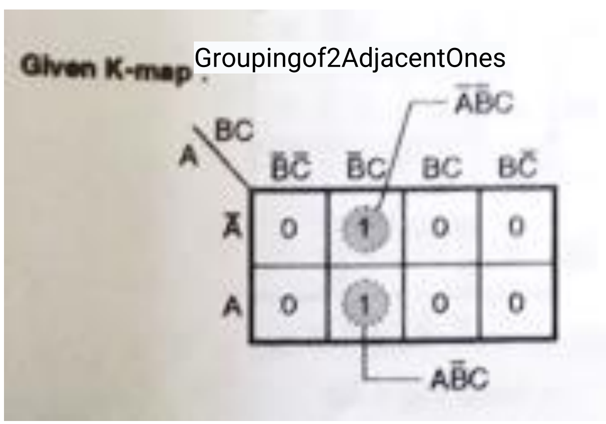
The grouping follows the binary rule i.e. we can group 1, 2, 4, 8, 16, 32 number of l's or 0's. We cannot group 3, 5, 7, ... number of 1's or 0's.

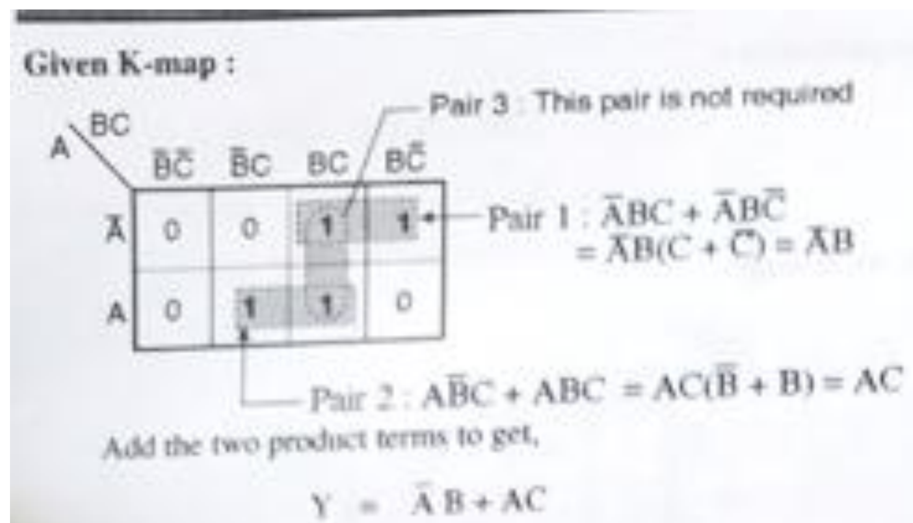
1. **Pairs:** A group of two adjacent 1's or 0's is called as a pair.
2. **Quad:** A group of four adjacent l's or 0's is called as a quad.
3. **Octet:** A group of eight adjacent l's or 0's is called as octet.

Grouping of 2 Adjacent Ones

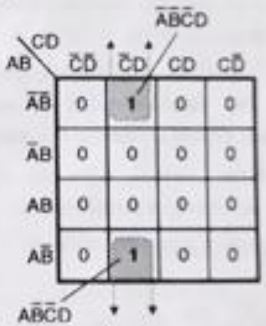








Given K-map :



(C-228)

Simplification :

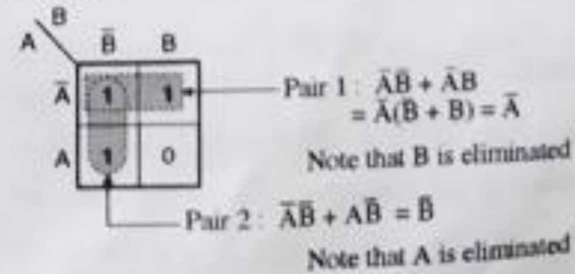
$$Y = \bar{A}\,\bar{B}\bar{C}\,D + A\bar{B}\bar{C}\,D = \bar{B}\bar{C}\,D\,(\bar{A} + A)$$

$$\therefore\ Y = \bar{B}\bar{C}\,D$$

Conclusion : A is eliminated.

Example of overlap :

Given K-map :



$\therefore$ Final expression $Y = \bar{A} + \bar{B}$

Note that in order to cover all the 1's, we have to overlap two pairs as shown.

Grouping of 4 Adjacent Ones

Given K-map :

$AB \backslash CD$	$\bar{C}\bar{D}$	$\bar{C}D$	CD	$C\bar{D}$
$\bar{A}\bar{B}$	0	1	0	0
$\bar{A}B$	0	1	0	0
AB	0	1	0	0
$A\bar{B}$	0	1	0	0

$\rightarrow Y = \bar{C}D$

(C-231)

Simplification :

$$Y = \bar{A}\,\bar{B}\bar{C}\,D + \bar{A}\,B\bar{C}\,D + AB\bar{C}\,D + A\,\bar{B}\bar{C}\,D$$

$$= \bar{C}\,D\,[\bar{A}\,\bar{B} + \bar{A}\,B + A\,B + A\bar{B}]$$

$$= \bar{C}\,D\,[\bar{A}\,(\bar{B} + B) + A\,(B + \bar{B})]$$

$$= \bar{C}\,D\,[\bar{A} + A] = \bar{C}\,D$$

Thus A and B are eliminated.

Given K-map :

$AB \backslash CD$	$\bar{C}\bar{D}$	$\bar{C}D$	CD	$C\bar{D}$
$\bar{A}\bar{B}$	0	0	0	0
$\bar{A}B$	0	0	0	0
AB	0	0	0	0
$A\bar{B}$	1	1	1	1

$\rightarrow Y = A\bar{B}$

(C-230)

Simplification :

$$Y = A\bar{B}\bar{C}\,\bar{D} + A\bar{B}\bar{C}\,D + A\bar{B}C\,D + A\,\bar{B}C\,\bar{D}$$

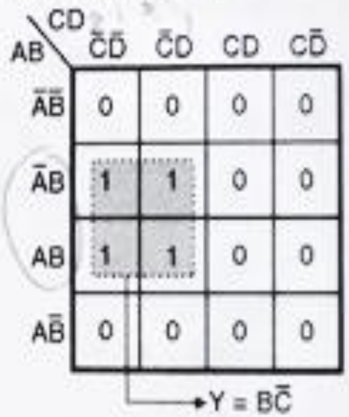
$$= A\bar{B}\,[\bar{C}\,\bar{D} + \bar{C}\,D + C\,D + C\,\bar{D}]$$

$$= A\bar{B}\,[\bar{C}\,(\bar{D} + D) + C\,(D + \bar{D})]$$

$$\therefore\ Y = A\bar{B}\,[\bar{C} + C] = A\bar{B}$$

Thus C and D are eliminated.

Four adjacent ones forming a square



(C-231(a))

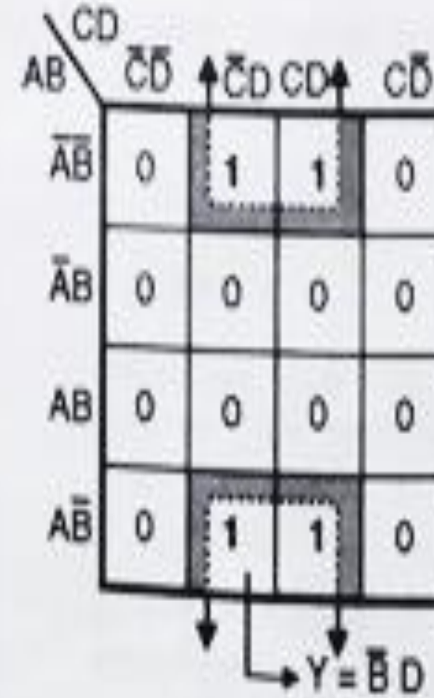
$$Y = \bar{A}B\bar{C}\bar{D} + AB\bar{C}\bar{D} + \bar{A}B\bar{C}D + AB\bar{C}D$$

$$= B\bar{C}\bar{D}(\bar{A} + A) + B\bar{C}D(\bar{A} + A)$$

$$= B\bar{C}\bar{D} + B\bar{C}D = B\bar{C}(\bar{D} + D)$$

$$\therefore Y = B\bar{C}$$

Thus A and D are eliminated



AB \ CD	$\bar{C}\bar{D}$	$\bar{C}D$	CD	$C\bar{D}$
$\bar{A}\bar{B}$	0	0	0	0
$\bar{A}B$	1	0	0	1
AB	1	0	0	1
$A\bar{B}$	0	0	0	0

$Y = B\bar{D}$

(C-231(c))

$$Y = \bar{A}B\bar{C}\bar{D} + AB\bar{C}\bar{D} + \bar{A}BC\bar{D} + ABC\bar{D}$$

$$= B\bar{C}\bar{D}(\bar{A} + A) + BC\bar{D}(\bar{A} + A)$$

$$= B\bar{C}\bar{D} + BC\bar{D}$$

$$Y = B\bar{D}(\bar{C} + C) = B\bar{D}$$

Thus A and C are eliminated

1's corresponding to corners forming a Quad :

AB \ CD	$\bar{C}\bar{D}$	$\bar{C}D$	CD	$C\bar{D}$
$\bar{A}\bar{B}$	1	0	0	1
$\bar{A}B$	0	0	0	0
AB	0	0	0	0
$A\bar{B}$	1	0	0	1

$Y = \bar{B}\bar{D}$

(C-232)

$$Y = \bar{A}\bar{B}\bar{C}\bar{D} + \bar{A}\bar{B}C\bar{D} + A\bar{B}\bar{C}\bar{D} + A\bar{B}C\bar{D}$$

$$= \bar{B}\bar{C}\bar{D}(\bar{A} + A) + \bar{B}C\bar{D}(\bar{A} + A)$$

$$= \bar{B}\bar{C}\bar{D} + \bar{B}C\bar{D} = \bar{B}\bar{D}(\bar{C} + C)$$

$\therefore Y = \bar{B}\bar{D}$ Thus A and C are eliminated

Overlapping of Quads and pairs

AB \ CD	$\bar{C}\bar{D}$	$\bar{C}D$	CD	$C\bar{D}$
$\bar{A}\bar{B}$	0	0	0	0
$\bar{A}B$	1	1	1	1
AB	0	1	1	1
$A\bar{B}$	0	0	0	0

Quad 1 : BC

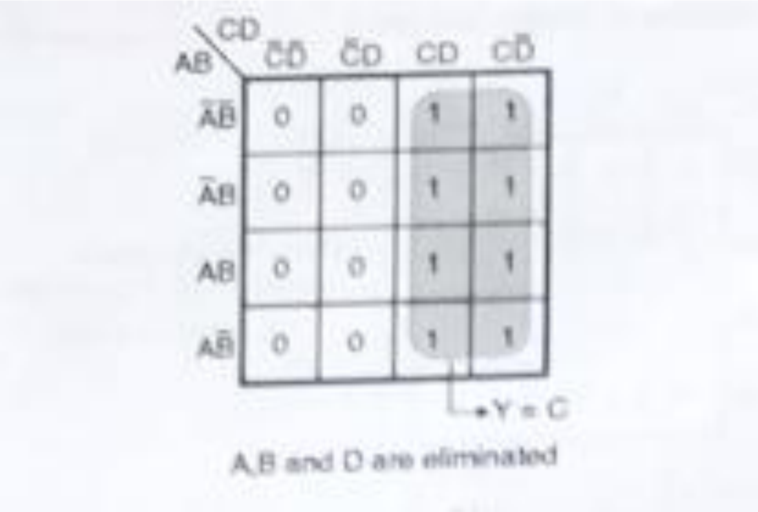
Quad 2 : BD

Pair : $\bar{A}B\bar{C}$

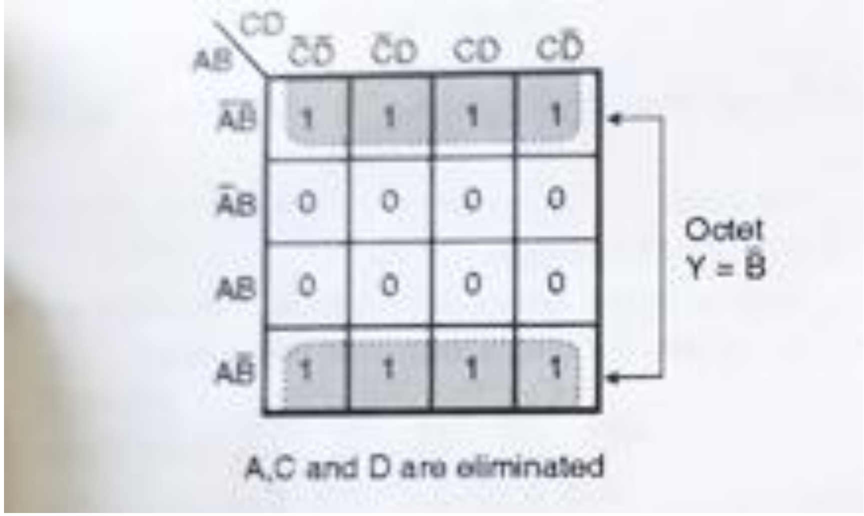
There are two quads (overlapping) and one pair

$\therefore Y = BC + BD + \bar{A}B\bar{C}$

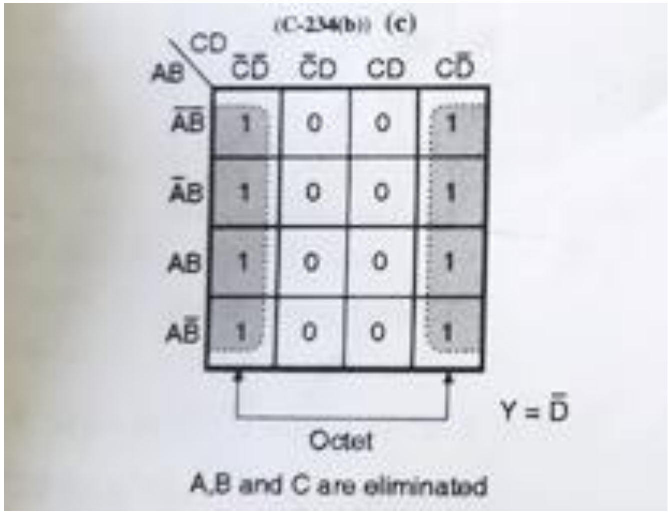
Grouping of 8 Adjacent Ones



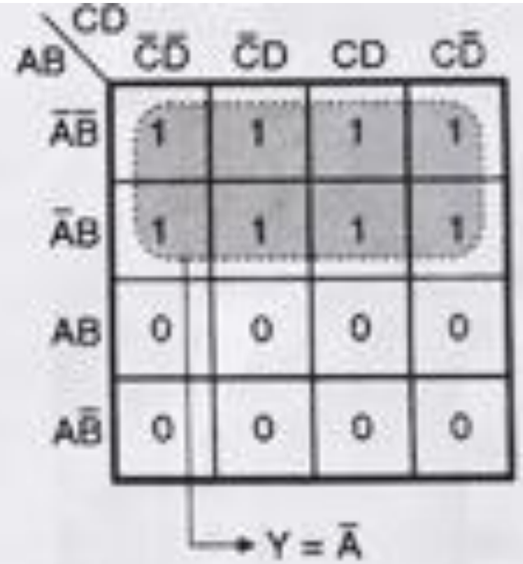
A,B and D are eliminated



A,C and D are eliminated



A,B and C are eliminated



B,C and D are eliminated

(C-234) **Fig. 4.6.6(a)**

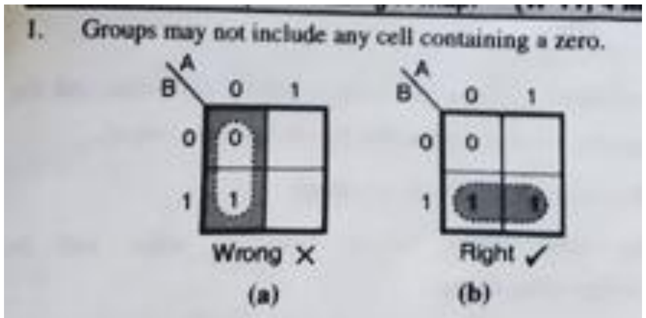
$$Y = \bar{A}\,\bar{B}\bar{C}\,\bar{D} + \bar{A}\,\bar{B}\bar{C}\,D + \bar{A}\,\bar{B}C\,D + \bar{A}\,\bar{B}C\,\bar{D}$$
$$+ \bar{A}\,B\bar{C}\,\bar{D} + \bar{A}\,B\bar{C}\,D + \bar{A}\,B\,C\,D + \bar{A}\,B\,C\,\bar{D}$$

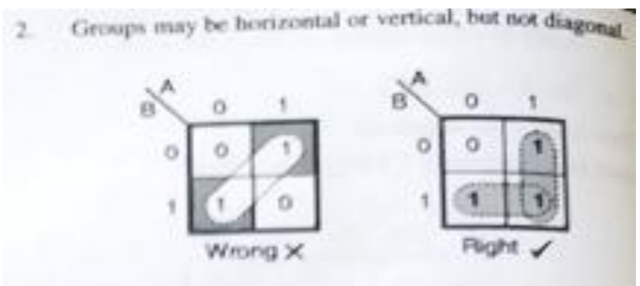
$$\therefore\ Y = \bar{A}\,\bar{B}\bar{C}\,(\bar{D} + D) + \bar{A}\,\bar{B}C$$
$$(D + \bar{D}) + \bar{A}\,B\,\bar{C}\,(\bar{D} + D) + \bar{A}\,B\,C\,(D + \bar{D})$$

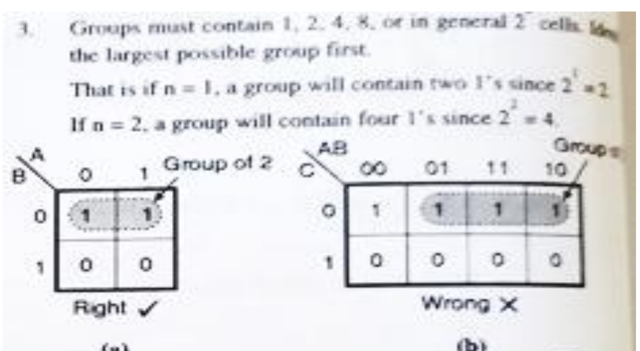
$$\therefore\ Y = \bar{A}\,\bar{B}\,(\bar{C} + C) + \bar{A}\,B\,(\bar{C} + C)$$

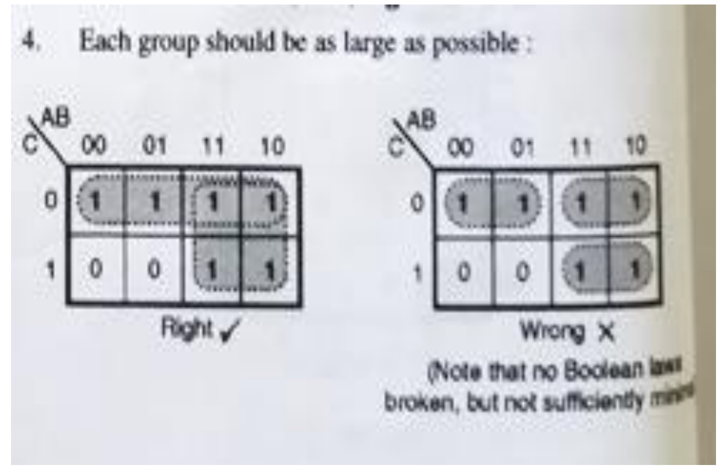
$$\therefore\ Y = \bar{A}\,(\bar{B} + B) = \bar{A}$$

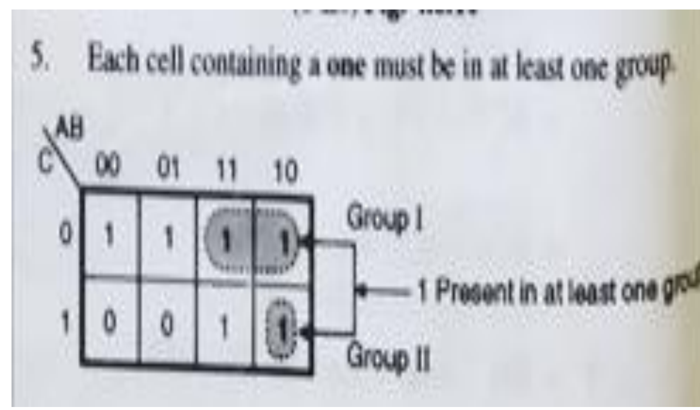
Rules followed for K-Map Simplification





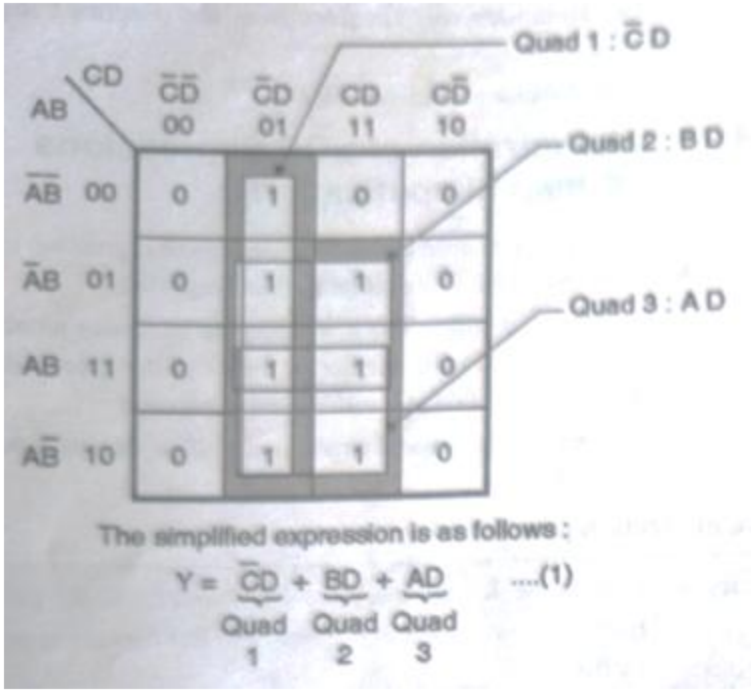






1. For the logical expression given below draw the K-map and obtain the simplified logical expression:

$Y = \Sigma m\ (1, 5, 7, 9, 11, 13, 15)$. Realize the minimized expression using the basic gates.



Examples

2. Using K-map realize the following expression using minimum number of gates : Σm (1, 3, 4, 5, 7, 9, 11, 13, 15)

3. Simplify the following expressions using K-map :

a. $f(A, B, C) = \Sigma m (0, 1, 3, 4, 6)$

b. $f(A, B, C, D) = \Sigma m (0, 1, 2, 4)$

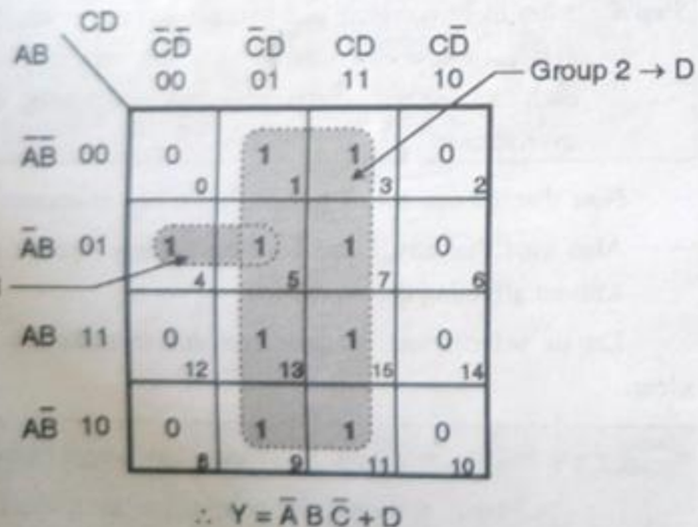
Ex. 4.7.2 : Using K-map realize the following expression using minimum number of gates :

$$Y = \sum m(1, 3, 4, 5, 7, 9, 11, 13, 15)$$

S-09. 4 Marks

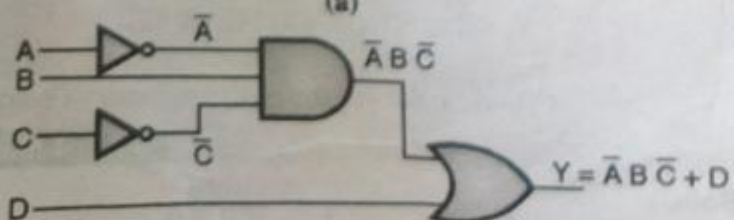
Soln. :

The required K-map is shown in Fig. P. 4.7.2(a) and the realization using minimum gates is shown in Fig. P. 4.7.2(b).



$\therefore Y = \bar{A}B\bar{C} + D$

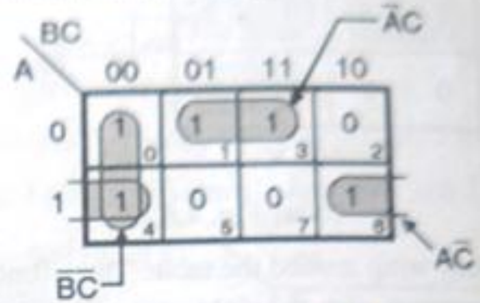
(a)



(b) Realization with minimum gates

1. $f(A,B,C) = \Sigma m(0,1,3,4,6)$:

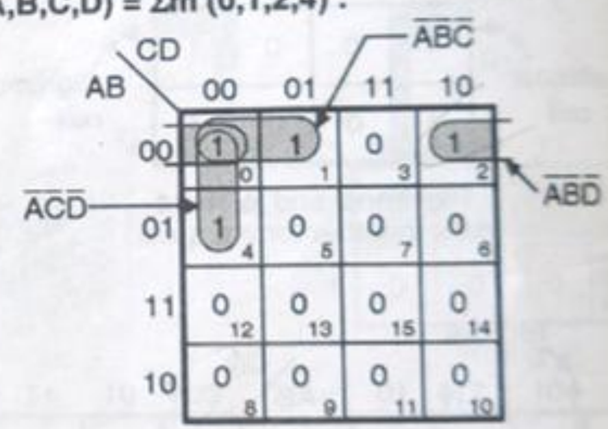
Simplification using K-map :



(C-2600) **Fig. P. 4.7.3(a)**

$$\therefore \quad f(A,B,C) = \bar{A}C + A\bar{C} + \bar{B}\bar{C}$$

2. $f(A,B,C,D) = \Sigma m(0,1,2,4)$:



(C-2601) **Fig. P. 4.7.3(b)**

$$\therefore \quad f(A,B,C,D) = \bar{A}\bar{B}\bar{C} + \bar{A}\bar{B}\bar{D} + \bar{A}\bar{C}\bar{D}$$

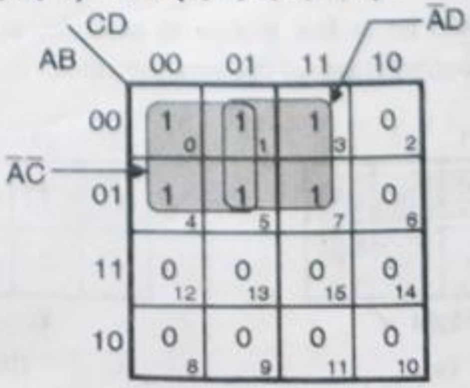
Ex. 4.7.4 : Solve the following SOP expression wi K-map :

1. $f(A, B, C, D) = \Sigma m\,(0, 1, 3, 4, 5, 7)$
2. $f(A, B, C) = \Sigma m\,(0, 1, 4, 5, 6, 7)$

W-11, W-17, 4 Marks

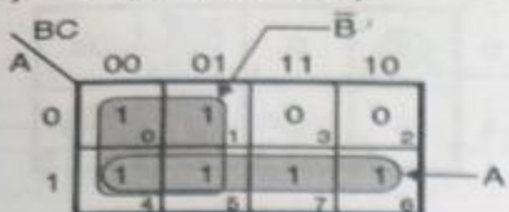
Soln. :

1. $f(A, B, C, D) = \Sigma m\,(0, 1, 3, 4, 5, 7)$:



$$f(A,B,C,D) = \bar{A}D + \bar{A}\bar{C}$$

2. $f(A, B, C) = \Sigma m\,(0, 1, 4, 5, 6, 7)$:



(C-2603) **Fig. P. 4.7.4(b)**

$$f(A,B,C) = A + \bar{B}$$

Ex. 4.7.5 : Convert the following functions into SOP form and plot the K map :
$Y = AB + AC + BC$. **W-11**

Soln. :

$$Y = AB + AC + BC$$

$$= AB\,(C + \bar{C}) + AC\,(B + \bar{B}) + BC\,(A + \bar{A})$$

$$= ABC + AB\bar{C} + ABC + A\bar{B}C + ABC + \bar{A}B$$

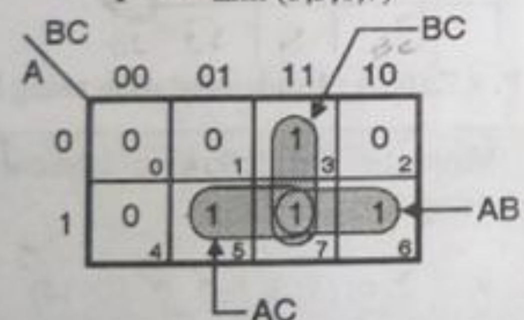
$$\therefore\ Y = ABC + AB\bar{C} + A\bar{B}C + \bar{A}BC$$

$$\therefore\ Y = \Sigma m\,(7,6,5,3)$$

This is the required standard SOP form (canonical

Simplification using K-map :

$$Y = \Sigma m\,(3,5,6,7)$$



(C-2604) **Fig. P. 4.7.5**

$$Y = AB + BC + AC$$

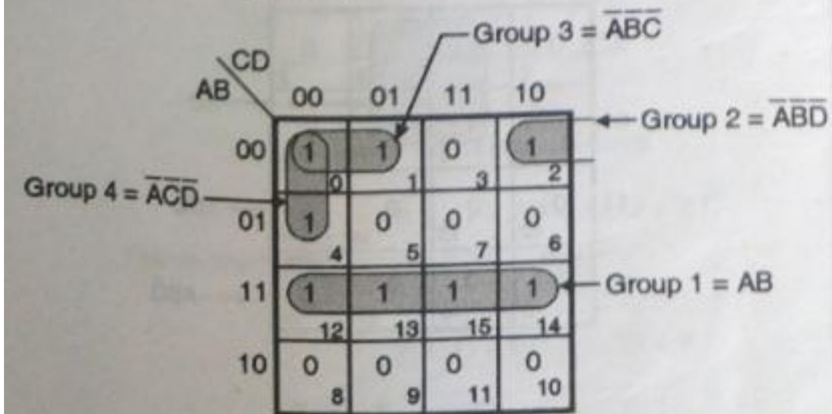
Ex. 4.7.6 : Solve the following Boolean expression using K-map :

$Y = \Sigma m\,(m_0, m_1, m_2, m_4, m_{12}, m_{13}, m_{14}, m_{15})$

S-12, 4 Marks

Soln. :

Simplification using K-map :



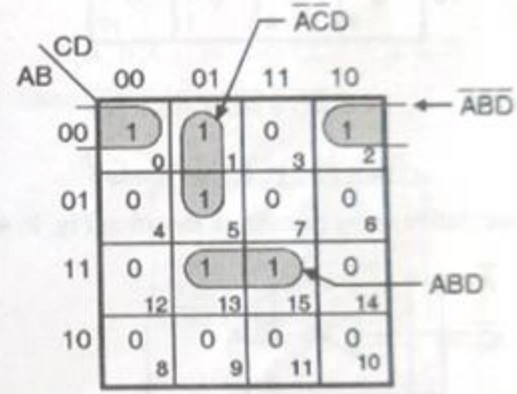
Ex. 4.7.8 : Simplify the following using K-map and realize using NAND – NAND gates.

$f(A, B, C, D) = \Sigma m\,(0, 1, 2, 5, 13, 15)$

W-12, 4 Marks

Soln. :

Step 1 : Minimization using K-map :



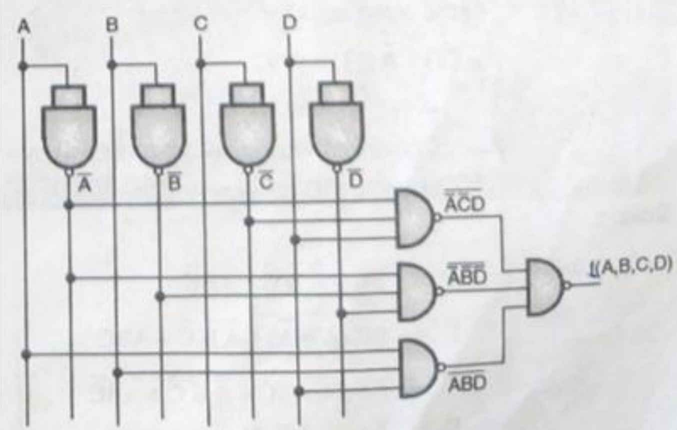
Step 2 : Realization using NAND gates :

$$f(A, B, C, D) = \overline{\overline{\bar{A}\,\bar{C}\,D + \bar{A}\,\bar{B}\,\bar{D} + ABD}}$$

...Take double inversion

$$= \overline{\overline{\bar{A}\,\bar{C}\,D} \cdot \overline{\bar{A}\,\bar{B}\,\bar{D}} \cdot \overline{ABD}}$$

...using De Morgan's laws.



.7.9 : Minimize the following Boolean expression using K-map.

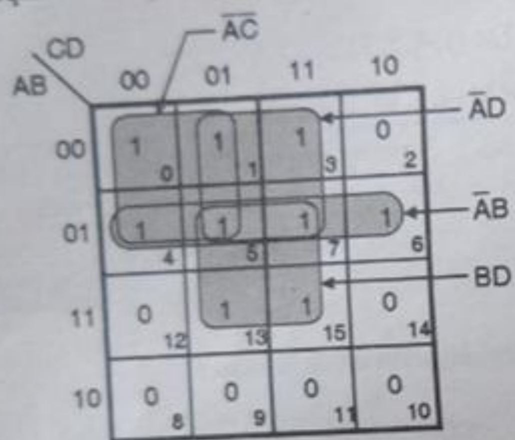
$Y = \Sigma m\,(0, 1, 3, 4, 5, 6, 7, 13, 15)$

Draw the logical circuits diagram of minimized expression using basic gates. **W-13, 4 Marks**

n. :

$$Y = \Sigma m\,(0, 1, 3, 4, 5, 6, 7, 13, 15)$$

The required K-map is as shown in Fig. P. 4.7.9.



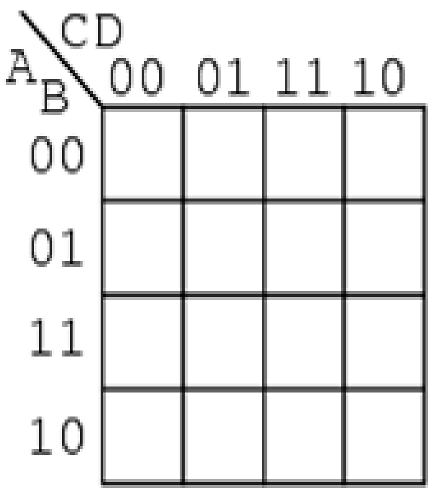
$$\therefore\ Y = \bar{A}\bar{C} + \bar{A}D + \bar{A}B + BD$$

4.Minimize the following expressions using K-map :

a. $Y = \Sigma m\,(1,5,6,7,11,12,13,15)$

b. $F = \Sigma m\,(0,1,2,3,11,12,14,15)$

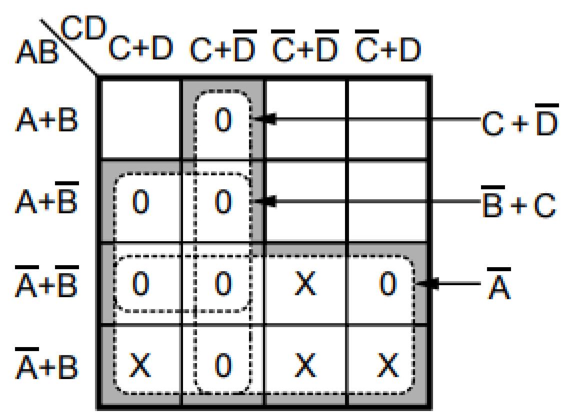
Soution:Simplification using K-Map is as shown below



$$F = \sum (0, 2, 3, 6, 7) + d\,(8, 10, 11, 15)$$

$$= \pi\,(1, 4, 5, 9, 12, 13, 14) + d\,(8, 10, 11, 15)$$

$$\therefore \quad F = (C + \overline{D}) \cdot (\overline{B} + C) \cdot \overline{A}$$



THANK YOU

Practice Questions

Numericals-

SOP POS- Standard to canonical conversion

Numericals covered in classroom (Refer ppt no 10)

K-map - SOP

K-Map- POS

Kmap with don’t care