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Set:→ A well defined collection of objects.

Finite set:→ A set containing finite elements.

Infite set:→ A set containing infinite number of elements.

Writting procudure of a set:→

1) Roaster method / tabular
2) Set builder form.

Exp:→ Set of first 10 natural numbers

$S = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$ → Roaster

$S = \{x : x \in N, 1 \leq x \leq 10\}$

Cardinality of a set:→ no. of different

$T = \{1, 1, 2, 3\} = \{1, 2, 3\}$, $|T| = 3$

→ no of different elements in a set A is the cardinality of the set A. It is denoted by $|A| = card(A)$

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infinite | Exp → $\mathbb{Q}$, $\mathbb{N}$, $\mathbb{R}$, $\mathbb{Q}^c$, infinite set.

$Q = \{ x : x = p/q ; p, q \text{ are integers}, q \neq 0 \}$

finite | Exps → $S = \{a, e, i, o, u\}$ → finite set $|S| = 5$

☆ **Subset :→** Let S be a non-empty set then set A is said to be subset of S iff all elements of A are elements of S also.

Note :– Element of a set : $S = \{a, b, c\}$
$a \in S$, $b \in S$,
$c \in S$, $d \notin S$

$\Omega = \{A, B, C, D, \ldots\ldots, Z\}$

$A = \Omega \ldots\ldots\ Z \in \Omega,\ a \notin \Omega$

→ A subset is denoted by $A \subseteq S$;
$A \subseteq S$, $A = \{x : x \in S\} \subseteq S$
$A \Rightarrow \{x : \text{for some } x \in S\}$

$S = \{a, b, c\}$

improper set

$A_1 = \{a\}$, $A_2 = \{a, b\}$, $A_3 = \{a, b, c\}$, $A_4 = \{b\}$, $A_5 = \{a, c\}$
$A_6 = \{b, c\}$, $A_7 = \{c\}$, $A_8 = \{\ \} = \emptyset$

improper set

$A_1 \subseteq S$, $A_8 \subseteq S$, $A_5 \subseteq S$, $A_3 \subseteq S$, $A_6 \subseteq S$

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☆ Proper subset :→ $A_i \subset S$, $i \neq 3, 8$.

☆ improper subset :→ set itself, and ϕ are improper subset.

Exp :→ $A = \{1,2,3,4,5\}$, $B = \{2,3,6\}$, $C = \{1,3,5\}$

$B = \{2,6\} \subseteq C$ $\quad B - \{6\} \subseteq A \quad C - \{1,5\} \subseteq B \quad C \nsubseteq B \quad \cancel{C \in} C \subseteq A$

$A \nsubseteq B$, $B \nsubseteq C$, $C \subseteq A$, $B \cancel{\subseteq} A$, $C \nsubseteq B$, $\cancel{C \in B}$, $\cancel{C \in A}$,

$6 \notin A$, $6 \nsubseteq A$ $\{2,3\} \nsubseteq C$, $6 \in A$

☆ Types of set :→

Null set :→ ϕ or $\{\}$ (Empty set)

Singleton set :→ $\{1\}$, $\{a\}$, $\{\{\}\}$, $\{\phi\}$, $\{\phi, \phi\}$, $\{\{\phi\}\}$, $\{\{a\}\}$

$C = \{x : 2x = 10 \ \& \ x \in N\}$

☆ Equality of two sets :→

$A = B$

iff $A \subseteq B$ & $B \subseteq A$

def $A \subseteq B$

$\forall x \in A \Rightarrow x \in B$

Disjoint set :→ $A \cap B = \emptyset$

☆ **Operations** :→

Intersection
$$A \cap B = \{x : x \in A \ \& \ x \in B\}$$

Union
$$A \cup B = \{x : x \in A \text{ or } x \in B\}$$

☆ Compliment of a set

If U is an universal set the compliment of A is defined and denoted as

$$A^c \text{ or } A' = \{x : x \notin A,\ x \in U\}$$

☆ Difference : $A - B = \{x : x \in A \ \& \ x \notin B\}$
$= \{x : x \in A \ \& \ x \in B'\}$
$= A \cap B^c$

☆ Algebric Law of Set :→

1. Idempotent Law :→ $A \cup A = A$, $A \cap A = A$.
2. Commutative Law :→ $A \cup B = B \cup A$, $A \cap B = B \cap A$
3. ~~Associative~~ Distributive law :→ 1) $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$
→ 2) $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$
4. Associative law :→ 1) $A \cup (B \cup C) = (A \cup B) \cup C$
2) $A \cap (B \cap C) = (A \cap B) \cap C$

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☆ De Morgan's Law :→

1) $(A \cup B)' = A' \cap B'$
2) $(A \cap B)' = A' \cup B'$

☆ Inclusion & Exclusion Principal :→

1) $n(A \cup B) = n(A) + n(B) - n(A \cap B)$
2) $n(A_1 \cup A_2 \cup A_3) = \sum_i n(A_i) - \sum_{i \neq j} (A_i \cap A_j) + n(A_1 \cap A_2 \cap A_3)$

$= n(A_1) + n(A_2) + n(A_3) - n(A_1 \cap A_2) - n(A_1 \cap A_3) - n(A_2 \cap A_3) + n(A_1 \cap A_2 \cap A_3)$.

ingeneral.

Q)→ In a group of 100 person 70 people can speak in English and 45 can speak in hindi ① find the number of person who can in english only ② can speak in Hindi only ③ can ~~to~~ speak both in English & Hindi.

Solun.:→ $n(E \cup H) = 100$
$n(E) = 70$
$n(H) = 45$

① $n(E - H) = ? \rightarrow n(E) - n(E \cap H) = 70 - 15 = 55$
② $n(H - E) = ? \rightarrow n(H) - n(E \cap H) = 45 - 15 = 30$
③ $n(H \cap E) = ? \Rightarrow n(H) + n(E) - n(H \cup E)$
$\Rightarrow 45 + 70 - 100 \Rightarrow 15$

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Q] A computer programmer must hire 20 programmers to handle system programming jobs and 30 programmers to handle application programming jobs of hired, 5 expected to perform jobs of both type. How many programmers must be hired

Q) Out of 250 students who were failed in exams, it was revealed that 128 failed in mathematics, 87 in Physics and 134 in aggregate. 31 failed in math and physics, 54 failed in aggregate and maths, 30 failed in aggregate and physics.
Find how many candidate failed

1. All 3 subs
2. in math not in physics.
3. In aggregate but Not maths
4. In aggregate but not maths nor physics
5. In physics but not in agregate or maths.

Solun:→

128 → M → n(A) = 128
87 → n(B) = 87
134 → Aggregate → n(C) = 134
31 → M & P. → $n(A \cap B) = 31$
54 → Agg & M → $n(C \cap A) = 54$
30 → Agg & Physics → $n(B \cap C) = 30$

$$n(A\cup B) = n(A) + n(B) - n(A\cap B)$$

250

$$n(A\cup B\cup C) = n(A) + n(B) + n(C) - n(A\cap B) - n(A\cap C) - n(B\cap C) - n(A\cap B\cap C)$$

a) $n(A\cap B\cap C) = 16$

b) $n(A) - n(A\cap B) = 97$

c) $n(C) - n(C\cap A) = 80$

d) $\cancel{n(B)}$ $n(B) - n((C\cup A)\cap B)$ $(C\cup A)\cap B$

$n(C\cup A) - n((C\cup A)\cap B)$

1) $n(M\cap P\cap A) = -n(M) - n(P) - n(A) + n(M\cap P) + (M\cap A) + n(P\cap A) + n(M\cup P\cup A).$

4) $n[(A\cup M) - P] = n(A\cup M) - n[(A\cup M)\cap P]$

$n[(A\cup M)\cap P] = n[(A\cap P)\cup(M\cap P)] = n(A\cap P) + n(M\cap P) - n[(A\cap P)\cap(M\cap P)]$

$= n(A\cap P) + n(A\cap P) - n[A\cap P\cap M]$

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☆ De Morgan's law :→

1) $(A \cup B)' = A' \cap B'$

i] $(A \cup B)' \subseteq A' \cap B'$
ii] $A' \cap B' \subseteq (A \cup B)'$

$\left.\right\} (A \cup B)' = A' \cap B'$

1) Let $x \in (A \cup B)' \Rightarrow x \notin A \cup B$
$\Rightarrow x \notin A$ and $x \notin B$
$\Rightarrow x \in A' \cap B' \Rightarrow (A \cup B)' \subseteq A \cap B$

→ $x \in A \cup B \Rightarrow x \in A$ or $x \in B$
⇒ $x \notin A \cup B \Rightarrow x \notin A$ and $x \notin B$
⇒ $x \in A \cap B \Rightarrow x \in A$ and $x \in B$
⇒ $x \notin A \cap B \Rightarrow x \notin A$ or $x \notin B$.

Aim 2 :→

Let $x \in A' \cup B'$
$\Rightarrow x \in A'$ or $x \in B'$
$\Rightarrow x \notin A$ or $x \notin B$
$\Rightarrow x \notin A \cap B$
$\rightarrow x \in (A \cap B)'$
$\Rightarrow A' \cup B' \subseteq (A \cap B)' \rightarrow$ (ii)

$\boxed{x \notin A' \Leftrightarrow x \in A}$

∴ from (i) & (ii) :→
$A' \cup B' = (A \cap B)'$

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1) $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

2) $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

3) $A \cup B = B \cup A$

4) $A \cap B = B \cap A$

4) $A - B \neq B - A$

$A = \{1, 2, 3\}$ $B = \{2, 3, 4\}$

$A - B = \{1\}$, $B - A = \{4\}$.

★ Symmentic Difference :→

$$A \Delta B = (A \cup B) - (A \cap B)$$

Another form $(A-B) \cup (B-A)$

$(B-A) \cup (A-B)$

Δ is commutative.

$A \Delta A = \phi$ does not follow idempotent law.

Q) If $|A| = 3$ $|B| = 5$ then find $|A-B|$, $|B-A|$, $|A \cup B|$, $|A \Delta B|$, $|A \cap B|$, $|A'|$, $|B'|$.

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* Power Set :→

The collection of all the subsets of a set A is known as power set It is denoted by $P(A)$, If $|A| = n$

Ex:→ $A = \{1,2\}$

$P(A) = \{\phi, A, \{1\}, \{2\}\}$

Q] Prove that :→

0):→ $A - B = A \cap B^c$

i):→ $A - (B \cup C) = (A-B) \cap (A-C)$

ii):→ $((A \cap B) - C) = (A-C) \cap (B-C)$

iii):→ $A \cap (B-C) = (A \cap B) - (A \cap C)$

iv):→ $A-(B-C) = (A-B) \cup (A \cap C)$

v):→ $A \cap (B-C) = (A \cup B) - (C-A)$

vi):→ $A \cap (B-C) = (A \cap B) - (A \cap C)$

vii):→ $A - B = B' - A'$

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Aim ①: $A-B \subseteq A \cap B'$ or if $x \in A-B \Rightarrow x \in A \cap B'$

Aim ②: $A \cap B' \subseteq A-B$ or if $x \in A \cap B \Rightarrow x \in A-B$

⇒ Proof of Aim 1:

As $x \in A-B \Rightarrow x \in A$ and $x \notin B$

$\Rightarrow x \in A$ and $x \in B'$

$\Rightarrow x \in (A \cap B')$

$\Rightarrow A-B \subseteq A \cap B' \longrightarrow ①$

⇒ Proof of Aim 2:

As $x \in A \cap B' \Rightarrow x \in A$ and $x \in B'$

$\Rightarrow x \in A$ and $x \notin B$

$\Rightarrow x \in A-B$

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Solu^n:→ 1)

$$LHS = A - (B \cup C)$$
$$= A \cap (B \cup C)' \quad (\because C - D = C \cap D')$$
$$= A \cap (B' \cap C') \quad (\text{demorgan's law})$$

$$RHS = (A - B) \cap (A - C)$$
$$= (A \cap B') \cap (A \cap C')$$
$$= (B' \cap A) \cap (A \cap C') \quad \text{using commutative law}$$
$$= B' \cap (A \cap A) \cap C'$$
$$= B' \cap A \cap C' \quad (\text{associative law})$$
$$= A \cap B' \cap C' \quad (\text{idompentant law})$$
$$= A \cap (B' \cap C')$$

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☆ Cartesian Product :→

$$A \times B = \{(x, y) : x \in A, y \in B\}$$

Exp :→

$$A = \{1, 2\}, \quad B = \{1, 3\}$$

$$A \times B = \{(1,1), (1,3), (2,1), (2,3)\}$$

$$2 \times 2 = 4$$

☆ Note :→ $|A| = m, \ |B| = n \qquad |A \times B| = m \times n$

☆ Note :→ $A \times B \neq B \times A$

$$B \times A = \{(1,1), (1,2), (3,1), (3,2)\}$$

☆ Note :→ If $|A \cap B| = n$

∴ A & B are finite set.

$$|(A \times B) \cap (B \times A)| = n^2$$

Proof's Exp :→

$$|A \cap B| = 1$$

$$(A \times B) \cap (B \times A) = \{(1,1)\}$$

$$|(A \times B) \cap (B \times A)| = 1^2$$

$2+(3\times 5) = (2+3)\times(2+5)$ ✗

$2\times(3+5) = (2\times 3)+(2\times 5)$ ✓

☆ <u>Properties</u> - for four sets, A, B, C & D :→

i) $(A\cap B)\times(C\cap D) = (A\times C)\cap(B\times D)$

ii) $(A-B)\times C = (A\times C)-(B\times C)$

iii) $(A\cup B)\times C = (A\times C)\cup(B\times C)$

iv) $A\times(B\cap C) = (A\times B)\cap(A\times C)$

☆ <u>Relation from A to B</u> :→

Any subset of $A\times B$ is known as a relation from A to B.

☆ <u>Relation on A</u> :→

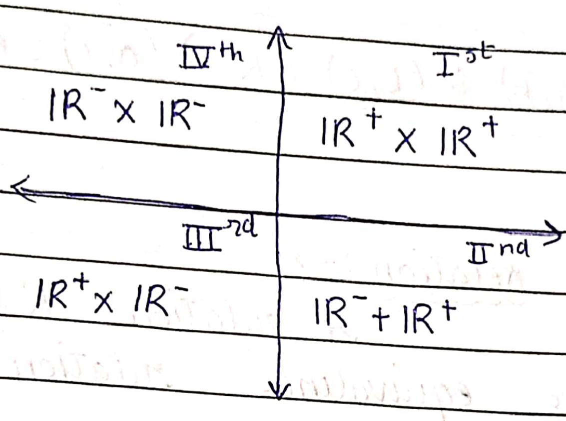
Any subset R of $A\times A$ is known as a relation on A.

Q1] How many no. of relations can be defined from A to B. if $|A| = m$ & $|B| = n$.

Q2] How many no. of relations can be defined on A, if $|A| = n$.

1^{st} quadrant

$$\mathbb{R}^+ \times \mathbb{R}^+ = \{(x, y) \; x, y, \in \mathbb{R}^+\}$$



☆ Relation from A to B :→ $R \subseteq A \times B$

☆ Relation on A :→ $R \subseteq A \times A$

☆ Reflexive Relation :→ A relation $R \subseteq A \times A$ is said to be reflexive

if $\forall \, a \in A \Rightarrow (a, a) \in R$

☆ Symmetric Relation :→ A relation $R \subseteq A \times A$ is symmetric if :→

$$\boxed{\text{if } (a, b) \in R \Rightarrow (b, a) \in R}$$

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☆ **Transitive Relation** :→ A relation $R \subseteq A \times A$ is said to be transitive.

$$\boxed{\text{If } (a,b) \ \& \ (b,c) \in R \Rightarrow (a,c) \in R}$$

☆ **Equivalence Relation** :→ A relation $R \subseteq A \times A$ is said to be equivalence relation if it is :-

i) Reflexive :- $\forall\ a \in A \Rightarrow (a,a) \in R$
ii) Symmetric :- $(a,b) \in R \Rightarrow (b,a) \in R$
iii) Transitive :- $(a,b)(b,c) \in R \Rightarrow (a,c) \in R$.

Q] 1) If $A = \{\alpha, \beta\}$ Find all the relations defined on A. Also identify that which relations are : →

1) Reflexive
2) Symmetric
3) Transitive
4) Equivalence
5) Reflexive but not symmetric.
6) Reflexive but not transitive.
7) Symmetric but not reflexive.
8) Symmetric but not transitive.
9) Transitive but not reflexive.
10) Transitive but not symmetric.
11) Neither reflexive nor symmetric nor transitive.

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Q2] Find all possible relation from A to B.
where :- $A = \{1,2\}$, $B = \{a,b\}$.

Soln 1 :→ $A = \{\alpha, \beta\}$ $\{\alpha, \beta\}$

$A \times A = \{(\alpha,\alpha), (\alpha,\beta), (\beta,\alpha), (\beta,\beta)\}$

$R_1 = \{\} \subseteq A \times A$, ${}^4C_0 = 1$

$R_2 = \{(\alpha,\alpha)\} \subseteq A \times A$, ${}^4C_1 = 4$

$R_3 = \{(\alpha,\beta)\} \subseteq A \times A$.

$R_4 = \{(\beta,\alpha)\} \subseteq A \times A$.

$R_5 = \{(\beta,\beta)\} \subseteq A \times A$.

$R_6 = \{(\alpha,\alpha), (\alpha,\beta)\} \subseteq A \times A$: ${}^4C_2 = \frac{\lfloor 4}{\lfloor 4-2 . \lfloor 2} = \frac{4 \times 3}{2} = 6$

$R_7 = \{(\alpha,\alpha), (\beta,\alpha)\} \subseteq A \times A$

$R_8 = \{(\alpha,\alpha), (\beta,\beta)\} \subseteq A \times A$.

$R_9 = \{(\alpha,\beta), (\beta,\alpha)\} \subseteq A \times A$.

$R_{10} = \{(\alpha,\beta), (\beta,\beta)\} \subseteq A \times A$.

$R_{11} = \{(\beta,\alpha), (\beta,\beta)\} \subseteq A \times A$.

$R_{12} = \{(\alpha,\alpha), (\alpha,\beta), (\beta,\alpha)\} \subseteq A \times A$. ${}^4C_3 = \frac{\lfloor 4}{\lfloor 3 . \lfloor 1} = 4$

$R_{13} = \{(\alpha,\beta), (\beta,\alpha), (\beta,\beta)\} \subseteq A \times A$.

$R_{14} = \{(\alpha,\alpha), (\beta,\alpha), (\beta,\beta)\} \subseteq A \times A$.

$R_{15} = \{(\alpha,\alpha), (\alpha,\beta), (\beta,\beta)\} \subseteq A \times A$.

$R_{16} = \{(\alpha,\alpha), (\alpha,\beta), (\beta,\alpha), (\beta,\beta)\} \subseteq A \times A$. ${}^4C_4 = 1$

Algebraic laws of set:

① Idempotent laws : $A \cup A = A$, $A \cap A = A$

② Commutative law : $A \cup B = B \cup A$, $A \cap B = B \cap A$

③ ~~Associative law :~~

④ Distributive law : (i) $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

(ii) $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

④ Associative law : $A \cup (B \cup C) = (A \cup B) \cup C$,

$A \cap (B \cap C) = (A \cap B) \cap C$

De-morgan's laws.

i) $(A \cup B) = A \cap B$

ii) $(A \cap B) = A \cup B$

inclusive & exclusive principle.

i) $n(A \cup B) = n(A) + n(B) - n(A \cap B)$

ii) $n(A_1 \cup A_2 \cup A_3) = \sum_i n(A_i) - n(A \cap B) - n(A \cap C) - n(B \cap C)$

$+ n(A \cap B \cap C)$

$= A_i \cap A_j$

$i \neq j$

$= n(A_1) + n(A_2) + n(A_3) - n(A_1 \cap A_2) -$

$n(A_2 \cap A_3) - n(A_1 \cap A_3) + n(A_1 \cap A_2 \cap A_3)$

Q. In ~~a~~ group of 100 person 70 people can speak english, 45 can speak in hindi
(1) Find the no. of person who can speak in english only
(2) Hindi only
(3) both eng. & Hindi.

→ $n(E \cup H) = 100$
$n(E) = 70$
$n(H) = 45$

1) $n(E-H) = n(E) - n(E \cap H) = 70 - 15 = 55$
2) $n(H-E) = n(H) - n(E \cap H) = \cancel{70-45}\ 45-15 = \cancel{25}\ 30$
3) $n(H \cap E) = n(H) - n(E) - n(H \cup E) = 15$

torial:
Q. How many expressions there in:
i) $n(A \cap B \cap C) \rightarrow {}^3C_1 + {}^3C_2 + {}^3C_3 \rightarrow 7 \rightarrow 2^3 - 1 = 7$
ii) $n(A \cap B \cap C \cap D) \rightarrow {}^4C_1 + {}^4C_2 + {}^4C_3 + {}^4C_4 \rightarrow 4 + 12 + 4 + 1 = 2^4 - 1 = 1$
iii) $n(A \cup B \cup C \cup D) \rightarrow 16 \rightarrow 2^4 - 1$

Q. In a class of 25 students 12 have taken maths 8 have taken mathematics but not biology. find no. of students who have taken maths & biology also find no. of students who have taken bio but not maths.
→ $n(M) = 12$ $n(M \cup B) = 25$ $n(M - B) = 8$
$= n(M) - n(M \cap B)$
$= 12 - 8 = 4$

$n(M \cap B)$?
$n(M \cup B) = n(M) + n(B) - n(B \cap M)$

$25 = n(B) + 8$

$n(B) = 17$

$n(B \cap M) = 17 + 12 - 25$

$\boxed{n(B \cap M) = 4}$

$n(B - M) = n(B) - n(M \cap B)$

$= 17 - 4$

$\boxed{n(B - M) = 13}$

Q. Out of 250 candidates who failed in an examination it was revealed that 128 failed in maths, 87 in physics, 134 failed in aggregate 31 failed in maths & physics, 54 failed in aggregate & maths, 30 failed in aggregate & physics. Find how many candidates failed in

i) All 3 sub.

ii) failed in maths but not in physics

iii) failed in aggregate but not in maths

iv) failed in physics but not in aggregate

v) failed in aggregate ~~and~~ or maths but not in physics.

→ $n(P \cup M \cup A) = 250$

$n(M) = 128$

$n(P) = 87$

$n(A) = 134$

$n(M \cap P) = 31$

$n(M \cap A) = 54$

$n(P \cap A) = 30$

$250 = 128 + 87 + 134 - 31 - 54 - 30 + n(P \cap M \cap A)$

i) $n(P \cap M \cap A) = 250 - 234 = 16$

iii) $n(A) - n(A \cap M) = 134 - 54 = 80$

ii) $128 - 31 = 97$

iv) $n(P) - P \cap (M \cup A)$

$n(P) - (P \cap M) \cup (P \cap A)$

$= n(P) - n(P \cap M) - n(P \cap A) + n(P \cap M \cap A)$

$= 87 - 31 - 30 + 16$

$= 42$

v) $n(A \cup M) \rightarrow n[A-(B\cup C)] = n(A) - n(A\cap(B\cup C))$

$n(A\cup M) - n(P\cap(A\cup M))$

$n(P\cap(A\cup M) = n(P\cup A) \cup n(P\cap M)$

$= 30 + 31 - 16$

$= 45$

$= 134 + 128 - 54 - 45$

$= 163.$

De Morgan's law:

1) $(A\cup B)' = A'\cup B'$

Two methods to find

i) $(A\cup B)' \subseteq A'\cap B'$

ii) $A'\cap B' \subseteq (A\cup B)'$

math (i) let $x \in (A\cup B)' \Rightarrow x \notin A\cup B \Rightarrow x\notin A$ and $x \notin B$

$\Rightarrow x \in A'$ and $x \in B'$

~~$\Rightarrow x \in A$ and~~

$\Rightarrow x \in A'\cap B' \Rightarrow (A\cup B)$

$(A\cap$

2) $(A\cap B)' = A'\cup B'$

i) $(A\cap B)' \subseteq A'\cup B'$ ii) $A'\cup B' \subseteq (A\cap B)'$

i) let $x \in (A\cap B) \Rightarrow x \notin A\cap B$

$\Rightarrow x \notin A$ or $x \notin B$

$\Rightarrow x \in A'$ or $x \in B'$

$\Rightarrow x \in A'\cup B'$

$\Rightarrow A\cap B \subseteq A'\cup B'$

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i) $x \in A \cup B \Rightarrow x \in A$ or $x \in B$
ii) $x \notin A \cup B \Rightarrow x \notin A$ and $x \notin B$
iii) $x \in A \cap B \Rightarrow x \in A$ and $x \in B$
iv) $x \notin A \cap B \Rightarrow x \notin A$ or $x \notin B$
v) $x \in A' \Rightarrow x \notin A$
vi) $x \notin A \Rightarrow x \in A'$, $x \notin A' \Rightarrow x \in A$

(ii) $A' \cup B' \subseteq (A \cap B)'$
Let $x \in A' \cup B'$
$\Rightarrow x \in A'$ or $x \in B'$
$\Rightarrow x \notin A$ or $x \notin B$
$\Rightarrow x \notin A \cap B$
$\Rightarrow x \notin (A \cap B)$
$\Rightarrow A' \cup B' \subseteq (A \cap B)'$

(i)(ii) $A' \cap B' \subseteq (A \cup B)'$
Let $x \in A' \cap B'$
$\Rightarrow x \in A'$ and $x \in B'$
$\Rightarrow x \notin A$ and $x \notin B$
$\Rightarrow x \notin A \cup B$
$\Rightarrow x \in (A \cup B)'$
$\Rightarrow A' \cap B' \subseteq (A \cup B)'$

proof the following
i) $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$
ii) $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$
iii) $A \cup B = B \cup A$
iv) $A \cap B = B \cap A$
v) $A - B \neq B - A$

Symmetric difference.

$B \Delta A = (B \cup A) - (B \cap A)$

$A \Delta B = (A \cup B) - (A \cap B)$

another form:

$(A - B) \cup (B - A)$

$(B - A) \cup (A - B)$

Δ is commutative

$A \Delta A = \phi$ does not follow idempotent law.

~~power sets~~

Q. If $|A| = 3$
$|B| = 5$, then find $|A - B|$, $|B - A|$, $|A \Delta B|$, $|A \cap B|$, $|A \cup B|$, $|A'|$, $|B'|$.

$|A \cup B|$, $|A \cap B|$ ⇒ insufficient info. (can decide its ran

$0 \le |A \cap B| \le 3$; $0 \le |A - B| \le 3$;

$5 \le |A \cup B| \le 8$.

power set.

The collection of all the subsets of a set A is known as power set. It is denoted by P(A

If $|A| = n$, then
$|P(A)| = 2^n$

Exp: $A = \{1, 2\}$ $|A| = 2$, $|P(A)| = 2^2 = 4$

$P(A) = \{\phi, A, \{1\}, \{2\}\}$

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Tutorial:

Q. How many elements in $A \times B$ and $B \times A$ are common. If n elements are common to A and B.

Cartesian product.

$A = \{1, 2\}$, $B = \{a, b\}$

$A \times B = \{(x, y) : x \in A \text{ and } y \in B\}$

$= \{(1, a), (1, b), (2, a), (2, b)\}$

Eg: $A = \{1, 2\}$, $B = \{1, a\}$

$\rightarrow$ $A \times B = \{(1,1), (1,a), (2,1), (2,a)\}$, $B \times A = \{(1,1), (1,2), (a,1), ($

$A \cap B =$

~~Q. $|A| = 2$, $|B| = 3$, $|A \cap B| = 1$ then $|A \times B| = ?$~~

Eg: $A = \{1, 2, a\}$, $B = \{1, a, 3\}$, $A \cap B = (?)$

$A \times B = \{(1,1), (1,a), (1,3), (2,1), (2,a), (2,3), (a,1), (a,a), (a,3)\}$

$B \times A = \{(1,1), (1,2), (1,a), (a,1), (a,2), (a,a), (3,1), (3,2), (3,a)\}$

$|A \times B \cap B \times A| = \{4\}$

~~# Eg:~~ If $|A| = n$, $|B| = m$

$|A \times B| = n \times m$.

Q. Find power set of $A = \{1, 2, \phi, \{1,2\}\}$

$\rightarrow$

$P(A) = 2^4 = 16$

$^4C_1 = \frac{L4}{1 \times 1} = 4$

$P(A) = \{A_1, A_2, A_3 \text{ ------ } A_{16}\}$

$A_1 = \{\} = \phi$, $A_2 = \{1\}$, $A_3 = \{2\}$, $A_4 = \{\phi\}$, $A_5 = \{\{1$

$A_6 =$

$\frac{4\times3\times2!}{2!\times2!}$, $\frac{4!}{2!\,2!}$, $\frac{n!}{(n-r)!\,r!}$ ^nC_r

$4C_2 = \frac{\lfloor 4}{\lfloor 2 \lfloor 2} = 6$ $A_6 = \{1, 2\}$, $A_7 = \{1, \phi\}$, $A_8 = \{1, \{1,2\}\}$, $A_9 = \{2, \phi\}$
$A_{10} = \{2, \{1,2\}\}$, $A_{11} = \{\phi, \{1,2\}\}$

$4C_3 = \frac{\lfloor 4}{\lfloor 1 \lfloor 3} = 4$ $A_{12} = \{1, 2, \phi\}$, $A_{13} = \{1, 2, \{1,2\}\}$, $A_{14} = \{2, \phi, \{1,2\}\}$
$A_{15} = \{1, \phi, \{1,2\}\}$

$4C_4 = 1$ $A_{16} = \{1, 2, \phi, \{1,2\}\}$

Q. Find power set of $A = \{(a,b), c\}$, $P(A) = ?$

Q.2 If $A = \{1, 2\}$, $B = \{2, 3\}$, $C = \{3, 5\}$.
find :
1) $A \times (B \cup C) \longrightarrow$ 6
2) $(A \times B) \cup (A \times C) \longrightarrow$
3) $A \times (B \cap C) \longrightarrow$
4) $A \times B \times C$.
Q. 3) $(A \times B) \cup (B \times A) \longrightarrow$ 7

→ 1. $P(A) = 2$, $\varepsilon P(A) = 2^2 = 4$

$A_1 = \{\}$, $A_2 = \{(a,b)\}$, $A_3 = \{c\}$, $A_4 = \{(a,b), c\}$

→ 2. $A \times B = \{(1,2), (1,3), (2,2), (2,3)\}$
$A \times C = \{(1,3), (1,5), (2,3), (2,5)\}$
$(A \times B) \cup (A \times C) = \{(1,2), (1,3), (2,2), (2,3), (1,5), (2,5$

→ 3. $B \cap C = \{3\}$ $A \times (B \cap C) = \{(1,3), (2,3)\}$

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class:

Cartesian products.

$A \times B = \{(x, y) : x \in A, y \in B\}$

Eg: $A = \{1, 2\}$, $B = \{1, 3\}$

$A \times B = \{(1,1), (1,3), (2,1), (2,3)\}$

NOTE: 1) If $|A| = m$, $|B| = n$ $|A \times B| = m \times n$

2) $A \times B \neq B \times A$, If A & B have diff. element.

3) If $|A \cap B| = n$, A & B are finite set, $[|(A \times B) \cap (B \times A)|$

for previous eg: $B \times A = \{(1,1), (1,2), (3,1), (3,2)\}$

$|A \cap B| = 1 = n$

$(A \times B) \cap (B \times A) = \{(1,1)\}$

$|(A \times B) \cap (B \times A)| = 1^2 \to n^2$

properties: for four sets, A, B, C & D.

1) $(A \cap B) \times (C \cap D) = (A \times C) \cap (B \times D)$
2) $(A - B) \times C = (A \times C) - (B \times C)$
3) $(A \cup B) \times C = (A \times C) \cup (B \times C)$
4) $A \times (B \cap C) = (A \times B) \cap (A \times C)$

Relation:

- Relation from A to B: Any subset R of $A \times B$ is known as a relation from A to B.
- Relation on A: Any subset R of $A \times A$ is known as a relation on A.

Q.1) How many no. of relations can be defined from A to B. if $|A| = m$ & $|B| = n$

Q.2) How many no. of relations can be defined on if $|A| = n$

unit - 2

A statement or proposition is a declarative sentence, which is either true or false but not both. Statement are denoted by small letters.

Eg: (1) The earth is round. ⟶ statement.

(2) Do you speak english. ⟶ sentence.

(3) $x + 2 = 5$ ⟶ sentence

If the value of x is given, then it will become statement.

(4) The sun will come out tomorrow. ⟶ stateme

This is the statement which is either true or false, although we have to wait until tomorrow.

T and F, called the truth values of the statement.

Negation: Let p be the statement. Then negation of p is denoted by $\sim p$ or $\neg p$.

Truth table

p	$\sim p$
T	F
F	T

Eg: p: Tea is hot.

$\sim p$: Tea is not hot.

Conjunction

If p & q are two statements, then their conjunction is the compound statement "p and q". It is denoted by $p \wedge q$.

e.g: let $p: 2+3=4$. & q: sky is blue.

Then $p \wedge q$: $2+3=4$ and sky is blue.

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TI-C

p	q	p∧q
T	T	T
T	F	F
F	T	F
F	F	F

Disjunction:
If p & q are two statements, then thei disjunction is a compound statement give by "p or q". It is denoted by p∨q.

e.g: p: I will go to college by bus.
q: I will go to college by bike.
Then p∨q : I will go to college by bu ~~by~~ bik

Truth Table:

p	q	p∨q
T	T	T
T	F	T
F	T	T
F	F	F

Conditional statement or implication
If p & q are 2 statements, then the compound statement "If p, then q" is ca conditional statement or one way implic It is denoted by $p \Rightarrow q$ (or $p \rightarrow q$).

Eg: let p: you work hard
& q: you will score good marks.
$p \Rightarrow q$: If you work hard, then you will sc good marks.

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The statement p is called antecedant / hypothesis and statement q is called consequent / conclusion.

Truth table:

p	q	$p \Rightarrow q$
T	T	T
T	F	F
F	T	T
F	F	T

Biconditional statements

If p and q are two statements then the compound statement "p if and only if q" is called biconditional. It is denoted by $p \Leftrightarrow q$. (i.e p implies and implied by q).
i.e $p \equiv q$ (i.e p and q are equivalent)

Note: $(p \Leftrightarrow q) \equiv (p \Rightarrow q) \wedge (q \Rightarrow p)$

p	q	$p \Leftrightarrow q$
T	T	T
T	F	F
F	T	F
F	F	T

Converse, Inverse & Contra positive

Let $p \Rightarrow q$ be the conditional statement
Then ① Its converse is $q \Rightarrow p$
② Its inverse is $\sim p \Rightarrow \sim q$
③ Its contra positive is $\sim q \Rightarrow \sim p$

Note: $(p \Rightarrow q) \equiv (\sim q \Rightarrow \sim p)$

Algebra of proposition

① commutative : $p \vee q \equiv q \vee p$
& $p \wedge q \equiv q \wedge p$

② Associative : $p \vee (q \vee r) \equiv (p \vee q) \vee r$
& $p \wedge (q \wedge r) \equiv (p \wedge q) \wedge r$

③ Distributive ① $p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$
② $p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$

i.e ① $\vee$ is distributive over and
② $\wedge$ is distributive over or

④ Idempotent prop : (i) $p \vee p \equiv p$ (ii) $p \wedge p \equiv p$

⑤ properties of negation : $\sim(\sim p) \equiv p$

$$\left.\begin{aligned} \sim(p \vee q) &\equiv \sim p \wedge \sim q \\ \sim(p \wedge q) &\equiv \sim p \vee \sim q \end{aligned}\right\} \text{De Morgan Law}$$

Q. prove that :
i) $(p \Leftrightarrow q) \equiv (p \Rightarrow q) \wedge (q \Rightarrow p)$
ii) $\sim(p \vee q) \equiv \sim p \wedge \sim q$
iii) $\sim(p \wedge q) \equiv \sim p \vee \sim q$
iv) $\sim(p \Rightarrow q) \equiv p \wedge \sim q$

unit-1

~~Ye8~~ ~~IXC~~ Yes IXC [Relation]

Reflexive Relation :
A relation $R \subseteq A \times A$ is said to be reflexive if $\forall a \in A \Rightarrow (a, a) \in R$

symmetric relation:
A relation $R \subseteq A \times A$ is symmetric if:
$(a, b) \in R \Rightarrow (b, a) \in R$

Transitive relation:
A relation $R \subseteq A \times A$ is said to be transitive. If (a, b) & $(b, c) \in R \Rightarrow (a, c) \in R$

Equivalence relation :
A relation $R \subseteq A \times A$ is said to be equivalence rela if it is :
i) Reflexive : $\forall a \in A \Rightarrow (a, a) \in R$
ii) Symmetric : $(a, b) \in R \Rightarrow (b, a) \in R$
iii) Transitive : (a, b) & $(b, c) \in R \Rightarrow (a, c) \in R$

Ques. If $A = \{\alpha, \beta\}$ find all the relations defined on A. Al identify that which relations are :
i) Reflexive
ii) symmetric
iii) Transitive
iv) Equivalence
v) Reflexive but not symmetric
vi) Reflexive but not transitive
vii) Symmetric but not reflexive
viii) Symmetric but not transitive
ix) Transitive but not re
x) Transitive but not sym
xi) neither reflexive nor sy nor transitive.

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Q.2) Find all possible relation from A to B
where $A = \{1, 2\}$ $B = \{a, b\}$

Q.1)
→ $A = \{\alpha, \beta\}$

$A \times A = \{(\alpha,\alpha), (\alpha,\beta), (\beta,\beta), (\beta,\alpha)\}$

$|A| = 2$, $|A \times A| = 4$, $(P(A\times A)) = 2^4 = 16$

$\underline{{}^4C_0 = 1}$ $R_1 = \{\} \subseteq A \times A$

$\underline{{}^4C_1 = 4}$ $R_2 = \{(\alpha,\beta)\}$, $R_3 = \{(\beta,\alpha)\}$, $R_4 = \{(\alpha,$

$R_5 = \{(\beta,\beta)\}$

$\underline{{}^4C_2 = 6}$

$R_6 = \{(\alpha,\beta), (\beta,\alpha)\}$ $R_9 = \{(\beta,\alpha), (\alpha,\alpha)\}$
$R_7 = \{(\alpha,\beta), (\alpha,\alpha)\}$ $R_{10} = \{(\beta,\alpha), (\beta,\beta)\}$
$R_8 = \{(\alpha,\beta), (\beta,\beta)\}$ $R_{11} = \{(\alpha,\alpha), (\beta,\beta)\}$

$\underline{{}^4C_3 = 4}$

$R_{12} = \{(\alpha,\alpha), (\alpha,\beta), (\beta,\alpha)\} \subseteq A \times A$
$R_{13} = \{(\alpha,\beta), (\beta,\alpha), (\beta,\beta)\} \subseteq A \times A$
$R_{14} = \{(\alpha,\alpha), (\beta,\alpha), (\beta,\beta)\} \subseteq A \times A$
$R_{15} = \{(\alpha,\alpha), (\alpha,\beta), (\beta,\beta)\} \subseteq A \times A$

${}^4C_4 = 1$

$R_{16} = \{(\alpha,\alpha), (\alpha,\beta), (\beta,\alpha), (\beta,\beta)\} \subseteq A \times A$

$\mathbb{Z} \to$ integer.

i) Reflexive $\to$ R_8, R_{14}, R_{15}, ~~R_{16}~~

ii) symmetric $\to$ R_6, R_{12}, R_{13}, R_{16}

iii) transitive $\to$ No

iv) equivalence $\to$ No.

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Q. If R be a relation on set of integers $\mathbb{Z}$ defined by —

$R = \{(x, y) : x, y \in \mathbb{Z}$ & $x - y$ is divisible by $5\}$
show that R is an equivalence relation.

$\to$ $R \subseteq \mathbb{Z} \times \mathbb{Z}$

i) Reflexive : Aim: $\forall x \in \mathbb{Z} \Rightarrow (x, x) \in R$
As $x \in \mathbb{Z} \Rightarrow (x, x) \in \mathbb{Z} \times \mathbb{Z}$
Here, $x - x = 0$, is divisible by 5
$\Rightarrow (x, x) \in R \Rightarrow$ R is reflexive.

ii) symmetric :
Aim: ~~if $\forall x \in \mathbb{Z} \Rightarrow (x, y) \in R$~~
If $(x, y) \in R \Rightarrow (y, x) \in R$
As $(x, y) \in R \Rightarrow x, y \in \mathbb{Z}$ & $x - y$ is divisible by 5
$\Rightarrow x, y \in \mathbb{Z}$ & $x - y = 5n_1$, n_1 is some integer
$\Rightarrow y, x \in \mathbb{Z}$ & $y - x = 5 \times (-n_1) = 5n_2$, n_2 is some integer
$\Rightarrow y, x \in \mathbb{Z}$ & $y - x$ is divisible by 5
$(y, x) \in R$

iii) Transitive.

Aim: (x,y) & $(y,z) \in R \Rightarrow (x,z) \in R$

As $(x,y), (y,z) \in R$

$\Rightarrow$ $x, y \in \mathbb{Z}$ & $(x-y)$ is divisible by 5
$y, z \in \mathbb{Z}$ & $(y-z)$ is divisible by 5

$\Rightarrow$ $x, y \in \mathbb{Z}$ & $x-y = 5n_1$
$y, z \in \mathbb{Z}$ & $y-z = 5n_2$ } n_1 & n_2 are some integer

$\Rightarrow$ $x, z \in \mathbb{Z}$, Here $x-z = (x-y)+(y+z)$

$= 5n_1 + 5n_2$

$= 5(n_1+n_2)$

$= 5n_3$

n is another complete integer

$\Rightarrow$ $x, z \in \mathbb{Z}$ & $x-z = 5 \times n_3$, n_3 is another integer

$\Rightarrow$ $x, z \in \mathbb{Z}$ & $x-z$ is divisible by 5

$\Rightarrow$ $(x,z) \in R$

$\therefore$ R is transitive

Hence, R is equivalence
It is denoted by $\mathbb{Z}_5$.
It is termed as $\mathbb{Z}$ modulus 5

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prove that

~~i) $(p \Leftrightarrow q) \equiv (p \Rightarrow q) \wedge (q \Rightarrow p)$~~

i) $(p \Rightarrow q) \equiv (\sim q \Rightarrow \sim p)$

p	q	① $p \Rightarrow q$	$\sim q$	$\sim p$	② $\sim q \Rightarrow \sim p$	① ⟺ ②
T	T	T	F	F	T	T
T	F	F	T	F	F	T
F	T	T	F	T	T	T
F	F	T	T	T	T	T

since, Every entry in the last coloumn is T, therefore the given statement is 'Tautology'.

$\therefore (p \Rightarrow q) \equiv (\sim q \Rightarrow \sim p)$

NOTE:

① If every entry in the last coloumn is F, therefore the given statement is contradiction.

② If some entries are T and some are F, then the given statement is called ~~Absar~~ Absardity or continge

verify :

$\sim (p \Rightarrow q) \equiv p \wedge \sim q$

p	q	$p \Rightarrow q$	① $\sim (p \Rightarrow q)$	$\sim q$	② $p \wedge \sim q$	① ⟺ ②
T	T	T	F	F	F	T
T	F	F	T	T	T	T
F	T	T	F	F	F	T
F	F	T	F	T	F	T

$\therefore$ It is tautology.

$\therefore \sim (p \Rightarrow q) \equiv p \wedge \sim q$

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Q. Write the negations of following statements

1. He is handsome but short.

→ p : He is handsome.
q : He is ~~tall~~ short.
∴ given statement is $p \wedge q$.
∴ It's negation is
$\sim(p \wedge q) \equiv \sim p \vee \sim q$
i.e, He is not handsome or he is not short.

Q. Write the negation of $2 > 3$.
→ $\sim(2>3) \equiv 2 \leq 3$

Q. Write the negation of
"If you work hard, ~~you~~ then you will score good marks"
→ p : you work hard.
q : you will score good marks.
∴ given statement is $p \Rightarrow q$
∴ It's negation is
$\sim(p \Rightarrow q) \equiv p \wedge \sim q$
i.e, you work hard and you will not score good marks

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Proof :

i) $\sim(p \vee q) \equiv (\sim p \wedge \sim q)$

p	q	$p \vee q$	$\sim(p \vee q)$ ①	$\sim p$	$\sim q$	$\sim p \wedge \sim q$ ②	① $\Leftrightarrow$ ②
T	T	T	f	f	f	f	T
T	f	T	f	f	T	f	T
f	T	T	f	T	f	f	T
f	f	f	T	T	T	T	T

$\therefore$ It is tautology

$\therefore \sim(p \vee q) \equiv (\sim p \wedge \sim q)$

ii) $\sim(p \wedge q) \equiv (\sim p \vee \sim q)$

p	q	$p \wedge q$	$\sim(p \wedge q)$ ①	$\sim p$	$\sim q$	$(\sim p \vee \sim q)$ ②	① $\Leftrightarrow$ ②
T	T	T	f	f	F	f	T
T	f	f	T	f	T	T	T
f	T	f	T	T	f	T	T
f	f	f	T	T	T	T	T

$\therefore$ It is tautology

$\therefore \sim(p \wedge q) \equiv (\sim p \vee \sim q)$

iii) Write the negation of

i) "If the sky is cloudy, than will rain".

→ Let p : the sky is cloudy.

q : It will rain.

$\therefore$ Given statement is $p \Rightarrow q$.

It's negation is

$$\sim(p \Rightarrow q) \equiv p \wedge \sim q$$

i.e sky is cloudy and it will not rain.

Validity of the argument.

Let $p_1, p_2, \dots, p_n$ be the hypotheses and q is the conclusion.
Then we say that q follows logically from $p_1, p_2, \dots, p_n$.

we write
$$\begin{array}{l} p_1 \\ p_2 \\ \vdots \\ p_n \\ \hline \therefore q \end{array}$$

Here we test the validity of
$$(p_1 \wedge p_2 \wedge \dots \wedge p_n) \Rightarrow q$$

Eg: If I drive to work, then I will arrive tired.
I am not tired, when I arrive at work.

$\therefore$ I do not drive to work

Soln: Let p: I drive to work.
q: I will arrive tired
$p \Rightarrow q$:

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congruence relation:

A relation R on Z defined as follows:

$R = \{ (x, y) \in Z \times Z : x-y$ is divisible by some +ve integer $m\}$ then the above relation is an equivalence relation on Z. It partitions the set Z into m equivalence classes. This equivalence relation is termed as Z modulo m, it is denoted by Z/m.

Defn: partition or Quotient set.
A partition or quotient of a non empty set A is a collection P of non empty subsets A_i of A $(A_i \subseteq A)$

s.t:
1) Each element of A must belongs to one of the subset of A_i of A.
2) If A_i and A_j are two distinct subsets of A then $A_i \cap A_j = \phi$

The sets A_i are called the cells or block of partitions p.

Defn: R relative set of x:
If R is a relation from A to B then for $x \in A$ we define R relation set of x and denote R(x) as follows:

$$R(x) = \{ y \in B : (x, y) \in R\}$$

Eg :

$A = \{1, 2, 3\}$, $B = \{a, b\}$

$R = \{(1, a), (2, b), (3, a)\}$

$R(1) = \{a\}$
$R(2) = \{b\}$
$R(3) = \{a\}$

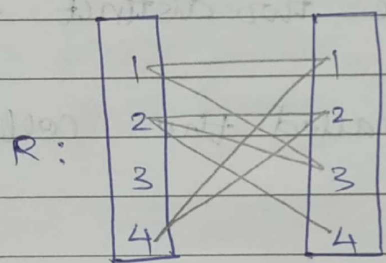
Representation of relation :

1) Graphical Representation.
2) Diagraph or directed graph
3) matrix representation of a relation.

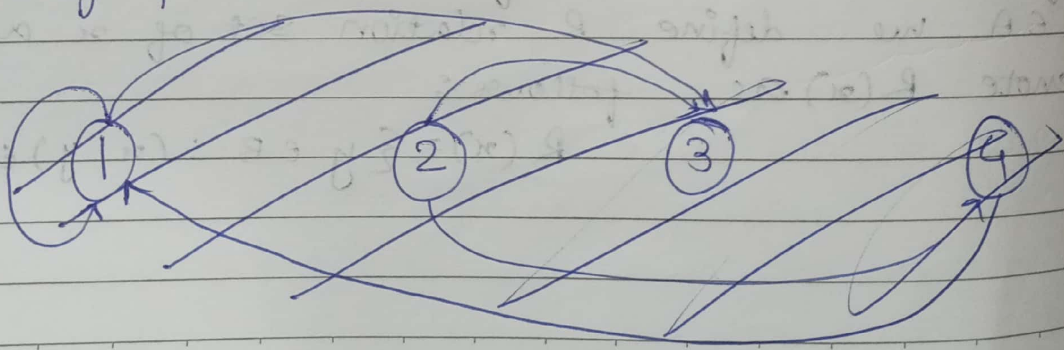
Q. find the representation of R depend on $A = \{1, 2, 3$
$R = \{(1,1), (2,3), (1,3), (2,4), (4,1), (4,2), (2,2)\}$

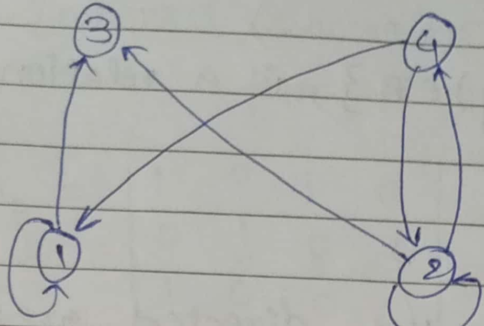
1) Graphical.

$R : A \to A$



2) Diagraph.





Not reflexive, not symmetric.

because, (1,1), (2,2) hora hai par, 3 & 4 me nahi hora hai toh not reflexive agar pura me hota toh reflexive.

because, (2,4), (4,2) waisa sab me nahi hai isliye not symmetric.

3) Matrix.

$$R : M_R = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \end{array}\begin{array}{c} \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 \end{bmatrix}_{4\times 4} \end{array}$$

also called Boolean matrix.
Not symmetric, not reflexive

- jo available hai usme 1, aur baki me 0.
- Not symmetric, because diagonal k ~~upo~~ upper and lower part same nahi hai, not symmetric.
- Not reflexive, because diagonal is not 1, whole diagonal should be 1.

Def^n: Domain of a relation:
It is denoted & defined by ~~Dom(R) = {~~

$$Dom(R) = \{x : x \in A \,\&\, (x, y \in R)\}$$

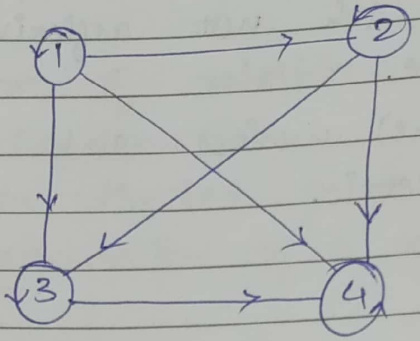
Range of a relation.
It is defined as.

$$Range(R) = \{y : y \in B \,\&\, (x, y \in R\}$$

Inverse relation.

$R^{-1} = \{(y,x) : (x,y) \in R\}$ is a relation from B to A.

Q. Determine whether the directed graph are reflexive, symmetric, transitive.



reflexive
transitive.
$R = \{1,2,3,4\}$

$R = (1,1), (1,2), (1,3), (2,2), (2,4), (2,3), (3,3), (3,4), ($

E:

composition of relation.

If R is a relation from A to B and S be relation from B to C, the R O S is a relation from A to C.

If M_R ; M_S are Boolean matrices then.

$\boxed{M_{R \circ S} = M_R \cdot M_S}$ where '.' is boolean matrix expression.

Note: If M_R is matrix of R then

$(M_R)^T$ is matrix corresponding to R^{-1}

i.e $M_{R^{-1}} = (M_R)^T$

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Q. let $A = \{1,2,3\}$ and $B = \{a, b, c, d\}$ let R and S be two relations from A to B with boolean matrices.

$$M_R = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 1 \end{bmatrix}$$

& $$M_S = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 \end{bmatrix}$$

find: ① $M_{R^{-1}}, M_{S^{-1}}$

② $M_{(R \cap S) \circ R^{-1}}$ & $M_{(R \circ R^{-1}) \cap (S \circ R^{-1})}$

③ $M_{S \circ (R^{-1} \cup S^{-1})}$

④ $M_{(S \circ R^{-1}) \cup (S \circ S^{-1})}$

Long que.

Q. R be a relation defined on Z set :
$R = \{(x,y) \in Z \times Z : x-y$ is divisible by $4\}$
prove that R is an equivalence relation. Also identify the equivalence classes.

Note : for Z modulo m : ~~we denote~~ Z/m.
we denote $a \equiv b \pmod m$ if $(a,b) \in R$ read as a is congruent to b modulo m.

~~SOP~~ Q let a relation R is : $x \equiv y \pmod 4$ defined on Z. then prove that it is an equivalence relation. find hence identify the equivalence classes.

Que. for $\mathbb{Z}_8$, prove that $a \equiv b \pmod 8$ is equivalence relation and hence find all the equivalence classes corresponding to relation R.

$R = \{(a,b) \in \mathbb{Z} \times \mathbb{Z} : a-b \text{ is divisible by } 8\}$

$A_0 = R(0) = \{0, \pm 8, \pm 16, \ldots\}$ $8n$

$A_1 = R(1) = \{9, 17, -7, -15\}$ $8n+1$

$A_2 = R(2) = \{10, 2, -6, -14, \ldots\}$ $8n+2$

$A_3 = R(3) = \{3, 11, -5, \ldots\}$ $8n+3$

Antisymmetric relation:

Let R be a relation defined on A, it is said to be antisymmetric if : $(a,b) \in R$ and $(b,a) \in R \Rightarrow a = b$

Eg: $R \subseteq \mathbb{Z} \times \mathbb{Z}$

Eg: $R \subseteq \mathbb{Z} \times \mathbb{Z}$

s.t $R = \{(a,b) : a \text{ divides } b\}$

$(1,3) \in R$, $(3,1) \in R$

$(2,3) \in R$, $(2,4) \in R$, $(2,-4) \in R$

proof: let $(a,b) \in R$ & $(b,a) \in R$

$\Rightarrow$ a divides b & b divides a

$\Rightarrow b = n_1 a$ & $a = n_2 b$

where n_1, n_2 belong to integers.

$\Rightarrow b = (n_1 . n_2) b \Rightarrow 1 = n_1 . n_2$ if $b \neq 0$

$n_1 = -1, n_2 = -1$ or $n_1 = 1, n_2 = 1$

for $n = -1$ $\left.\begin{array}{l} b = -a, \\ a = -b \end{array}\right\} \boxed{b = -a}$

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for | $b = n_1 a \Rightarrow b = a$
$a = n_2 b \Rightarrow a = b$ $\boxed{a=b}$

$\Rightarrow$ $R = \{(a,b) : a$ divides $b\}$ on Z^+ is not antisymmetri

Q. R be a relation defined on Z^+ s.t :
$R = \{(a,b) \in Z^+ \times Z^+ : a$ divides $b\}$
It is antisymmetric. → prove.

→ $(a,b) \in R$ & $(b,a) \in R$
$\Rightarrow$ $a = n_1 b$ $\quad b = n_2 a$
$n_1, n_2 \in Z^+$

It is antisymmetric

partial order relation:

A relation R defined on A is said to be partial ord relation if
i) R is reflexive
ii) R is antisymmetric
iii) R is transitive.

we write these relation as (A, R) and call it as POSET (partial order set).

Ex:1 $(Z^+, \leq)$ is a POSET, '$\leq$' stands for "less than or equ

Ex:2 $(N, |)$ is a POSET, '|' stands for "divides"

~~(A, R)~~

Rec

Ex 3: $(S = \{1, 2, 3, 6\}, |)$ is POSET where '|' stands for divides. Draw also its Hasse diagram.

partial order set.

$(S, R) \longrightarrow$ POSET

$\hookrightarrow$ partial order ~~set~~ relation
- $\hookrightarrow$ Ref.
- $\hookrightarrow$ antisymm.
- $\hookrightarrow$ Transitive

a|b

$R \rightarrow$ divides '|' $\Rightarrow$ a divides b

$R \rightarrow$ less than or equal $\Rightarrow$ a is less than or equal to b

$R \rightarrow \leq \longrightarrow a \leq b$

Q. Let S be a set of divisions of 6 and R be a relat[ion] defined on S

$R = \{(a,b) \in S \times S : a \text{ divides } b\}$

[Note: Here R stands for divides and in genera[l] it is denoted by symbol '|'].

Sol^n: $S = \{x : x \text{ is a divisor of } 6\}$

$S = \{1, 2, 3, 6\}$

$R \subseteq S \times S$

$R = \{(a,b) : a|b\}$

$= \{(1,1), (1,2), (1,3), (1,6), (2,2), (2,6), (3,3), (3,6)$

Note: $\frac{6}{3} \rightarrow 6/3 \rightarrow$ 6 divided by 3

& $3|6 \rightarrow$ 3 divides 6

i) Reflexive as we have $\forall a \in S \Rightarrow (a,a) \in R$ as a|

ii) Antisymmetric: Let $(a,b) \in R$ & $(b,a) \in R$

$\Rightarrow$ a divides b & b divides a

$\Rightarrow b = n_1 a$ & $a = n_2 b$ —— (i)

n_1 & $n_2 \in N$

$\Rightarrow b = n_1 (n_2 \cdot b)$

$\cancel{b} = (n_1 \cdot n_2) \cancel{b}$ $b \neq 0$

$1 = n_1 n_2$

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$\therefore$ (i) $\Rightarrow b = 1 : a \Rightarrow b = a$ $\qquad a = b$

$\Rightarrow \boxed{a = b}$

$\Rightarrow$ R is antisymmetric.

iii) Transitive

Let $(a,b) \in R$ & $(b,c) \in R$

$\Rightarrow$ a divides b & b divides c

$\Rightarrow b = n_1 a$ & $c = n_2 b$

n_1 & $n_2 \in \mathbb{N}$

$c = n_2 b = (n_2 \cdot n_1) a = n \cdot a$, $n \in \mathbb{N}$

$\Rightarrow c = n.a \Rightarrow$ a divides c $\Rightarrow \boxed{(a,c) \in R}$

it is transitive.

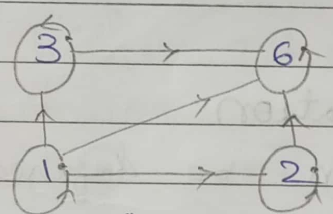
Hence, R is Reflexive, antisymmetric, transitive.

~~$\therefore$ (S,R) is POSET. thus~~

$\therefore$ R is partial order relation and thus (S, |) is a POSET.

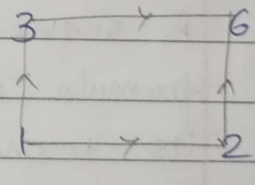
Hasses dig.

1) Reflexive



Diagraph

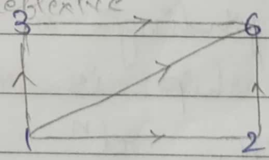
3) antisymmetric.



start = starting point

end = vertex

line = edge

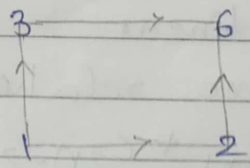


reflexive

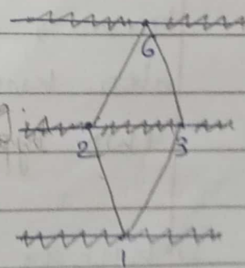
is kind k

* Reflexive hata isme se hata is diagonal

2) Transitive



* Transitive hata dena



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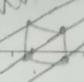
mm → removed lines.

is antisy

* with lines i.e 

* without lines i.e □ is a hash dig.

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Q. prove that $(P(A), \supseteq)$ is a POSET. where $A = \{1,2,3\}$

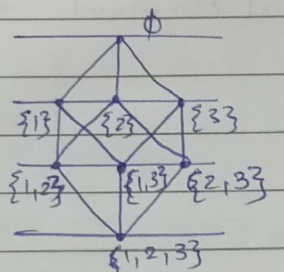
$(P(A) = \{ \phi, \{1\}, \{2\}, \{3\}, \{1,2\}, \{1,3\}, \{2,3\}, \{1,2,3\}$

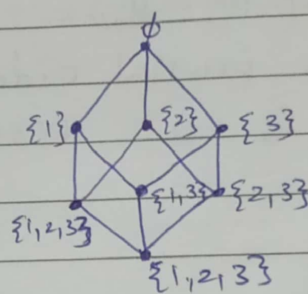
Sol^n

$R = \{(\phi, \phi), (\{1\}, \phi), (\{1\}, \{1\}), (\{2\}, \phi), (\{2\}, \{2\}),$
$(\{3\}, \phi), (\{3\}, \{3\}), (\{1,2\}, \phi), (\{1,2\}, \{1\}), (\{1,2\},$
$\{2\}), (\{1,2\}, \{1,2\}), (\{1,3\}, \phi) (\{1,3\}, \{1\}), (\{1,3\}, \{3\}),$
$(\{1,3\}, \{1,3\}), (\{2,3\}, \phi), (\{2,3\}, \{2\}), (\{2,3\}, \{3\}),$
$(\{2,3\}, \{2,3\}), (\{1,2,3\}, \phi), (\{1,2,3\}, \{1\}), (\{1,2,3\},$
$\{2\}), (\{1,2,3\}, \{3\}), (\{1,2,3\}, \{1,2,3\}), (\{1,2,3\}, \{1,2\}),$
$(\{1,2,3\}, \{1,3\}), (\{1,2,3\}, \{2,3\})\}$

Total elements = 27

Hasse dig.





Recurrence relation

A sequence can be defined by giving a general formula of its n^{th} term or by writing few of its terms. An alternative approach is to write a sequence, by finding a relationship among its terms, such relationship is called recurrence relation also known as diff. rel^n.

Two types $\longrightarrow$ Linear
Non linear

1. Linear : A recurrence relation of the ~~teg~~ form

$$C_0 a_n + C_1 a_{n+1} + C_2 a_{n-2} + \cdots + C_k \cancel{a_{n-k}} = f(n)$$

where c_i's are coefficients of the recurrance relation
note that the coefficients c_i may be functions of n.

Linear recurrance relation with constant coefficients:
A recurrance relation of the form eq^n (*) is known as linear recurrance with const. coeff. if all coeff. are const. Otherwise l r r with variable coeff.

Linear homogenous recurrance relation:
A recurrance relation of form eq^n (*) is known as linear homogenous recurrance relation if RHS $\rightarrow f(n) = 0$, otherwise if $f(n) \neq 0 \Rightarrow$ non-homogenous linear recurrance relation.

Order and degree of LRR
A RR of form eq^n (*) is of order 'a' because a_n c be expressed in terms of previous k terms of sequen i.e the order is the diff. between the highest a lowest subscript of a.

In eq (*) highest subscript of $a = n$
lowest $= n-k$

Order of eq^n (*) $= n-(n-k) = k$

The degree of a RR is defined to be the highe power of a_n.

methods for solving LRR.

1) iteration method

2) characteristics' roots method.

(*) 3) Generating functions method.

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Q. Solve the recurrence relation. $a_n = a_{n-1} + 2$, $n \geq 2$ with initial condition $a_1 = 3$

$$a_n = a_{n-1} + 2 \quad \text{--- (1)}$$

$$a_{n-1} = a_{n-2} + 2 \quad \text{--- (2)}$$

$$a_{n-2} = a_{n-3} + 2 \quad \text{--- (3)}$$

eq^n (2) in (1) $\Rightarrow a_n = (a_{n-2} + 2) + 2$

$a_n = ((a_{n-3} + 2) + 2) + 2$ (by 3)

$$\boxed{a_n = a_{n-3} + 3 \times 2}$$

in general

$$\boxed{a_n = a_{n-k} + k \times 2}$$

$k = n-1$:

$$a_n = a_{n-(n-1)} + (n-1).2$$

$$a_n = a_1 + (n-1).2 \Rightarrow \boxed{a_n = 3 + (n-1).2}$$

$n \geq 2$

Verify,

$a_1 = 3$, $a_2 = 5$, $a_3 = 7$, $a_4 = 9$, $a_5 = 11$, $a_6 = 13$

3, 5, 7, 9, 11, 13 - - -

$a_n = a_{n-1} + 2$, $a_1 = 3$

add +2 to the series

same series we'll get

3, 5, 7, 9 - - - ✓

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Characteristics root method
The basic approach to solve homogenous equation is to find solution of the form a.

First we create the auxillary eq^n of given recurrence relation by putting ~~eq^n~~ $a_n = r^n$ to the ~~then find the roots~~ corresponding homogeneous recurrence relation. Then find the root for the ~~Note that~~ auxillary eq^n.
Note that a characteristic eq^n of a^{th} order has k characteristics roots. Depending on the following cases we form the complementary function corresponding to the given homogenous eq^n

Case 1: Distinct roots
If the Charact... eq^n have ~~different~~ distinct root, r_1, r_2, r_3, r_4 then the sol^n (c.f) is given by

$$a_n = b_1 r_1^n + b_2 r_2^n + \cdots + b_k r_k^n$$

b_i's are constants can be obtained using the initial conditions.

Q. $a_n = a_{n-1} + 2a_{n-2}$, $n \geq 2$
given $a_0 = 0$, $a_1 = 1$

$a_n - a_{n-1} + 2a_{n-2} = 0 \longrightarrow$ Homogeneous

It's auxillary eq^n.

$$a_n = r^n$$

$$r^n - r^{n-1} - 2r^{n-2} = 0$$

$$r^{n-2}(r^2 - r^1 - 2) = 0$$

$$r^2 - r^1 - 2 = 0$$

$r^2 - 2r + r - 2$

$r^2 - 2 \Rightarrow r(r-2) + 1(r-2)$

$r = 2, -1$

$r_1 = 2, r_2 = -1$

$\therefore a_n = b_1 2^n + b_2 (-1)^n$

let $n = 0$

$a_0 = b_1 + b_2$

but $a_0 = 0$ $\qquad \therefore b_1 + b_2 = 0$

$b_1 = -b_2$

$n = 1,$

$a = 2b_1 - b_2$

but $a_1 = 1 \Rightarrow 2b_1 - b_2 = 1$

$-b_2 = 1 - 2b_1$

$\therefore b_1 = 1 - 2b_1$

$3b_1 = 1$

$b_1 = \frac{1}{3}$ $\qquad \therefore b_2 = \frac{-1}{3}$

Puting (*) $\longrightarrow a_n = \frac{1}{3} 2^n - \frac{1}{3}(-1)^n$

solution method.

A linear recurrence relation is :

$C_0 a_n + c_1 a_{n-1} + c_2 a_{n-2} + \cdots + c_k a_{n-k} = f$

its soln consists of two parts :

$a_n = a_n^{(h)} + a_n^{(p)}$

when $a_n^{(h)}$ is soln corresponding to the homogeneous recurrance relation (*) and $a_n^{(p)}$ is particular soln obtained by using trial functions.

calculating $a_n^{(h)}$:

first of all constant the auxillary eqⁿ corresponding to eqⁿ (*) by putting $a_n = r^n$ after equating $f(n) = 0$ solve for r :- we get the characteristic roots depending on nature of characteristic roots.

case-1 If all are distinct : say — $r_1 \neq r_2 \neq r_3$ then :

$$a_n^{(h)} = b_1 r_1^n + b_2 r_2^n + b_3 r_3^n$$

when b_i's are unknown constants.

case-2 If ~~roots~~ are equal : $r_1 = r_2 = r_3$

$$a_n^{(h)} = (b_1 + b_2 n + b_3 n^2)\, r_1^n$$

case-3 mixed : If $r_1 = r_2$ & $r_3 \neq r_1$

$$a_n^{(h)} = (b_1 + b_2 n)\, r_1^n + b_3 r_3^n$$

calculating : $a_n^{(P)}$

(*) Depending on nature of $f(n)$ we use the following trial functions :

form of $f(n)$	Trial functions.
① b^n (b is not a root of characteristic eqⁿ) →	$A_0 b^n$
② $p(n)$ is a polynomial of degree m →	$[A_0 + A_1 n + A_2 n^2 + \cdots A_m n^m]$
③ $c^n p(n)$ (c is not a root of ch. eqⁿ) →	$c^n [A_0 + A_1 n + A_2 n^2 + \cdots + A_m n^m]$
④ b^n (b is a root of ch. eqⁿ of multiplie by s) →	$A_0 n^s b^n$
⑤ $c^n p(n)$ (c is a root of ch. eqⁿ of multiplied by s) →	$c^n n^s [A_0 + A_1 n + A_2 n^2 + \cdots + A_m n^m]$

Q.1) solve : $a_{n+2} - 5a_{n+1} + 6a_n = 2$ with intial conditi
$a_0 = 1$, $a_1 = -1$

Q.2) solve : $y_{n+2} - y_{n+1} - 2y_n = n^2$

Q.3) solve : $a_{n+2} - 4a_{n+1} + 4a_n = 2^n$

Q.4) solve : $a_n - 7a_{n-1} + 12a_{n-2} = n \cdot 4^n$

Q.5) solve : $a_n - 4a_{n-1} + 4a_{n-2} = n + 4^n$

Q.6) fibnocii series

$$f_n = f_{n-1} + f_{n-2} \quad , n \geq 2$$

given $f_0 = 1$, $f_1 = 1$

Soln :1 As given eqn is non homogenous linear recurrance relation with constant coefficient so its soln. is given by :

$$a_n = a_n^{(h)} + a_n^{(p)}$$

calculating $a_n^{(h)}$:
Auxillary eqn is given by $a_n = r^n$

$$r^{n+2} - 5r^{n+1} + 6r^n = 0$$

$$r^n [r^2 - 5r + 6] = 0 \Rightarrow \boxed{r^2 - 5r + 6 = 0}$$

$\Rightarrow r_1 = 6 , r_2 = -1$ are distanct

$$\therefore \boxed{a_n^{(h)} = b_0 6^n + b_1 (-1)^n}$$

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calculating $a_n^{(p)}$: $f(n) = 2n^0 \rightarrow$ constant polynomial

Trial sol^n: $a_n^{(p)} = A_0$

$a_{n+2} = A_0, \; a_{n+1} = A, \; a_n = A$

Putting in eq^n (1):

$A_0 - 5A_0 + 6A_0 = 2 \Rightarrow \boxed{A_0 = 1}$

$\therefore$ complete sol^n $a_n = a_n^{(h)} + a_n^{(p)} = b_0 6^n + b_1 -1^n + 1$

As given $a_0 = 1 \Rightarrow 1 = b_0 6^0 + b_1 (-1)^0 + 1$

given $a_1 = -1$

$-1 = b_0 6^1 + b_1(-1) + 1$ —— (ii)

solve for b_0 & b_1

8) $a_n - 8a_{n-1} + 21a_{n-2} - 18a_{n-3} = 3^n$ $\quad a_0 = 0, a_1 = 1, a_2 = 0$

non-homogeneous, linear constitution

sol^n: the sol^n of eq^n (1)

$a_n = a_n^{(h)} + a_n^{(p)}$

calculating $a_n^{(h)}$: Auxilary eq^n $\quad a_n = r^n$

$r^n - 8r^{n-1} + 21r^{n-2} - 18r^{n-3} = 0$ —— (2)

$r^{n-3}(r^3 - 8r^2 + 21r - 18) = 0$

$r^3 - 8r^2 + 21r - 18 = 0$ —— (3)

$r^2(r-2) - 6r^2 + 21r - 18 = 0$

$r^2(r-2) - 6r(r-2) + 9r - 18 = 0$

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$r^2(r-2)-6r(r-2)+9(r-2)=0$

$(r-2)[r^2-6r+9]=0$

$(r-2)(r-3)^2=0$

~~$r=2,\ r=3,3$~~

$r=2,3,3$

$\therefore$ corresponding homogenous form

$\therefore a_n^{(h)}=b_0 2^n+(b_1+b_2 n)3^n$ —— (4)

b_i's are unknown

calculating $a_n^{(p)}$:

$f(n)=3^n$

$a_n^{(p)}=A_0 n^2 . 3^n$ is trial soln of (1) (x, y

$\therefore$ from eqn (1) : $a_n = A_0 n^2 3^n$

$A_0 n^2 3^n - 8A_0(n-1)^2 3^{n-1} + 21 A_0 (n-2)^2 3^{n-2} - 18 (n-3)^2 3^{n-3} = 3^n$

$A_0 3^{n-3}[n^2 \times 3^3 - 8(n-1)^2 \times 3^2 + 21(n-2)^2 \times 3 - 18(n-3)^2] =$

~~$A_0[27n^2 - 72(n^2-1$~~

$A_0[27n^2-72(n^2+1-2n)+63(n^2+4-4n)-18(n^2+9-6n)]$

$A_0[18]=27$

$A_0=\frac{3}{2}$

$\therefore a_n^{(p)}=\frac{3}{2}n^2 . 3^n$ —— (5)

$\therefore$ complete solution:

$a_n=b_0 2^n+(b_1+b_2 n)3^n+\frac{3}{2}n^2 . 3^n$

Given: $a_0=0, a_1=1, a_2=0$

18 × 6 = 108

63 × 4 = 252

72 × 2 = 144

-35

73

$a_0 = 0$ $\boxed{n=0}$ $\Rightarrow$ $0 = b_0 + (b_1)$ — (i)

$a_1 = 1$ $\boxed{n=1}$ $\Rightarrow$ $1 = 2b_0 + 3(b_1 + b_2) + \frac{9}{2}$ — (ii)

$a_2 = 0$ $\boxed{n=2}$ $\Rightarrow$ $0 = 4b_0 + 9(b_1 + 2b_2) + 54$ — (iii)

$b_0 =$

$b_1 =$

$b_2 =$

16/8

unit: 3 permutation & combination.

In many distinct problems, when ~~have~~ we concern with the counting problems.

fundamental principles.

1. sum Rule :

If an event can occur in 'm' no. of ways and another event can occur in 'n' no. of ways and if this two events can't occur simultaneously then one of the two events can occur in $m+n$ no. of ways.

2. product Rule :

If an event can occurs in 'm' no. of ways & 2nd event can occurs in 'n' no. of ways. if the no. of ways the 2nd event occurs doesn't depend upon on the occurence of the 1st event. then the two events can ~~when the~~ occur simultaneously in $m \times n$ no. of ways.

3. factorials :

$$n! = \underline{|n} = n(n-1)(n-2) \cdots 3.2.1$$

$$\underline{|6} = 6 \times 5 \times 4 \times 3 \times 2 \times 1$$

$$\underline{|0} = 1$$

4. permutation :

The different arrangment which can be made out of given set of things by taking some or all of them at a time are called their permutation. It is denoted by ^nP_r given n distinct objects we have to choose r distinct members.

$$^nP_r = \frac{\lfloor n}{\lfloor n-r}$$

5. combination:

The different groups ~~of~~ or selections that can be made out of given sets of things while taking some or all of them at a time irrespective of the order are called their combination.

The no. of combination of 'n' different things taken at a time. is given by $^nC_r = \frac{\lfloor n}{\lfloor (n-r) . \lfloor r}$

6. Difference between permutation and combination.

1. In a combination only selection is made whereas in a permutation not only a selection is made but also an arrangement in a definite order is considered

2. In the combination the ordering of a selected objects is immaterial. In a permutation this ordering is essential.

Q. If there are 14 boys and find the no. of ways of selecting 1 class representative.

Q. 3 persons enter into a car where there are 5 seats. In how many they ~~ca~~ take up their seats.

Q. For a set of 6 true or false questions find the no. of ways of answering ~~of~~ all the questions.

Q. in How many ways one can select 2 books for deff. sub from among 6 distinct C.S books, 3 distinct maths, 2 distinct chemistry book.

Q. How many deff. nos lying between 100 and 1000 can be formed with the digits 1, 2, 3, 4, 5

i) If ~~no~~ no digits is repeated
ii) If a digit can repeat more than on[illegible]

Q. 3 prizes are to be awarded among 10 c[illegible] in how many ways can the prize be g[illegible] in such a way.
i) no candidate may get more than one p[illegible]
ii) a candidate can get more than one pri[illegible]

Q. In how many ways can 7 boys and 5 girls can be seated in a row so that no two girls may sit together.

Q In how many can 4 boys and 4 girls can be seated in a row so that boys and girls are alternate.

Q. Given 10 people $P_1, P_2, P_3 --- P_{10}$ in how many ways can a people can be lined up in a way.
How many line ups are there if P_2, P_8 & P_9 want to stand together.
How many line ups are there if P_2, P_8 & P_9 do not want to stand together.

Q. How many no. of 4 digit can be form with the digit 1, 2, 3, 4, 5.
i) If the repeation of the digit is not allow[illegible]
ii) If the repeation of the digit is allowe[illegible]

Q. How many no. betn 400 & 1000 can be formed with the digits 2, 3, 4, 5, 6, 0
If repeatation of digit is not allowed
" " " " is allowed.

Q. Find the no. of permutations that can be made from the letters of the word DAUGHTER.
i) Taking all the letters together
ii) Beginning with 'D'
iii) Beginning with 'D' & ending with 'R'
iv) vowels being always together.
v) ~~Non-~~ Not all vowels together
vi) Not even 2 vowels together.
vii) vowels occuping even places.

~~Que: CAT → 2 letters CC, AA, TT nahi ∴ no repeatation~~
~~in digits repeatation is allowed~~

* Restricted permutations.
i. The no.s of permutations of n diff. objects taken r at a time in which k particular objects do not occur is

$^{n-k}P_r$

ii) The no. of permn of n diff objects taken r at a time in which k particular objects are present.

$$\boxed{^{n-k}P_{r-k} \times {}^{r}P_k}$$

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permutations of things not all different.

$$\frac{\lfloor n}{\lfloor p \lfloor q \lfloor r}$$

n objects in which one object is repeated p times, 2nd object is repeated q times & 3rd objects are repeated r times, rest objects are distinct different.

Q. write diff. words from "ALLAHABAD"

$$\frac{\lfloor 9}{\lfloor 4 \lfloor 2} = \frac{9\times8\times7\times6\times5\times4\times3\times2\times1}{4\times3\times2\times1\times2\times1}$$

$= 7560$

circular permutation:

A circular permutation is an arrangement of object around a circle.

The no. of circular arrangement of 'n' diff objects is $\lfloor (n-1)$

Note: when clock wise and anticlock wise arrangements are not different, Eg: arrangement of beats in neck
" flowers in garl

~~The no. of diff circular permutations~~

The no. of circular permutation of 'n' different things is $\frac{\lfloor n-1}{2}$

Q. How many ways 8 different beats can be arrange to form a necklace

→ $\frac{\lfloor 7}{2}$ → $\frac{\lfloor 8-1}{2}$

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Q. How many ways 5 boys & 4 girls can be seated in round table.

i) No restriction

ii) all 4 girls seat together → $\lfloor 5 \times \lfloor 4$ (6−1)

iii) all 4 " not " "

iv) no 2 girls seat together.

Q. In how many ways can a party of 4 girls & 4 boys be seated at circular table, no 2 boys are adjusted.

Q. Find the no. of diagonals that can be drawn by joining angular points of the hexagon.

Q. Their are 50 students in each of the senior & junior classes. Each class has 25 mal & 25 female. In how many ways can 8 students committe member can be form so that their are 4 females & 3 junior in the committee

→

	Junior		Senior		total
possibilities	25m	25 f	25 m	25 f	
1	−	3	4	1	8
2	1	2	3	2	8
3	2	1	2	3	8
4	3	0	1	4	8

Ans: $25C_1 \times 25C_4$

$\checkmark \rightarrow$ allowed

$\times \rightarrow$ ~~to~~ doesn't matter

combination with repetation is allowed

	ordering	repetation
${}^nP_r$ permutaⁿ	$\checkmark$	$\times$
${}^nC_r$ combinaⁿ	$\times$	$\times$
~~kea~~ product rule	$\checkmark$	$\checkmark$
${}^{n+r-1}C_r$	$\times$	$\checkmark$

eg: ${}^{3+2-1}C_2 = {}^4C_2 = \frac{\lfloor 4}{\lfloor 2 . \lfloor 2} = \frac{\cancel{4}^2 \times 3 \times \cancel{2 \times 1}}{\cancel{2} \times 1 \times \cancel{2 \times 1}} = 6$

Result : 1 The no. of ~~all~~ an unordered choices of 'r' different objects from 'n' different objects with repetation allowed. is given by $\boxed{{}^{n+r-1}C_r}$

Result : 2 The no. of solution of $x_1 + x_2 + x_3 + \cdots x_n = r$ where : x_i's are non negative integers is given by $\boxed{{}^{n+r-1}C_r}$.

eg: $x+y+z=3$ (ordering doesnot matter, repetation allo

$\rightarrow$ (3,0,0) | (0,1,2)

(0,3,0) | (2,1,0)

(0,0,3) | (1,0,2)

(1,1,1) | (2,0,1)

(1,2,0)

(0,2,1)

Verify: ~~over~~ ${}^{3+3-1}C_3$

$= {}^5C_3$

$= \frac{\lfloor 5}{\lfloor 3 . \lfloor 2}$

$= \frac{5 \times \cancel{4}^2 \times \cancel{3 \times 2 \times 1}}{\cancel{3 \times 2 \times 1} \times \cancel{2} \times 1}$

$= 10$

10 possible

Date:

Q. consider the set {a, b, c, d}. In how many ways we can select two of these letters in which repetation is allowed.

n = no. of objects = 4 {a, b, c, d}

r = 2

$^{4+2-1}C_r$

$^{5}C_{2}$

$= \frac{\lfloor 5}{\lfloor 2 \cdot \lfloor 1}$

$= \frac{5\times 4\times 3\times 2\times 1}{2\times 1}$

= 60

piegen hole principle.

Q. If n piegens are assigned to m ~~with~~ no. of piegens holes with m<n. Then atleast o piegens hole contains two or more pieg

so that If any 5 integers are chosen from {1, 2, 3, 7, 8} then at least 2 of them have su 90.

{1, 2, 3, 4, 5, 6, 7, 8}

$^8C_5 = \frac{\lfloor 8}{\lfloor 5 \lfloor 3} = \frac{8\times 7\times \cancel{6}}{\cancel{6}} = 56 \rightarrow$ su

Q. In a group of 13 children there must be atleast 2 c who were born on same month.

Date:

Q. consider the set {a, b, c, d}. In how many ways we can select two of these letters in which repetition is allowed.

$n =$ no. of objects $= 4$ {a, b, c, d}
$r = 2$

$$^{4+2-1}C_r$$
$$^{5}C_{2}$$
$$= \frac{\lfloor 5}{\lfloor 2 . \lfloor 1}$$
$$= \frac{5 \times 4 \times 3 \times 2 \times 1}{2 \times 1}$$
$$= 60$$

piegen hole principle.

Q. If n piegens are assigned to m ~~with~~ no. of piegens holes with $m < n$. Then atleast one piegens hole contains two or more piegens so ~~that~~ If any 5 integers are chosen from {1, 2, 3, 4, 5, 6, 7, 8} then at least 2 of them have sum 9.

$$\{1, 2, 3, 4, 5, 6, 7, 8\}$$
$$^{8}C_{5} = \frac{\lfloor 8}{\lfloor 5 \lfloor 3} = \frac{8 \times 7 \times 6}{6} = 56 \rightarrow \text{subsets}$$

Q. In a group of 13 children there must be atleast 2 childs who were born on same month.

Generalised piegen hole principle.

~~If n pieg~~
If n piegens holes are occupied by kn+1 or more peigen where k is positive integer then atleast one piegen hole is occupied by k+1 or more piegens hole.

Q. A bag contains black, blue & white socks find the minimum no. of socks that one may choose in order to get ~~same~~ two pairs of socks of same colour.

→ $k = 3, \; k+1 = 4$

$kn+1 = 10$

Q. Show that if any 20 people are selected then we may choose a subset of 3 so that all 3 were born on the same day of the week.

Q. What the minimum no. of students in a class to be sure that 4 out of of them are born in the same month.

Extended pigen hole principle.

Note: $\lfloor x \rfloor$ is the greatest integer less than or equal to x.

floor function.

$\lfloor 2.1 \rfloor = 2$

$\lfloor -2.1 \rfloor = 3$

Q. If n pegens are assign to m plegen holes and n is greater than m then 1 of the plegen hole must contain atleast $\left\lfloor \frac{n-1}{m} \right\rfloor + 1$ piegens.

→ $m = 12$

least n $\left\lfloor \frac{n-1}{m} \right\rfloor + 1 = 4$

$\left\lfloor \frac{n-1}{12} \right\rfloor = 3 \Rightarrow 3 \leq \frac{n-1}{12} < 4$

$3 = \frac{n-1}{12} \Rightarrow n = 37$

$\frac{n-1}{12} = 4$

$\Rightarrow \boxed{n = 49}$

$37 \leq n < 49$

Chap-4:

Graph Theory.

Some basic Terminologies.

A graph G consist of two sets

1) A non-empty set 'v' whose elements are called vertices, nodes, or points

2) The set 'E' of edges such that each edge $e \in E$ associated with some elements of v. In general

graph 'G' is algebraic structure of (V, E, ψ) which V is vertex set, E is set of edges ψ is mapping from set E to elements of V. In general we denote G(V, E) as graph.

finite and Infinite graph.

Graph G(V, E) is said to be finite if it has finite no. of vertices and finite no. of edges otherwise it is infinite graph.

NOTE : $|E(G)|$ = size of graph = n

$|V(G)|$ = order of graph = m

then G(V, E) is a finite graph of (m, n) graph.

undirected and directed graph.

A directed graph G consist of set V of vertices and the set E of edges such that edge $e \in E$ is associated with an ordered pair of vertices

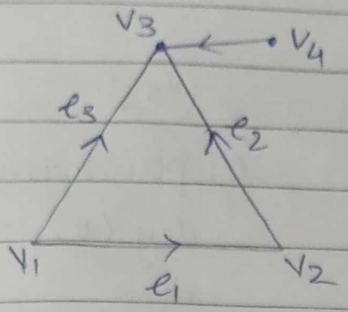
eg: $v_1 \xrightarrow{e} v_2$

undirected grap G consists of vertex V and set E of edges such that each edge $e \in E$ is associated with an unordered pair of vertices.

$V_2 \overset{e}{\text{———}} V_1$ $\quad e \rightarrow \{v_1, v_2\}$

or $e \rightarrow \{v_2, v_1\}$

Incident / Incidence & Adjacent



v_1 & v_3 are adjacent
v_1 & v_2 " "
v_2 & v_3 " "
v_1 & v_4 are not adjacent
v_2 is not adjacent to v_1

self loop or loop.

An edge of the graph that join the vertice to itself is called a loop.

Multiple or parallel edges:

In some graph, if a pair of vertices is join by more than one edge such edges are called vertical edges or parallel edge

Simple graph.

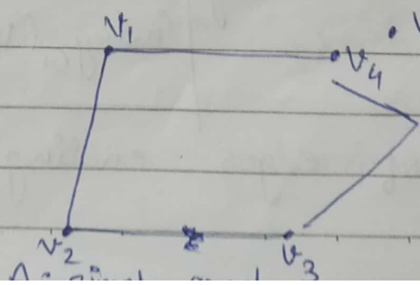
A graph which has either loops nor multiple ~~be~~ edges known as simple graph.

Multi graph.

A graph which contain some multiple edges called multiple edges. No loops are allowed.

pseudo graph.

A graph which contain self loop & multiple edges.



(deg 1 for v_3 & v_4 pendent vertex)

$deg_A(v_5) = 0$

(directed & undirected graph)

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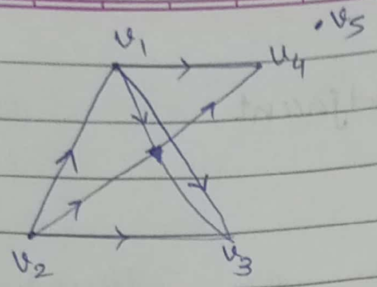
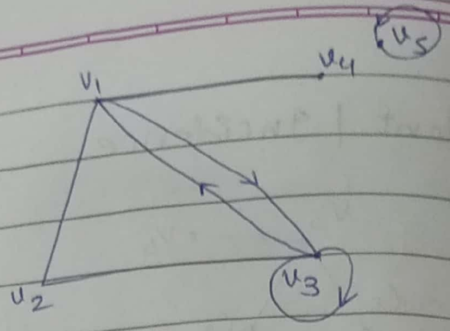




B: Multigraph

$deg_B(v_1) = 4$

$deg_B^-(v_1) = in\ deg_B(v_1) = 1$

$deg_B^+(v_1) = out\ deg_B(v_1) = 3$

C: pseudograph

$deg(v_3) = 5$

~~deg~~

$deg_C^-(v_3) = 2$

$deg_C^+(v_3) = 3$

degree of vertex

The degree of vertex of undirected graph is the no. of edges incident with it. except that a loop at a vertex contribute twice to the degree of that vertex. It is denoted by $\boxed{deg_G(v)}$

Indegree and outdegree

In the directed graph 'G' the outdegree of vertex (v) is denoted by

$\boxed{out\,deg_G(v) \mid or \mid deg_G^+(v)}$

It is the no. of ~~ve~~ edges beginning from (v)

The indegree of vertex (v) is denoted by

~~Indeg~~ $\boxed{indeg_G(v) \mid or \mid deg_G^-(v)}$

It is the no. of edges ending at v.

The sum of indegree & outdegree of vertex is known as total degree of vertex.

source & sink.
A vertex with 0 indegree is known as source
A " " 0 outdegree " " " sink.

Isolated vertex.
A vertex with degree 0 is known as isolated vertex.

pendent vertex.
A vertex with degree 1 is known as pendent vertex

Odd and even vertex.
A vertex of a graph is called odd vertex If its degree is odd no.
A vertex of a graph is called even vertex if its degree is even no.

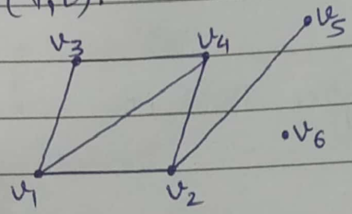
Result 1:
A simple graph with atleast ~~2~~ (2 vertices) of same degree
A " " " with " " 2.
Result 2 : <u>Handshaking theorem</u>
If $G=(V,E)$ is undirected graph with E no. of edges. $|E| = e$, then $\sum_{v} deg_G(v) = 2e$
that is the sum of degrees of the vertices in a undirected graph is even.
<u>NOTE</u>: ~~I~~ The above theorem is applicable ~~if~~ ~~ea~~ even if multiple edges & loops also present.

Result 3:
In an undirected graph the total no. of odd degree vertices is even.

Result 4: $G=(V,E)$ directed $|E|=e$ no. of degree edges. then $\sum deg^{+}_{G}(v) = \sum_{v} deg_{G}^{-}(v) = e$

Degree sequence of a graph
If $G(V,E)$ be a graph then the degrees of vertices arranged in ascending order gives the degree sequence of $G(V,E)$.

Exp.



$deg_G(v_1) = 3$ | $deg_G(v_4) = 3$
$deg_G(v_2) = 3$ | $deg_G(v_5) = 1$
$deg_G(v_3) = 2$ | $deg_G(v_6) = 0$

the degree sequence $(0,1,2,3,3,3)$

Q. 1) Is it possible the following degree sequences for a simple graph.
a) $(1,1,1,6)$ ✗
max degree is 3 $\therefore$ 6 degree cannot be possible.

Result 5: The max degree of a vertex in a simple graph with n vertices cannot exceed 'n−1'.

Result 6: The max. no. of edges in a simple graph with vertices is $\frac{n(n-1)}{2}$

Note: Max. degree of 1 vertex 'n−1'.

Q.2) Is it possible the following degree sequences for a simple graph.

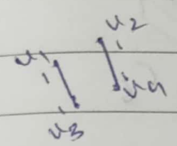
a) $(1,1,2,3) \longrightarrow R_1$ ✓ R_2 ✗

b) $(2,2,4,6) \longrightarrow R_1$ ✓ R_2 ✓ R_3 ✓ R_5 ✗

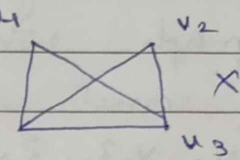
c) $(1,1,1,1) \longrightarrow R_1$ ✓ R_2 ✓ R_3 ✓ R_4 ✓ R_5 ✓ R_6 ✓

d) $(1,3,3,4,5,6,6) \longrightarrow R_1$ ✓ R_2 ✓ R_3 ✓ R_4 ✓ R_5 ✓ R_6 ✓

→ complete graph



e) $(1,1,3,3)$ ✓ all



✗ atleast degree should be with 2, ∴ lowest 1 is not possible.

Types of graph.

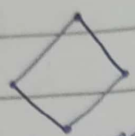
null graph: A graph which contains only isolated points, it is represented by N_n, n is no. of vertices.

complete graph: A simple graph $G(V, E)$ is said to be complete graph if every vertex in G is connected with every other vertex. It is denoted by K_n, n is no. of vertices.

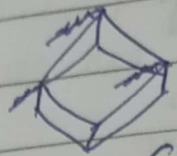
Imp: NOTE: that K_n has exactly $\frac{n(n-1)}{2}$ vertices.

→ Regular graph : A graph in which every vertex has the same degree is said to be regular graph.

Imp# NOTE: that every null graph is a regular graph of degree 0. also complete graph is a regular graph of degree $(n-1)$. for a regular graph G of degree r, the no. of edges is $\frac{r \times n}{2}$



is a regular graph of order 2.



C_6

→ cycle graph : The cycle C_n $(n \geq 3)$ of length n is a connected graph which consists of n vertices $\{v_1, v_2, \ldots v_n\}$ and associated n no. of edges $\{v_1, v_2\}, \{v_2, v_3\}, \{v_3, v_4\} \ldots \{v_{n-1}, v_n\}, \{v_n, v_1\}$ is known as a cycle graphs.

Imp# Note that every cycle is a regular graph of degree

→ wheel graph : The wheel graph W_n $(n \geq 4)$ is obtained from cycle (C_{n-1}) while adding a vertex (C_{n-1}) and connecting it to every vertex in (C_{n-1}). W_n is always a regular graph.

→ Bipartite graph : A graph $G(V, E)$ is said to be bipartite if the vertex set V can be partitioned into 2 disjoint subsets (V_1, V_2) such that $V_1 \cup V_2 = V$, $V_1 \cap V_2$ in such a way if each every edges of V connect one vertex from V_1 & one from V_2. (V_1, V_2) is called as bipartition of G.

→ Regular graph : A graph in which all vertices have same degree is said to be regular graph.

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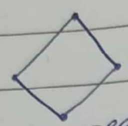

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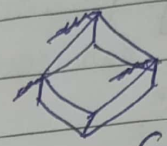

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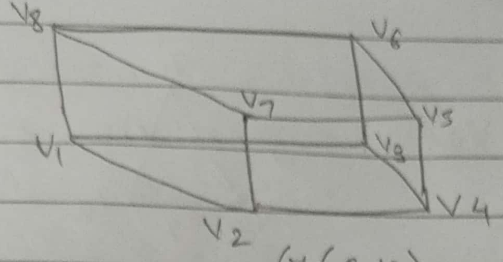
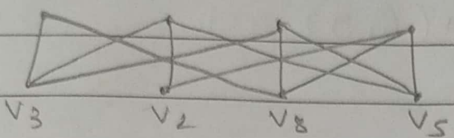


G(8,12)

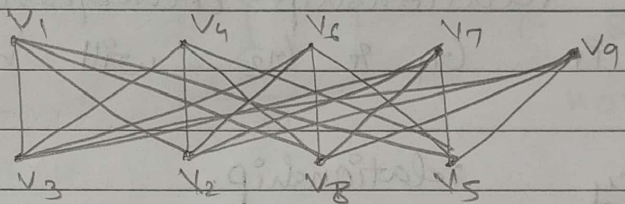
$V = \{V_1 \cdots V_8\}$
$V_1 = \{V_1, V_4, V_6, V_7\}$
$V_2 = \{V_3, V_2, V_8, V_5\}$
~~eg~~ $(V_1, V_2) \rightarrow$ bipartition of G(8,12)



→ Complete biparticle graph (G(V, 6))
A biparticle graph is said to be cbg if each vertex V_1 is connected to each vertex V_2.

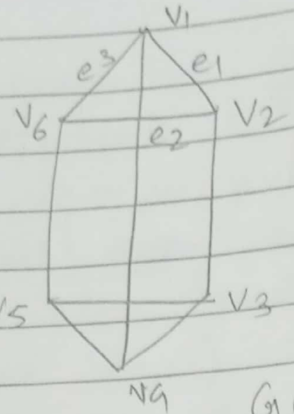
Imp # Note that it is denoted by $K_{m,n}$, $|V_1| = m$ & $|V_2| = n$

Imp Note: The no. of edges in $K_{m,n} = \frac{\text{sum of degrees}}{2}$

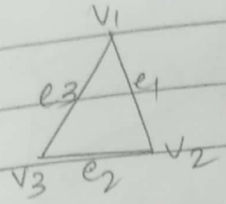


subgraphs & Isomorphic graph:
consider a graph G(V, E), A graph H (V', E') is said to be subgraph of G if the vertices & edges of H are contained in the vertices & edges of G.
i.e $V' \subseteq V$, $E' \subseteq E$

so if H_1 is a subgraph of G then
i) all vertices of H are in G
ii) all edges of H are in G
Imp iii) Each edge of H has some end points in H as in G.



G(6,9)



H(3,3)
is a subgraph of G

Isomorphic graph.

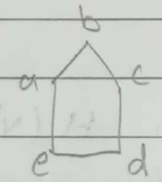
Two graphs $G_1(V_1, E_1)$, $G_2(V_2, E_2)$ is said to be isomorphic if theis exists a functionf from V_1 to V_2 s.t

i) f is bijective i.e one-one & onto

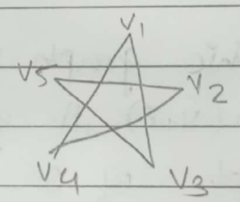
ii) If two vertices a & b are adjacent in G_1 & f(a) & f(b) must be adjacent in G_2.

In other words the function f reserves the adjacent relationship consequently the correspon vertices in G_1 & G_2 will have the same de

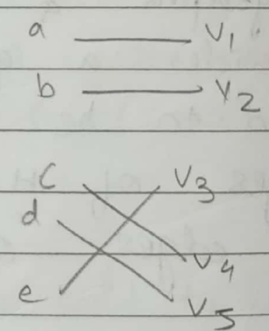
Adjacency relationship.



G(5,5)



H(5,5)



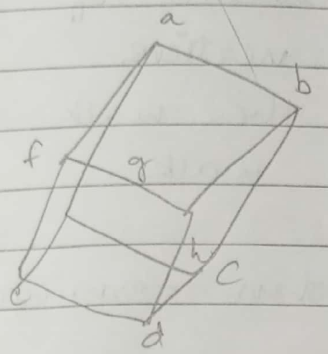
$V_1 = \{a, b, c, d, e\}$

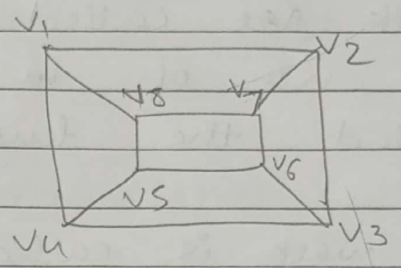
$V_2 = \{v_1, v_2, v_3, v_4, v_5\}$

s.t $f(a) = v_1$, $f(b) = v_3$, $f(c) = v_5$,
$f(d) = v_2$, $f(e) = v_4$ is one-one onto

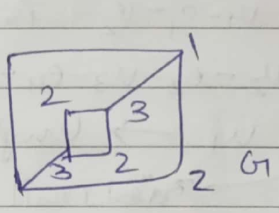
a — v_1
b — v_3
c — v_5
d — v_2
e — v_4

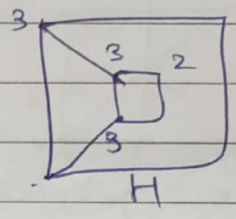
Hence, presenting adjacency of vertices. Hence, it is isomorphic





Q. find whether iso-morphic





G & H are not isomorphic

Walk.

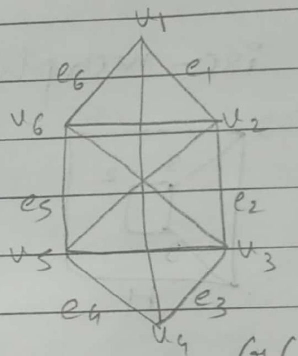
A walk in a graph $G(V,E)$ is a finite alternating sequence of vertices & edges $v_0 - e_1 - v_1 - e_2 - v_2 - e_3 - \cdots e_n - v_n$.

The end vertices v_0 & v_n are the terminal vertices of the walk. Other vertices of the walk are called internal vertices.

The no. of edges in the walk is called the length of the walk.

Note: 1. A walk is called open when terminal vertices are distinct.

2. When terminal vertices are same are called close walk.



$G(6,11)$

walk:

$v_1 - e_1 - v_2$ is of length 1

$v_2 - e_2 - v_3 - e_{11} - e_5 - e_7 - v_2$

$v_1 \rightarrow$ length 4

$\rightarrow$ ~~Trail~~ Trail, open

3. A walk may repeat both vertices & edges.

Trail

A special type of walk is call a trial if all edges are distinct.

circuit

A close trail is called a circuit.

path.

A walk is called a path if all its edges & vertices are distinct.

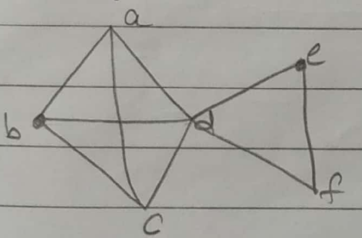
Cycle
A closed path is called a cycle.

Connected in an undirected graph
A graph is connected if it is possible to travel ~~go~~ from any vertex to any other vertex ~~to~~ along a sequence of adjacent edges of the graph.

A vertex (v, u) are called reachable from one another if there is a path from u to v.

Disconnected graph
A graph is disconnected if their exist atleast 2 vertices having no path b/w them.

Distance & diameter ~~of~~ after a graph
In a connected graph (G) the distane b/w the vertices is denoted by d (u, v) is the length of shortest path.
The diameter of the connected graph is written as diam (G) is the max. dist. b/w any 2 vertices in G.



$\underset{min}{d(b,e)} = \cancel{8}\,2$

G (6, 9)

diam (G) = 5
max (c, f)

u \ v	a	b	c	d	e
a	0	1	1	1	2
b	1	0	1	~~2~~ 1	~~3~~ 2
c	~~2~~ 1	1	0	1	2
d	1	1	1	0	1
e	2	2	2	1	0
f	2	2	2	1	1

diam (G) = 2 (highest)

Result 1: If a graph (connected / disconnected) has exactly two vertices of odd degree, there must be a path joining these two vertices, ~~the min. no.~~

Result 2: The ~~no.~~ min. no. of edges in connected graph with n vertices is $(n-1)$.

Result 3: The min. no. of ~~vertices~~ edges in a simple graph with 'n' vertices is $(n-k)$, where k is the no. of connected components of graph.

Result 4: The simple graph with n vertices & k components cannot have more than $\frac{(n-k)(n-k+1)}{2}$ edges.

Connected components

A disconnected graph is the ~~unique~~ union of 2 or more connected subgraphs. Each pair of which has no vertex in common this disjoint connected sub graph are called the connected component of the sub graph.

13/9/24 (Tutorial)

Quantifiers

predicate is denoted by $p(x)$

It is the property satisfied by x we have two types of quantifiers.

1. universal
2. Existential

eg ① "Every student in this class is intelligent". is universal quantification of the predicate.

$p(x)$: student x in this class is intelligent.

② The existential quantification of the predicate $p(x)$ is

"There is a student x in the class who is intelligent".

Symbolically universal quatification of $p(x)$ is writen as :

$\forall x, p(x)$ is true or $\forall x, p(x)$

($\forall \equiv$ all or every)

Symbolically existential quantification of $p(x)$ is $\exists x$, $p(x)$ is true or $\exists x, p(x)$

Negation :

$$\sim(\forall x, p(x)) \equiv \exists x, \sim p(x)$$

$$\sim(\exists x, p(x)) \equiv \forall x, \sim p(x)$$

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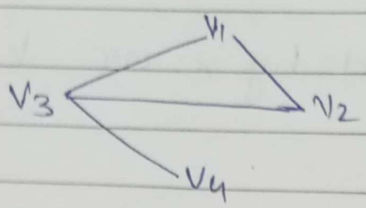
$$\sim(\forall x, p(x)) \equiv \exists x, \sim p(x)$$
$$\sim(\exists x, p(x)) \equiv \forall x, \sim p(x)$$

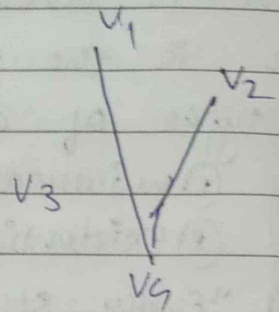
20/9/24 class

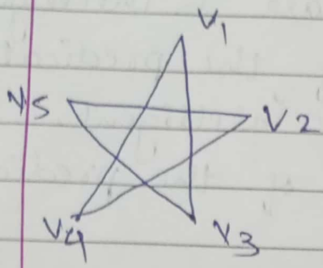
Complement of a graph

G_1:

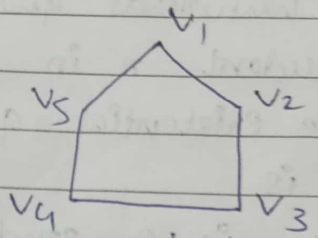


G_1':





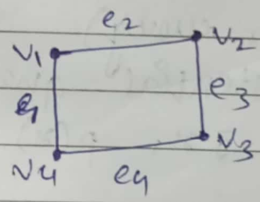
G_1



G'

Eulerian Graph.

A circuit in a connected graph is a euler graph if it contains every edge of graph exactly once. A connected graph with an euler circuit is known as euler graph.

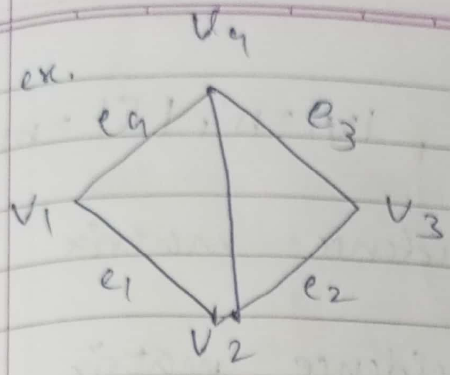


Hamaltonian graph

A circuit in a graph G that contains each vertex in G exactly once is known as hamaltonian circuit / cycle.

A graph G contains hamaltonian graph if it contains hamonian cycle or circuit.

ex.



$v_1 - v_4 - v_3 - v_2 - v_1$

Matrix Representation of a graph:

① Adjacency matrix.

a) for undirected graph with no parallel edges
$G(V, E)$, $|V| = n$.
then, $A = [a_{ij}]_{n \times n}$

$$a_{ij} = \begin{cases} 1, & \text{if there is an edge b/w } i\text{th and } j\text{th vertices.} \\ 0, & \text{otherwise} \end{cases}$$

b) for directed graph:

$A [a_{ij}]_{n \times n}$

$$a_{ij} = \begin{cases} 1, & \text{if arc } (v_i, v_j) \text{ is present in graph} \\ 0, & \text{otherwise} \end{cases}$$

(2) Incidence matrix : $G(V, E)$, $|V| = n$, $|E| = m$

a) undirected graph : Incidence matrix

$B = [b_{ij}]_{n \times m}$, is incidence matrix

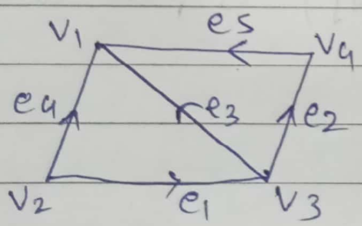
$$b_{ij} \begin{cases} 1, & \text{when edge } e_j \text{ is incident with } v_i \\ 0, & \text{otherwise.} \end{cases}$$

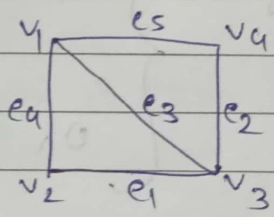
b) For directed graph :

$$B = [b_{ij}]_{n \times m}$$

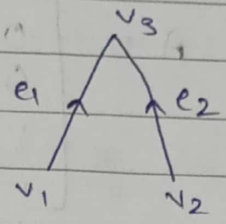
$$b_{ij} = \begin{cases} 1, & \text{if arc } j \text{ is directed away from vertex } v_i \\ -1, & \text{if arc } j \text{ towards vertex } v_i \\ 0, & \text{otherwise.} \end{cases}$$

Q. Find the adjacency matrix for the following graphs also find incidence matrix.





Q. Adjancy matrix =



undirected

$$\begin{array}{c|c} & \begin{matrix} v_1 & v_2 & v_3 \end{matrix} \\ \begin{matrix} v_1 \\ v_2 \\ v_3 \end{matrix} & \begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix} \end{array}$$

directed

$$\begin{array}{c|c} & \begin{matrix} v_1 & v_2 & v_3 \end{matrix} \\ \begin{matrix} v_1 \\ v_2 \\ v_3 \end{matrix} & \begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix} \end{array}$$

Incidence matrix

directed

$$I = \begin{array}{c|c} & \begin{matrix} e_1 & e_2 \end{matrix} \\ \begin{matrix} v_1 \\ v_2 \\ v_3 \end{matrix} & \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ -1 & -1 \end{bmatrix} \end{array}$$

undirected

$$I = B = \begin{array}{c|c} & \begin{matrix} e_1 & e_2 \end{matrix} \\ \begin{matrix} v_1 \\ v_2 \\ v_3 \end{matrix} & \begin{bmatrix} 0 & 1 \\ 0 & 1 \\ 1 & 1 \end{bmatrix} \end{array}$$

Q. Draw the undirected graph represented by adjacency matrix A.

$$A = \begin{bmatrix} 0 & 1 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 \\ 1 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 1 \end{bmatrix}$$

Q. Draw the graph G corresponding to adjacency matrix.

$$A = \begin{bmatrix} 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 \\ 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 0 \end{bmatrix}$$

Q. Draw the graph who's incidence matrix

$$B = \begin{bmatrix} 0 & 1 & 0 & 0 & 1 & 1 \\ 1 & 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \end{bmatrix}$$

Q. incidence matrix

$$B = \begin{bmatrix} 0 & 0 & 1 & -1 & 1 \\ -1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 \\ 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 1 & 0 \end{bmatrix}$$

→ matrix representation of disconnected graph

Adjacency : let $G(V, E)$ has component $G_1(V, E)$ and $G_2(V_2, E_2)$ then adjacency matrix of G is

$$A(G) = \begin{bmatrix} A(G_1) & 0 \\ 0 & A_2(G_2) \end{bmatrix}$$

where $A_1(G_1)$ is adjacency matrix of G_1 of order $n_1 \times n_2$ $|V_1| = n_1$

$A_2(G_2)$ is adjacency matrix of G_2 of order $n_2 \times n_2$, $|V_2| = n_2$

0 is null matrix

Incidence matrix

If $G(V, E)$ having two components $G_1(V_1, E_1)$ & $G_2(V_2, E_2)$ with $|V| = n$, $|E| = m$ & $|V_1| = n_1$ $|E_1| = m_1$, $|V_2| = n_2$, $|E_2| = m_2$

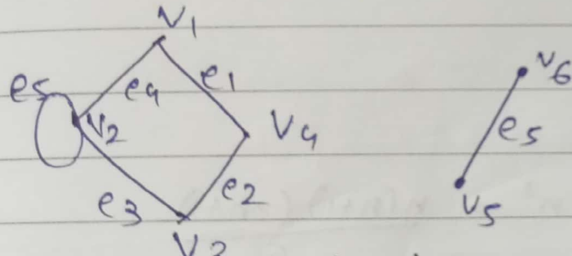
then incidence matrix of G is

$$I(G) = \begin{bmatrix} I(G_1) & 0 \\ 0 & I_2(G_2) \end{bmatrix}$$

where $I_1(G_1)$ is incidence matrix of G_1 of order $n_1 \times m_1$, $I_2(G_2)$ is incidence matrix of G_2 of order $n_2 \times m_2$

0 is null matrix.

Q. Find adjacency matrix and incidence matrix



$G_2(2,1)$

$G_1(4,5)$

$I(G_1) = [b_{ij}]_{4\times 5}$

$I(G_2) = [b_{ij}]_{2\times 1}$

$$I(G) = \begin{array}{c|ccccc|c} & e_1 & e_2 & e_3 & e_4 & e_5 & e_6 \\ \hline v_1 & 1 & 0 & 0 & 1 & 0 & 0 \\ v_2 & 0 & 0 & 1 & 1 & 1 & 0 \\ v_3 & 0 & 1 & 1 & 0 & 0 & 0 \\ v_4 & 1 & 1 & 0 & 0 & 0 & 0 \\ \hline v_5 & 0 & 0 & 0 & 0 & 0 & 1 \\ v_6 & 0 & 0 & 0 & 0 & 0 & 1 \end{array}$$

Q. find the graph corresponding to following incidence matrix.

$$I(G) = \begin{bmatrix} 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 \end{bmatrix}$$

UNIT I :

proof
Mathematical Induction:
let $s(n)$ be a given statement for nth value

Step ① : verify : for $n=1$ that $s(1)$ is true

Step ② : suppose that the statement $s(k)$ is true for $n=k$, for any k

Step ③ : verify, $s(k+1)$ is true with help of given $s(k)$.

Q. show that

① $$1^2+2^2+3^2+\cdots\cdots n^2 = \frac{n(n+1)(2n+1)}{6}$$

② $$\frac{1}{1\cdot 2}+\frac{1}{2\cdot 3}+\cdots\cdots+\frac{1}{n(n+1)} = \frac{n}{(n+1)}$$

③ $$3+33+333+\cdots\cdots+33\cdots\cdots3 = \frac{(10^{n+1}-9n-10}{27}$$

Q.4) show that : $n^2 > 2n+1$ for $n \geq 3$, using mathematical induction

$s(n) = 1^2 + 2^2 + 3^2 + \cdots\cdots n^2 = \frac{n(n+1)(2n+1)}{6}$

$S(1)$ i.e $n=1$.

$$1^2 = \frac{1(2)(2+1)}{6}$$

$1 = 1$

LHS = RHS

$S(1)$ is true.

$$S(k) = \frac{k(k+1)(2k+1)}{6}$$

(1)

→ Let $s(n)$ begun statement checking

$S(1)$ is true :

$$S(1) : 1^2 = \frac{1(1+1)(2\times1+1)}{6} = 1 \quad \text{—— (1)}$$

~~Hene~~ Here, LHS = RHS ⇒ $S(1)$ is true

suppose. $S(k)$ is true

i.e,

$$S(k) : 1^2 + 2^2 + 3^2 + \cdots\cdots + k^2 = \frac{k(k+1)(2k+1)}{6} \quad \text{—— (2)}$$

Now, verifying $S(k+1)$ is true

$$S(k+1) : 1^2 + 2^2 + 3^2 + \cdots\cdots + (k+1)^2 = \frac{(k+1)(k+2)\{2(k+1)+}{6} \quad \text{—— (3)}$$

$$\text{L.H.S} = \{1^2 + 2^2 + 3^2 + \cdots\cdots + k^2\} + (k+1)^2$$

$$= \frac{k(k+1)(2k+1)}{6} + (k+1)^2 \quad (\text{by eq}^n \text{ (2)})$$

$$= \frac{k(k+1)(2k+1) + 6(k+1)^2}{6}$$

$$= \frac{(k+1)[k(2k+1) + 6(k+1)]}{6}$$

Date:

$$= \frac{(k+1)[2k^2+7k+6]}{6}$$

$$= \frac{(k+1)[2k^2+4k+3k+6]}{6}$$

$$= \frac{(k+1)[2k(k+2)+3(k+2)]}{6}$$

$$= \frac{(k+1)(k+2)(2k+3)}{6}$$

= RHS of eq^n (2)

$\Rightarrow$ $S(k+1)$ is true

Hence by mathematical induction $S(n)$ is true

4) $\rightarrow$ $S(n) : n^2 > 2n+1$, $n \geq 3$

$S(3) : 3^2 > 2 \times 3 + 1$

$\left.\begin{matrix} LHS = 9 \\ RHS = 7 \end{matrix}\right\} \Rightarrow LHS > RHS$

$S(3)$ is true

~~let S(3) is~~ let $S(k)$ is true :

$S(k) = k^2 > 2k+1$ —— (2)

verify : $S(k+1)$

$(k+1)^2 > 2(k+1)+1$ —— (3)

$\cancel{LHS = (k+1)^2 = k^2 + 1^2 + 2k > k^2}$

$LHS = k^2 + 1 + 2k$

$RHS = 2k+3$

Adding both sides of eq^n (2) $(2k+1)$

$k^2 + (2k+1) > 2k+1+(2k+1)$

$k^2 + 2k + 1 > 4k + 2$

Theory : isomorphic
27/21st sept notes

$LHS = k^2 + (2k+1) > (2k+1) + (2k+1) = 2k+2+2k >$
$2k+2+1$
as $2k > 1$

Q. prove by mathematical induction that $6^{n+2} + 7^{2n+1}$ is divisible by 43.

→

$S(n) : 43 \mid 6^{n+2} + 7^{2n+1}$

step 1: verify $S(1)$ is true

$S(1) : 43 \mid 6^{1+2} + 7^{2\times1+1}$ is true
as $6^3 + 7^3 = 216 + 343 = 559 = 43 \times 13$

step 2: let $S(k)$ is true.
$S(k) : 43 \mid 6^{k+2} + 7^{2k+1} \Rightarrow 6^{k+2} + 7^{2n+1}$
$\Rightarrow 43 \times \alpha ; \alpha \in Z^+$

step 3: verify $S(k+1)$ is true:
Aim: $43 \mid 6^{k+1+2} + 7^{2(k+1)+1}$

Aim: $6^{k+3} + 7^{2k+3} = 43\beta, \beta \in Z^+$

consider, $6^{k+3} + 7^{2k+3} = 6 \times 6^{k+2} + 7^2 \times 7^{2k+1}$
$= 6 \times 6^{k+2} + (6+43) 7^{2k+1}$
$= 6 \times 6^{k+2} + 6 \times 7^{2k+1} + 43 \times 7^{2k+1}$
$= 6[6^{k+2} + 7^{2k+1}] + 43 \times 7^{2k+1}$
$= 6 \times 43 \times \alpha + 43 \times 7^{2k+1}$ by eqⁿ ②
$= 43[6\alpha + 7^{2k+1}]$
$= 43 \times \beta$
$\beta = 6\alpha + 7^{2k+1} \in Z^+$

i.e $43 \mid 6^{k+3} + 7^{2k+3} \Rightarrow$ By mathematical induction $S(n)$ is true for each of n.

Q) prove by M.I that $7^{n+2} + 8^{2n+1}$ is divisible by 57 : $n \geq 1$

Aim: $57 \mid 7^{k+3} + 8^{2k+3}$ given $57 \mid 7^{k+2} + 8^{2k+1}$

$$7^{k+3} + 8^{2k+3} = 7 \times 7^{k+2} + 8^2 \times 8^{2k+1}$$
$$= 7 \times 7^{k+2} + (7+57)\, 8^{2k+1}$$
$$= 7\left[7^{k+2} + 8^{2k+1}\right] + 57 \times 8^{2k+1}$$
$$= 7 \times 57\alpha + 57 \times 8^{2k+1}$$
$$= 57\left[7\alpha + 8^{2k+1}\right] = 57$$

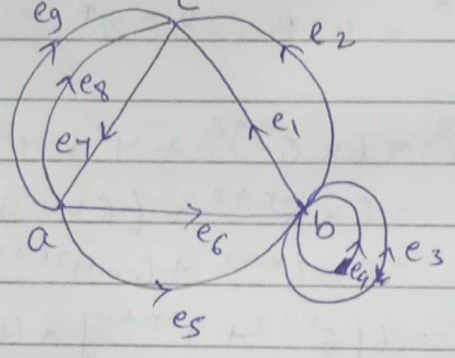
Imp: for Matrix Representation

Note:

1. ~~for~~ The matrix representation of ~~for~~ multi graph & pseudo graph for the no. of edges connected b/w v_i & v_j gives the value of a_{ij}.

2. for the incident matrix for self loop we count 2, ~~for~~ ~~each~~ incident vertex v_i to v_i itself.

Q.



Adjacency matrix

undirected

$$\begin{array}{c|ccc} & a & b & c \\ \hline a & 0 & 2 & 3 \\ b & 2 & 2 & 2 \\ c & 3 & 2 & 0 \end{array}$$

self loop = 2 always

directed

$$\begin{array}{c|ccc} & a & b & c \\ \hline a & 0 & 2 & 2 \\ b & 0 & 2 & 2 \\ c & 1 & 0 & 0 \end{array}$$

Incident matrix.

undirected

$$\begin{array}{cc} & \begin{matrix} e_1 & e_2 & e_3 & e_4 & e_5 & e_6 & e_7 & e_8 & e_9 \end{matrix} \\ \begin{matrix} a \\ b \\ c \end{matrix} & \begin{bmatrix} 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 2 & 2 & 1 & 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 & 0 & 1 & 1 & 1 \end{bmatrix} \end{array}$$

directed

$$\begin{array}{cc} & \begin{matrix} e_1 & e_2 & e_3 & e_4 & e_5 & e_6 & e_7 & e_8 & e_9 \end{matrix} \\ \begin{matrix} a \\ b \\ c \end{matrix} & \begin{bmatrix} 0 & 0 & 0 & 0 & \cancel{0}1 & 1 & -1 & 1 & 1 \\ 1 & 1 & 2 & 2 & -1 & -1 & 0 & 0 & 0 \\ -1 & -1 & 0 & 0 & 0 & 0 & -1 & -1 & -1 \end{bmatrix} \end{array}$$

Shortest path:-

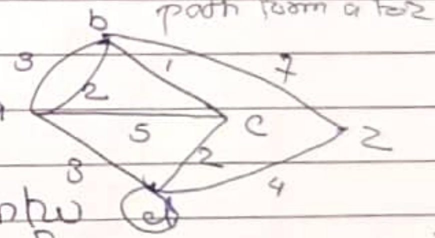
The length of the path in the graph without weights denotes the number of edges in the path and the shortest path is a path b/w two vertices u and v which uses the least number of edges.

Graphs that have a number assigned to each edge are called weighted graph the number may represent computer processing time or other quantity. The length of ~~graph~~ path in the weighted graph is the sum of the ~~edges~~ weights of the edges of this path.

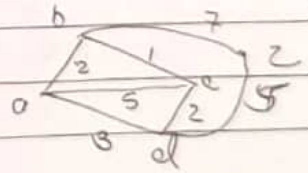
In weighted graph:-

1) ~~Dij~~ Dijkshtra's Algorithm

To find the shortest path from a to z



Step (0):- A connected weighted graph G have self loop then delete them. If G has parallel edges b/w two vertices keep only the edge of minimum weight



	a	b	c	d	z
a	$\boxed{0}$	∞	∞	∞	∞
a	$\boxed{0}$	$\boxed{2}$	5	3	∞
2 = b	$\boxed{0}$	$\boxed{2}$	$\boxed{3}$	3	9
3 = c	$\boxed{0}$	$\boxed{2}$	$\boxed{3}$	$\boxed{3}$	9
3 = d	$\boxed{0}$	$\boxed{2}$	$\boxed{3}$	$\boxed{3}$	8

Shortest distance

Step ① Initially set the starting vertex ~~with~~ of permanent label is zero and others as temporary label ∞

Step ②:- let u be a vertex in temporary label for with length is minimum mark them as a permanent label. if u = z then stop

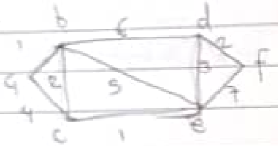
I am Kesare Persistence Mathematics Include

Step ③ For every edge $e = (u, v)$ incident with u and v is temporary label then change the length of v to minimum of

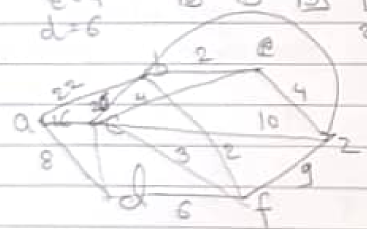
$$\min\{L(v), L(u) + L(e)\}$$

Step ④ :- Reduce the temporary label set by removing the vertex u and repeat step ②

Ex ① :-

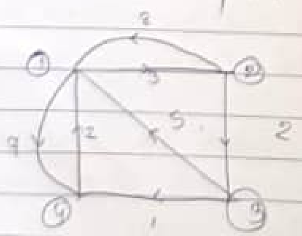


	a	b	c	d	e	f
a	$\boxed{0}$	∞	∞	∞	∞	∞
a = 0	$\boxed{0}$	$\boxed{1}$	4	∞	∞	∞
b = 1	$\boxed{0}$	$\boxed{1}$	$\boxed{3}$	6	5	∞
c = 3	$\boxed{0}$	$\boxed{1}$	$\boxed{3}$	6	$\boxed{4}$	∞
e = 4	$\boxed{0}$	$\boxed{1}$	$\boxed{3}$	$\boxed{6}$	$\boxed{14}$	7
d = 6			2			7



Floyd Warshall shortest path Algorithm

Q1 find the shortest path is has F-W Algor



$$A = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \end{array} \begin{array}{c} \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ \begin{bmatrix} 0 & 3 & \infty & 7 \\ 8 & 0 & 2 & \infty \\ 5 & \infty & 0 & 1 \\ 2 & \infty & \infty & 0 \end{bmatrix} \end{array}$$

$$A^1 = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \end{array} \begin{array}{c} \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ \begin{bmatrix} 0 & 3 & \infty & 7 \\ 8 & 0 & 2 & 15 \\ 5 & 8 & 0 & 1 \\ 2 & 5 & \infty & 0 \end{bmatrix} \end{array}$$

$$A^2 = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \end{array} \begin{array}{c} \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ \begin{bmatrix} 0 & 3 & 5 & 7 \\ 8 & 0 & 2 & 15 \\ 5 & 8 & 0 & 1 \\ 2 & 5 & 7 & 0 \end{bmatrix} \end{array}$$

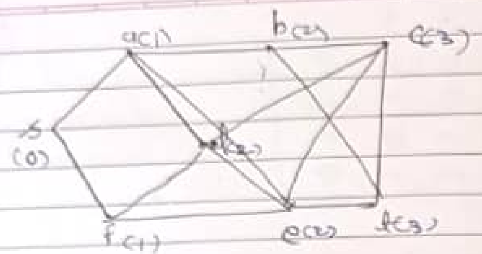
$$A^3 = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \end{array} \begin{array}{c} \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ \begin{bmatrix} 0 & 3 & 5 & 6 \\ 7 & 0 & 2 & 3 \\ 5 & 8 & 0 & 1 \\ 2 & 5 & 7 & 0 \end{bmatrix} \end{array}$$

$$A^4 = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \end{array} \begin{array}{c} \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ \begin{bmatrix} 0 & 3 & 5 & 6 \\ 5 & 0 & 2 & 3 \\ 3 & 6 & 0 & 1 \\ 2 & 5 & 7 & 0 \end{bmatrix} \end{array}$$

The Breath First Search Algorithm

For the unweighted graph one of the Algorithm to find the shortest path is Breath first Search algorithm. The basic idea behind the BFS Algo is we start from the initial vertex (S) then we process the neighbour of that vertex (S)

R Rahul



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Then we process all neighbours of neighbours of s and so on. Generally we ~~kee~~ neep to keep track of neighbour of vertex and we need to guarantee that no vertex process twice this algo involves assign labels to the vertex

Step ① :-
label vertex s with 0 and i = 0

Step ② :- find all unlabelled vertices in G which are adjacent to vertices label i if there are no such vertices then i is not connected to s if there are such vertices label them i+1

Step 3 :- If t is labelled Go to step no 4 if not increase i to i+1 Go to step ②

Step 4 :- The length of shortest path form s to t is i+1

The backtracking algorithm for shortest path

Step 1 :- Set $i = \lambda(t)$

length, vertex

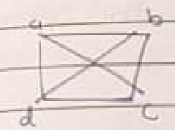
and assign v_i ~~$\lambda(t)$~~ $= t$

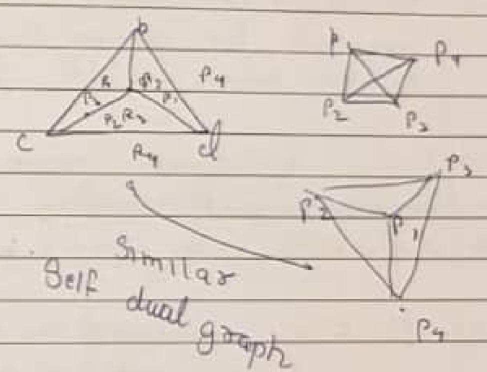
Step ② :- find the vertex v adjacent to v_i such that ~~labe~~ $\lambda(v) = i-1$

Assign $v_{i-1} = v$

Step ③ :- If i = 1 Stop
if not
decrease $i = i-1$, go to Step ②

In general there may be many short path form a to b





Note :-

The cromatic No. of Null graph is 1

2) If their are n vertices then cromatic

3) A simple connected graph with one or more edge is having chromatic number ≥ 2

$\chi(G) \geq 2$

4) If degree of vertex $v = d$, Then atmost d colors are required for proper colouring of the vertex adjacent to V

5) Every k-chromatic graph has atleast k-vertices with degree of each vertices

If $\chi(G) = k$ then $\deg(V_i) \geq k-1$ $V_i = V_1, V_2 - - - V_k$

6) If H is subgraph of G then

$\chi(H) \leq \chi(G)$

7) If K_n is n ie $\chi(K_n) = n$

8) The chromatic no of cycle with n vertices is either C_n is 2 or 3 if n is even or odd respectively

Note :-

1) If C_n is a subgraph of graph (G)

$\chi(G) \geq 2$ if n is even

$\chi(G) \geq 3$ if n is odd

Result :-

for any graph $\chi(G) \leq \Delta(G)+1$

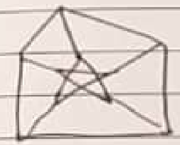
$\Delta(G)$ is that maximum degree of vertex in G

Brook's theorem :-

If G is a connected graph other than a complete graph with

$\Delta(G) \geq 3$, then $\chi(G) \leq \Delta(G)$

Q petersen graph : find its chromatic no



G is connected and not complete graph

$\Delta(G) = 3$ Thus by brook's theorem

$\chi(G) \leq \Delta(G)$

$\chi(G) \leq 3$ — ②

As C_5 is a subgraph of G

$\chi(C_5) \leq \chi G$

$3 \leq \chi(G)$ — ①

form ① & ②

$\chi(G) = 3$

Independent Set :-

A set of vertices in a graph is said to be Independent set if not two vertices in a set are adjacent & is also known as stable set

Note:-

- A Single vertex ~~is~~ a graph is always an a independent Graph
- Every subset of Independent set is an independent set
- Vertices in an independent set can have the Same Colour

Maximal independent Set :-

An independent set of G is not a ~~graph~~ which ~~of~~ Subset of any independent set is known as Maximal Independent Set

Maximal independent set of a graph can be more than one set in graph

Independence Number:-

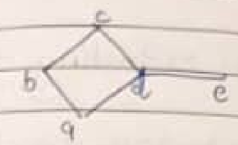
A Graph may have many maximal independent set and they are of different sizes the No of vertices in the largest M.I.S of G is called the Independence Number of the Graph

Re-defining the chromatic number

The minimal no of all maximal I.S

or

$\nu(G) = \gamma(v)$



$V = \{a, b, c, d, e\}$

Independent set : $S_1 = \{b\}, \{c\}, \{d\}, \{e\}, \{a\},$
$\{b,d\}, \{b,e\}, \{a,c,e\}, \{a,e\}$
$\{a,c\}$

Maximal independent set $= \{a,c,e\}, \{b,e\}, \{b,d\}$
A B C

Independence No $= 3$

Chromatic no of G :-

3

$A \cup B \cup C = V$

$2 = A \cup C = V$

$A \cup B \neq V$

$B \cup C = V$ ×

$B \cup A \neq V$

$\therefore A \cup C = V$

Chromatic no $= 2$

Method of finding all maximal ~~sets~~ I.S in a Graph using boolean Algebra

We define,

1) ~~a+b~~ first treat all vertices as boolean variables

2) ~~a~~ a+b denotes the operation of including vertex a or b or both

3) a.b denotes the operation of including vertex both a and b

4) a' denotes the operation of excluding vertex A

5) Express an is (x, y) as boolean product xy

6) $\phi = \Sigma xy$ for all (x, y) in G to get a Boolean expression

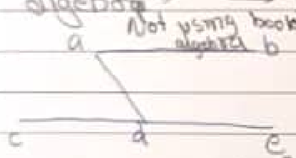
7) $\phi' = \Pi(x_i + y_i) = (x_1 + y_1)(x_2 + y_2) \cdots (x_n + y_n)$

$= f_1 + f_2 + \cdots + f_n$

Where ϕ' is a boolean Complement of ϕ and each f_i is a maximal independent set of G in its Compliment form for a vertex set is maximal independent set if and only if $\phi = 0$

but $\phi = 0$ if and only if $\phi' = 1$, ϕ iff $f_i = 1$ for some i

Imp* Q Find all possible maximal I S of following graph using boolean algebra ⇒ Not using boolean algebra



→ Independent Set $V = \{a, b, c, d, e\}$

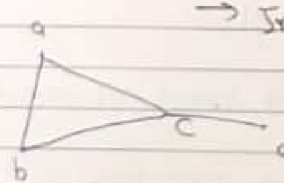
$\{b, c\}, \{b, d\}, \{b, e\}, \{a, c, e\}, \{a\}$

$\{a, e\}, \{b, c, e\}$

→ Maximal independent set

$S_1 = \{a, c, e\}$ $S_2 = \{b, c, e\}$ $S_3 = \{b, d\}$

Q find maximal I S



→ Independence No r

$\beta = \beta(G)$

$\chi(G)$

$V = S_1 \cup S_2 \cup S_3$ (3)

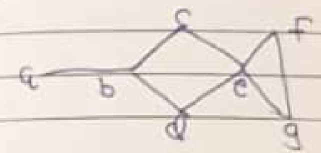
$V = S_2 \cup S_3$ (2)

$\chi(G) = 2$

Chromatic partitioning :-

If G be a Simple Connected graph let S be the all disjoint independent set of G whose union is V then S is the partition of the vertex set V of G. the smallest possible number of partioning of vertices of G is called as a chromatic partitioning

Q find the chromatic partitioning of the following graph



1) $\{a, c, d, f\}$

$\{b, g\}, \{e\}$

2) $\{a, c, d, g\}, \{f, b\}, \{e\}$

3) $\{a, c, d, f\}, \{b, e\}, \{g\}$

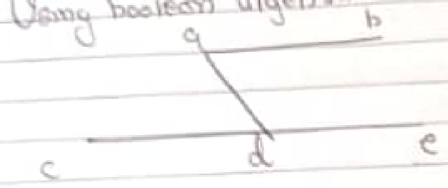
4) $\{a, c\}, \{b, e\}, \{d, g\}, \{f\}$

5) $\{a, d\}, \{b, e\}, \{c, g\}, \{f\}$

These are the ~~chromatic~~ partitioning of the graph

1), 2), 3) are the chromatic partitioning of the Graph

Using boolean algebra



let us Consider

$\phi = ab + ad + cd + de$

$\phi' = (ab + ad + cd + de)'$

$= (a'+b')\cdot(a'+d')(c'+d')\cdot(d'+e')$

$= a'd' + a'c'e' + b'd'$

$= f_1 + f_2 + f_3$

Here $f_1 = a'd'$ is a maximal independent set in the form of complement

$= v - \{a, d\}$

$= \{b, c, e\}$

lly,

$f_2 = \{a', c', e'\}$

$= v - \{a, c, e\}$

$= \{b, d\}$

$f_3 = \{b', d'\}$

$= v - \{b, d\}$

$= \{a, c, e\}$

Edge Coloring:-

An edge coloring of graph G is an assignment of colours to the edges of G so that no two edges with a common vertex receive same colour. the minimum no of colours required to edge coloring G is called chromatic index and it is denoted by $\chi'(G)$

k-edge colorable Graph:-

If in a graph G k colours are used the G is said to be k-edge colourable graph

11 Lower & upper bound of $\chi'(G)$: the lower bound of $\chi'(G) \geq \Delta(G)$ where ΔG is the largest vertex degree of a vertex in G

12 If Graph G has m number of edges then $\chi'(G) \leq m$, where $|E| = m$

13 Vizing theorem:-

If G be a simple graph with maximum vertex degree $\Delta(G)$ then $\Delta(G) \leq \chi'(G) \leq \Delta(G) + 1$

14 Kong's theorem:

If G be a bipartite graph then chromatic index of G

$\chi'(G) \leq \Delta(G)$

15 the chromatic index of a complete graph

$$\chi'(K_n) = \begin{cases} n, & \text{if } n \text{ is odd} \\ n-1, & \text{if } n \text{ is even} \end{cases}$$

Vidyalekhan

Chromatic index of cycle C_n

$$\chi'(C_n) = \begin{cases} 2, & \text{if } n \text{ is even} \\ 3, & \text{if } n \text{ is odd} \end{cases}$$

Matching :-

$G(V,E)$ $M \subseteq E$ is said to be a Matching if no two edges in ~~capital~~ M are incident on the Same vertex ~~that~~ ie no two edges are adjacent

A set Containing single edge of a graph is a Matching

A vertex is Said to be matched or saturated if it is an end point of one of the edges in the matching otherwise the vertex is Said to be unmatched

Maximal Matching :-

A maximal matching is a matching to which new edge can be added without violating the matching property ie ~~on~~ M is maximal matching if it is not a proper subset in any other matching in G

Note :-

① A Graph may have many different maximal matching with different Sizes

② In K_3 every single edge forms a maximal matching

Vidyalekhan

Maximum matching :-

A Maximum matching is a maximal matching that Contains the largest possible no of Edges

Matching number :-

There may exists more then one maximum matching in a graph the size of maximum matching is called as the matching number of G it is denoted by $\gamma(G)$

Perfect matching :-

A Matching is Said to be — " — iff every vertex of G is Saturated ie iff every vertex of G is incident to exactly one edge of the matching

① Not all graphs are PM

② Every PM is maximum matching but Converse not true

③ the number of perfect matching in a Complete K_n is 0 when n is odd $(K_n) = (n-1)(n-3) \cdots 5.3.1$ if n is even

Q find the no of perfect matching in the Complete bipartite graph in $K_{n,n}$

Q write down the perfect matching of K_6

q1 Find no of perfect matching in K_{mn} graph (complete bipartite)

q2 Write down perfect matching of K_6 graph.

Graph covering

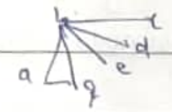
q1 An edge (u,v) of graph g covers vertices u & v. A vertex of G is said to be cover the edges with which it is incident. Covering with vertices and edges closly related with independent set & matching

Vertex covering

VC of graph G is subset of vertices $C \subseteq V$ such that each edge of g is incident to atleast one vertex in C. The set C is set to cover the edges of g.

Minimal covering

A vertex covering $C \subseteq V$ of graph g is said to minimal covering is no vertex can be removed without distroying its ability to cover g.



set is {a,b} or {f,b}

Minimum vertex covering.

A graph may have many minimal covering of different sizes.
A ~~mine~~ mvc is a minimal vc of smallest possible sizes.

Vertex Covering no

It is size of the minimum vertex covering

The complete biperatte graph has minimum covering size = ~~m~~
K_{mn} = min {m,n}

mvcn it is denoted by $T(G)$.

For Any graph G(V,E) $T(G)$ + max indp set = no of vertices in V.

g1

Edge covering
$G(V,E)$ is subset of edges $h \subseteq E$ such that each vertex in G is incident with atleast one edge in h. Then set h is set to cover the vertices of G.

minimal edge covering
An edge covering h in G is set to be minimal covering if no edge of h can be removed without destroying its ability to cover the graph

Minimum edge covering:
It is minimal covering of smallest possible size.

Edge covering no:
$\rho(G)$ is the size of minimum edge covering.

Result: If g is a bipartite graph then max size of matching in g is equal to the minimal size of vertex covering g.

The complete graph K_{mn} has edge covering no max of $\{m, n\}$.
Every edge covering of g includes all pendent edges of graph.

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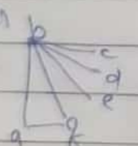
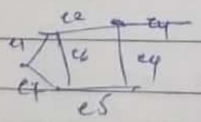
G1 

G2 

Find $\rho(G1)$ & $\rho(G2)$

Applicants a1 – a5 apply for post P1 – – P5 ie a1 {P1, P2} – – – etc
a2 {P1, P3, P5}, a3 {P1, P2, P3, P5} a4 {P3, P4} using graph theory find whether there is any perfect matching (ii) find matching