Unit-I (Mathematical Logic)

★ Statement: [proposition] An assertive sentence which is either true or false but not both is called a statement or proposition.

Example :-

(i) The earth is round → this is a statement which is true.

(ii) $x+2=5$ → this is a sentence not a statement unless the value of x is given, we cannot decide whether it's a statement or not or simply sentence.

(iii) Take two asperines → this is a sentence.

(iv) $3+1=9$ → this is a statement which is false.

(v) The sun will come out tomorrow → This is a statement which is either true or false but not both although we would have to wait till tomorrow.

★ Logical connectives :

→ Negation : If p is a statement, then the negation of p is the statement denoted by $\sim p$ or $\neg p$.

Ex. :- p: Tea is hot.

$\sim p$: Tea is not hot.

OR

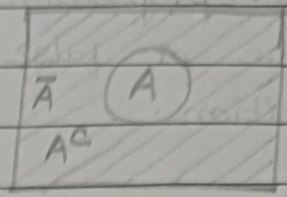
It is not the case that Tea is hot.

q: $2 \leq 3$

$\sim q$: $2 > 3$ OR $2 \leq 3$ (It is not the case that $2 \leq 3$)

Truth Table:

p	$\sim p$
T	F
F	T

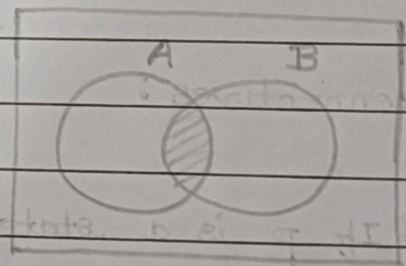


→ Conjunction: If p & q are two statements, then their conjunction is a compound statement = "p and q". It is denoted by $p \wedge q$.

Ex:- p: Sky is blue.
q: $2-1=4$
Then, $p \wedge q$: Sky is blue and $2-1=4$.

★ NOTE: T & F are called Truth values of the statement.

p	q	$p \wedge q$
T	T	T
T	F	F
F	T	F
F	F	F



$A \cap B$
$(p \wedge q)$

Unit-I

1) Negation:

p	~p
T	F
F	T

2) Conjunction:

and ∧ (conjunction)

p	q	$p \wedge q$
T	T	T
T	F	F
F	T	F
F	F	F

3) Disjunction: If p and q are two statements, then their disjunction is a compound statement "p or q". It is denoted by $p \vee q$.

Ex.- let $p: 2>7$ & q: sky is blue.
Ans: $p \vee q$: 2>7 or sky is blue.

p	q	$p \vee q$
T	T	T
T	F	T
F	T	T
F	F	F

★ Conditional statement: (one way implication)
If p&q are two statements, then the compound statement "If p, then q" is called conditional statement. It is denoted by $p \Rightarrow q$ (or $p \rightarrow q$).

Here, p is called antecedent or hypothesis and q is called conclusion or consequent.

Ex.- If you work hard, then you will score good marks.

Here, p : You work hard (Hypothesis)
& q : You will score good marks.

p	q	$p \Rightarrow q$
T	T	T
T	F	F
F	T	T
F	F	T

★ **Bi-conditional statement :**

If p & q are two statements, then "p if and only if q", i.e., "p iff q" is called biconditional statement. It is denoted by $p \Leftrightarrow q$ (or $p \leftrightarrow q$).

$$p \Leftrightarrow q \equiv (p \Rightarrow q) \wedge (q \Rightarrow p)$$

p	q	$p \Leftrightarrow q$
T	T	T
T	F	F
F	T	F
F	F	T

★ **Converse, inverse, contra-positive of the given conditional statement :**

If $p \Rightarrow q$ is the conditional statement, then :-

(i) Converse is $q \Rightarrow p$
(ii) Inverse is $\sim p \Rightarrow \sim q$
(iii) Contra-positive is $\sim q \Rightarrow \sim p$

Note : $(p \Rightarrow q) \equiv (\sim q \Rightarrow \sim p)$

→ Verify $(p \Rightarrow q) \equiv (\sim q \Rightarrow \sim p)$.

$\equiv$, $\Leftrightarrow$ are same.

p	q	1 $p \Rightarrow q$	$\sim q$	$\sim p$	2 $\sim q \Rightarrow \sim p$	1 $\Leftrightarrow$ 2
T	T	T	F	F	T	T
T	F	F	T	F	F	T
F	T	T	F	T	T	T
F	F	T	T	T	T	T

Note:

i) If all the entries in last column are T, then the given statement is **Tautology**.

ii) If all the entries in last column are F, then the given statement is **contradiction**.

iii) If some entries are T and some are F, then the given statement is **absurdity / contigency**.

To be continued in new notebook...