classmate

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③ Elementary Combinatorics
(Permutations and Combinations)

★ **Combinatories :**

In many discrete problem techniques for counting are important in Mathematics in the analysis of algorithms.

In combinatorics, we will discuss the basics of counting and their application.

★ **The Fundamental Principles :**

→ Sum Rule :- If an event can occur in 'm' number of ways an another event can occur in 'n' number of ways and if these events cannot occur simultaneously, then one of the two events can occur in m+n number of ways.

→ Product Rule :- If an event can occur in 'm' number of ways an 2nd event can occur in 'n' number of ways if the number of ways the 2nd event occur does not depend upon the occurrence of 1st, then the two events can occur simultaneously in mn number of ways.

Factorial :-

$$n! = \lfloor n = n\cdot(n-1)\cdot(n-2)\cdot\cdot\cdot\cdot 3\cdot 2\cdot 1$$

$$0! = 1 \qquad 1! = 1$$

★ **Permutation :**

→ The different arrangements which can be made out of a given set of things, by taking some or all of them at a time are called their permutation.

→ The number of permutations of n different things taken 'r' objects ($r \le n$) at a time is denoted and defined as:

$$\boxed{{}^nP_r = \frac{\lfloor n}{\lfloor (n-r)}}$$

★ Combination:

→ The different groups that can be made out of given set of things by taking some or all of them at a time irrespective of the order are called their combination.

→ The number of combinations of 'n' different things taken 'r' objects ($r \le n$) at a time is denoted and defined as:-

$$\boxed{{}^nC_r = \frac{\lfloor n}{\lfloor (n-r) \cdot \lfloor r}}$$

★ Difference between Permutation and Combination:

→ In a combination, only selection is made where as in permutation, not only a selection is made but also arrangement in definite order is considered.

→ In a combination, the ordering of the selection object is immaterial whereas in a permutation, the ordering is essential.

permutation (use only single object at a time)

00
01 } not counted.

★ Example:

CAT (2 letters)	$P_1 P_2 P_3$ (2 groups)	1, 3, 5 (2 numbers)
CA	P_1, P_2	1 3
AC	P_2, P_3	3 1
TA	P_1, P_3	1 5
AT		5 1
CT		3 5
TC		5 3
		1 1
$^3P_2 = \frac{\lfloor 3}{\lfloor (3-2)}$	$^3C_2 = \frac{\lfloor 3}{\lfloor (3-2) \cdot \lfloor 2}$	3 3
$= \frac{3 \cdot 2}{1}$	$= \frac{3 \cancel{\lfloor 2}}{\lfloor 1 \cancel{\lfloor 2}}$	5 5
$= 6$	$= 3$	

★ Circular Permutations:

A circular permutation is an arrangement of objects around a circle. The circular permutations are different only when the relative order of the object is changed. The no. of circular arrangements of n different things = $(n-1)!$

When clockwise and anti-clockwise arrangements are not different, for example, arrangement of beads in a necklace, then the no. of circular arrangements of n different things is $\frac{(n-1)!}{2}$.

(1) In how many ways 5 boys and 4 girls can be seated in a round table?

1) no restriction
2) all the 4 girls sit together
3) all the 4 girls don't sit together
4) No two girls sit together

(2) In how many ways 8 different beads can be arranged to form a necklace?

(3) In how many ways 7 person sit around a table so all shall not have the same neighbours in any 2 arrangements?

(4) There are 50 students in each of the senior or junior classes. Each class have 25 male and 25 female students. In how many ways can 8 students committee be formed that there are 4 female and 3 juniors in the committee?

$^{25}C.$

$^{25}C_1 \times {}^{25}C_4 \times {}^{25}C_3 +$

class room (50)

25M 25F

Seniors (50)

25M 25P 4P+3J

1 2
2 1
1 0

3 2
2 3
2 4

ALLAHABAD → $\frac{19}{14 \times 12}$

★ Combination with Repetition:

The no. of unordered choices of r from n objects with repetition allowed is ${}^{n+r-1}C_r$.

Result:-

The no. of solutions of $x_1+x_2+x_3+----+x_n=r$ where x_i's are non-negative integers is given by:- ${}^{n+r-1}C_r$ or ${}^{n+r-1}C_{n-1}$

(1) How many ~~d no. of~~ two digits can be possible from the digits $\{1,3,5\}$ when a digit can be repeated and we assume ordering of digits in a number doesn't matter? 6

(2) Consider the set A, B, C, D. In how many ways can we select two of these letters when repetition is allowed and order doesn't matter? 10

(3) In how many different ways ~~can~~ 20 identical apples can be distributed among 4 persons?

(4) How many solutions does the eq^n $x+y+z=17$ for x, y, z as non-negative integers ~~a~~ have?

(5) For a set of 6 T and F questions, find the no. of ways of answering all the questions? 2^6

60

(6) How many different no. lie between 100 and 1000 can be formed with the digits 1, 2, 3, 4, 5 without digit being repeated?

7. In how many ways can 7 boys and 5 girls be seated in a row so that no 2 girls may sit together?

8. Given 10 people :- $\{P_1, P_2, P_3, \text{-----}, P_{10}\}$
   1) In how many ways can the people lined up in a row?
   2) if P_2, P_6, P_9 want to stand together?
   3) If P_2, P_6, P_9 do not want to stand together?

9. Find the no. of permutations that can be held from the letters of 'daughter' taking all words together:
   1) taking all the letters
   2) beginning with D
   3) beginning with d ending with r
   4) all vowels together
   5) not even 2 vowels together
   6) vowels occupying even places

→ $x+y+z=3$; $x\geq 0, y\geq 0, z\geq 0$, $x,y,z \in Z^+$

No. of possible $sol^n = {}^{3+3-1}C_3 = {}^5C_3 = \underline{10}$

$\because n=3$, $r=3$

★ Combination of things not all different :

By taking some or all of $(p+q+r)$ things where p are alike, q are alike, r are alike, then the possible no. of selections can be made in $(p+1)(q+1)(r+1)-1$ no. of ways.

① In how many ways a selection be made out of 3 mangoes, 5 oranges and 2 apples? 71

② There are 5 questions in a Q.P.. In how many ways can a boy solve 1 or more questions?

★ Pigeonhole principle :

If n pigeons are assigned to m pigeon holes, then at least one pigeon hole contains 2 or more pigeons, if $m < n$.

→ Generalised Pigeonhole Principle :-

If n pigeon holes are occupied by $kn+1$ or more pigeons, where $k \in \mathbb{Z}^+$, then at least one pigeon hole is occupied by $k+1$ or more pigeons.

① Find the minimum no. of students in a class to be sure that 4 out of them are born in the same month? 37

$$\left\lfloor \frac{n-1}{m} \right\rfloor + 1 = 4 \Rightarrow 3 = \frac{n-1}{12}$$

$$\left\lfloor \frac{n-1}{12} \right\rfloor = 3 \Rightarrow \boxed{n = 37} \text{ (L.B.)} \to \text{min.}$$

$$3 \le \frac{n-1}{12} < 4 \qquad \frac{n-1}{12} < 4$$

$$\Rightarrow \boxed{n < 49}$$

$$\Rightarrow n = 48 \text{ (U.P.)}$$

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→ Extended Pigeonhole Principle :-
If n pigeons are assigned to m pigeon holes ; if $n > m$, then one of the pigeon holes must contain atleast $\left\lfloor \frac{n-1}{m} \right\rfloor + 1$.

Note: $\lfloor x \rfloor$ = greatest integer less than or equal to x

(1) If 9 books are to be kept in 4 shelves, there must be atleast 1 shelf which contains 3 books.

$$\left\lfloor \frac{9-1}{4} \right\rfloor + 1 = \lfloor 2 \rfloor + 1 = \underline{3}$$

(2) * If 20 people are selected, then we may choose a subset of 3 so that all 3 were born on the same day of the week.

(3) If there are 12 persons in a party and if each of them with each other, then how many people shake hand in a party?

~~find~~