Syllabus of the subject [Credits: 3, Max. Marks: 75 (CA: 30 + ESE: 45), Contact Hours: 3]

Number System:

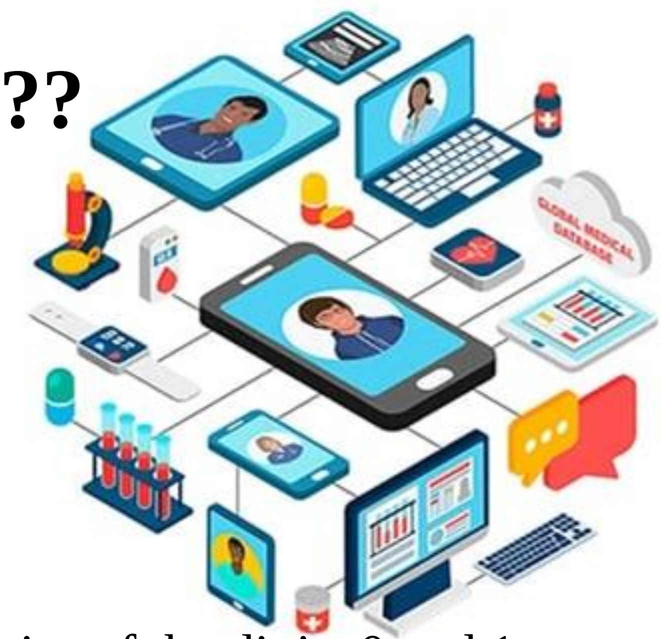
Binary numbers, Decimal numbers, hexadecimal numbers, octal numbers and number conversion, signed binary number representation: signed magnitude, 1's complement and 2's complement representation, Arithmetic operations: binary addition, binary subtraction using 1's complement and 2's complement

binary multiplication and division, 2's complement arithmetic, octal addition, Octal subtraction using 8's complement hexadecimal addition, Hexadecimal subtraction using 16's complement

Sr. No.	Topic	Actual Teaching Hours	Contact Hours Equivalence
1	**Number System:** Binary numbers Decimal numbers hexadecimal numbers octal numbers and number conversion signed binary number representation: signed magnitude 1's complement and 2's complement representation Arithmetic operations: binary addition binary subtraction using 1's complement and 2's complement binary multiplication and division 2's complement arithmetic octal addition Octal subtraction using 8's complement hexadecimal addition Hexadecimal subtraction using 16's complement	10	10

Unit 1- Number System:

Digital means??



a signal expressed as series of the digits 0 and 1

Today, the digital technology is used not only in computers but also in many other applications such as TV, Radar, Military systems, Medical equipment, Communication systems, Industrial process control etc.

Identify digital device and analog device


60
80
40
100
20
120
0
°C


TRACEABLE
26.0
14.7


0 25 50 75 100 125 150
AC VOLTS
BLUE SEA SYSTEMS


Digital Voltmeter
Line.1
Line.2
220
V
AH-076

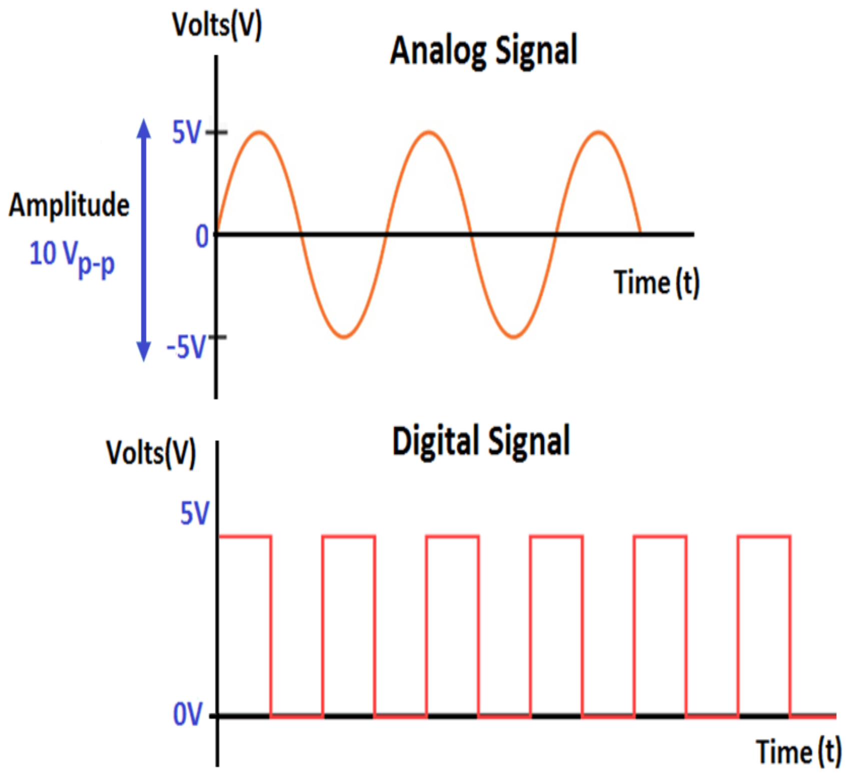
SIGNALS-

It is a physical quantity which contains some information

- The signal are classified into two categories.

1. Analog signals:-

They vary continuously with time.



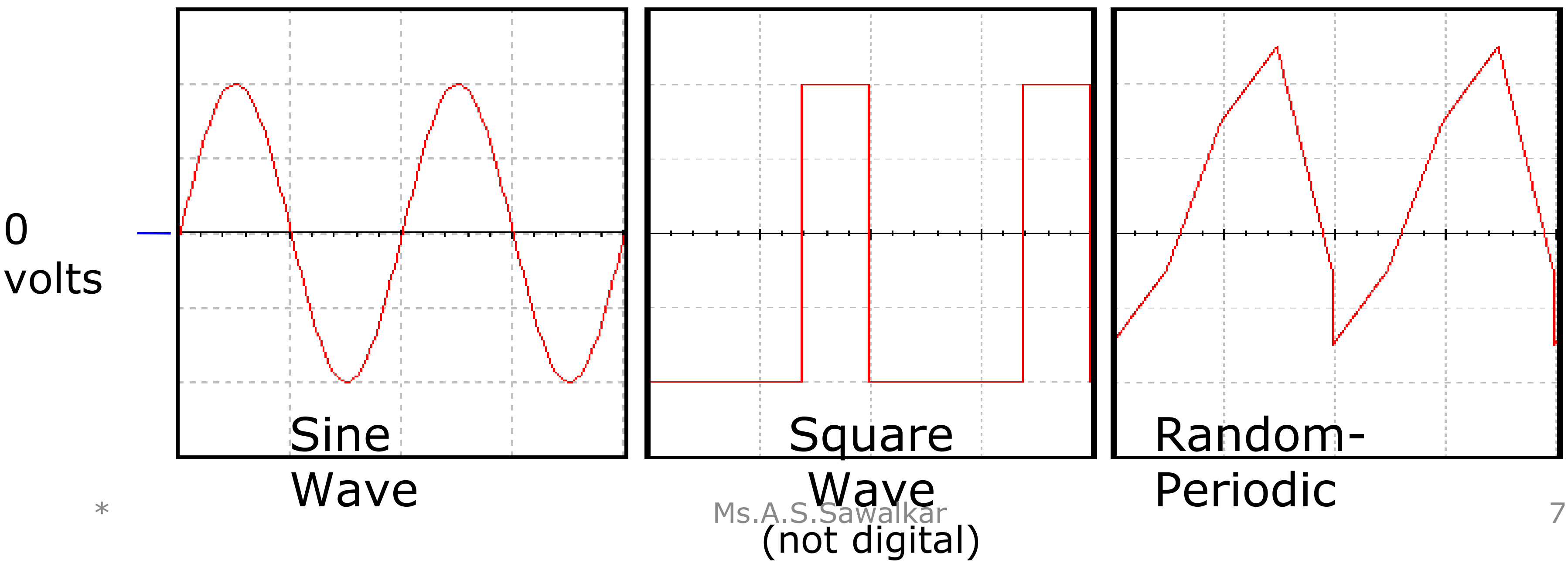
2. Digital signals.

They do not vary continuously with time.

They are discrete with time.

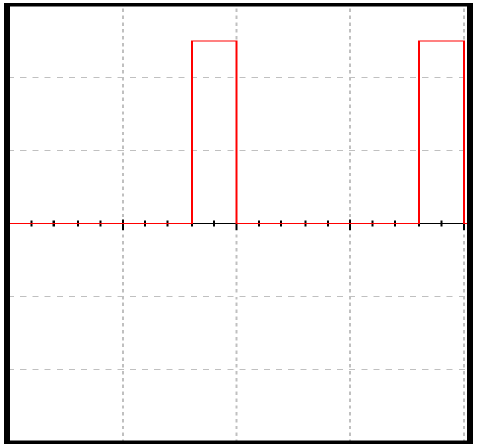
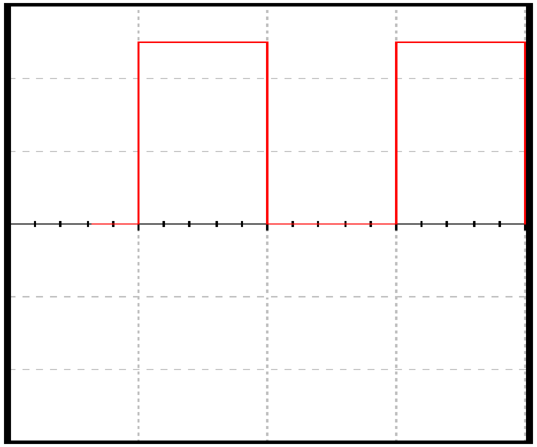
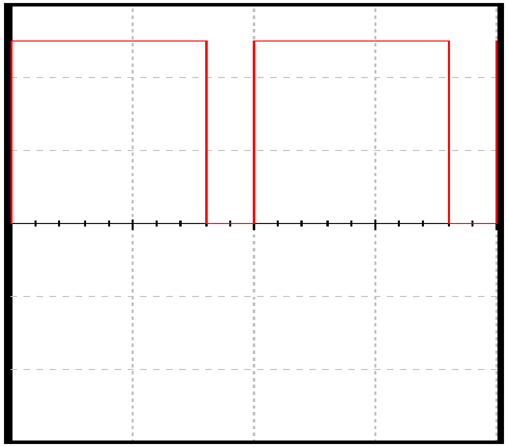
Example of Analog Signals:

- An analog signal can be any time-varying signal.
- Minimum and maximum values can be either positive or negative.
- They can be periodic (repeating) or non-periodic.
- Sine waves and square waves are two common analog signals.
- Note that this square wave is not a digital signal because its minimum value is negative.
- Video and Audio



Example of Digital Signals:

- Digital signal are commonly referred to as square waves or clock signals.
- Their minimum value must be 0 volts, and their maximum value must be 5 volts.
- They can be periodic (repeating) or non-periodic.
- The time the signal is high (t_H) can vary anywhere from 1% of the period to 99% of the period.
- Text and Integers.

Advantages of Digital Signal

- Digital signal can be processed and transmitted more efficiently and reliably than analog signals.
- It is possible to store the data
- The effect of noise(unwanted voltage fluctuations) is less. So digital data does not get corrupt.

Comparison of Analog and Digital Signal

Sr.No	Parameter	Analog Signal	Digital Signal
1	Number of values	Infinite	Finite
2	Nature	Continuous	Discrete
3	Sources	Transducers, Signal generator	Computers, A to D Converters
4	Examples	Sinewave,triangular qwave	Binary signal
5	Applications	Operational Amplifiers,telephone	Television,Military systems,Microprocessors

Systems or Circuits

- Analog Systems:

  The system which required **analog** signals.

  Ex- filters, amplifiers.

- Digital Systems:

  The system which required **Digital** signals.

  Ex- flip flop, shift register, timer, counter.

Sources of Digital Signal:

Computer-All the data used by the computers is digital

A to D Converter

Analog system

- **<u>Disadvantages:-</u>**

1. Less accurate
2. Less reliability
3. Storage & processing data is not possible
4. Their parameter change with time.
5. System affected due to noise.

Digital system

- **Advantages**

1. The have more accuracy.
2. Highly reliable system
3. Less affected by noise.
4. Digital systems have memory hence Information and data can be stored and processed.
5. Design is easier.
6. The are less affected by variation of temperature.
7. They are cost efficient
8. Communication between the digital systems is much easier

Comparison

Sr no	Parameter	Analog systems	Digital systems
1.	Type of signals processed	Analog signals	Digital signals
2.	Type of display	Analog meters	Digital displays using LED and LCD
3.	Accuracy	Less	More
4.	Design complexity	Difficult to design	Easier to design
5.	Memory	No memory	They have Memory
6.	Storage of information	Not Possible	Possible
7.	Effect of noise	More	Less
8.	Versatility	Less	More
9.	Distortion	More	Less
10.	Effect of temperature and ageing on performance	More	Less
11.	Communication between systems	Not easy	Easy
12.	Examples	Filters, amplifiers, power supplies, signal generators	Counters, resisters, microprocessors, Computers

Questions

1) What is signal? Define analog and digital signal.
2) State advantages of digital signal.
3) State disadvantages of analog signal
4) State advantages of digital system.
5) Compare analog and digital signal.
6) Compare analog and digital system.

Binary Logic and Logic levels

Statement which is true if some condition is satisfied and false if not satisfied-------Logic

Logic Denoted by two voltage levels—

1)High level (logic 1)

2)Low level (logic 0)

- **Positive logic** is defined as a high voltage level representing a logic 1 and a low voltage level representing a logic 0.

+ve logic	-ve logic
high voltage correspond to logic '1'	high voltage correspond to logic '0'
'+5V' = LOGIC "1"	'+5V' = LOGIC "0"
'0V' = LOGIC "0"	'0V' = LOGIC "1"

- **Negative logic is the reverse**, i.e., a low voltage level represents a logic 1 and a high voltage level represents a logic 0

1.1 Terms- Bit, Byte, Nibble

A **bit** is a single **0** or **1** on its own.

1

A group of 4 bits together is called a **nibble**

1010

A group of 8 bits together is called a **byte** and this is one of the main units of measurement in all of computing

11011010

Important Definition

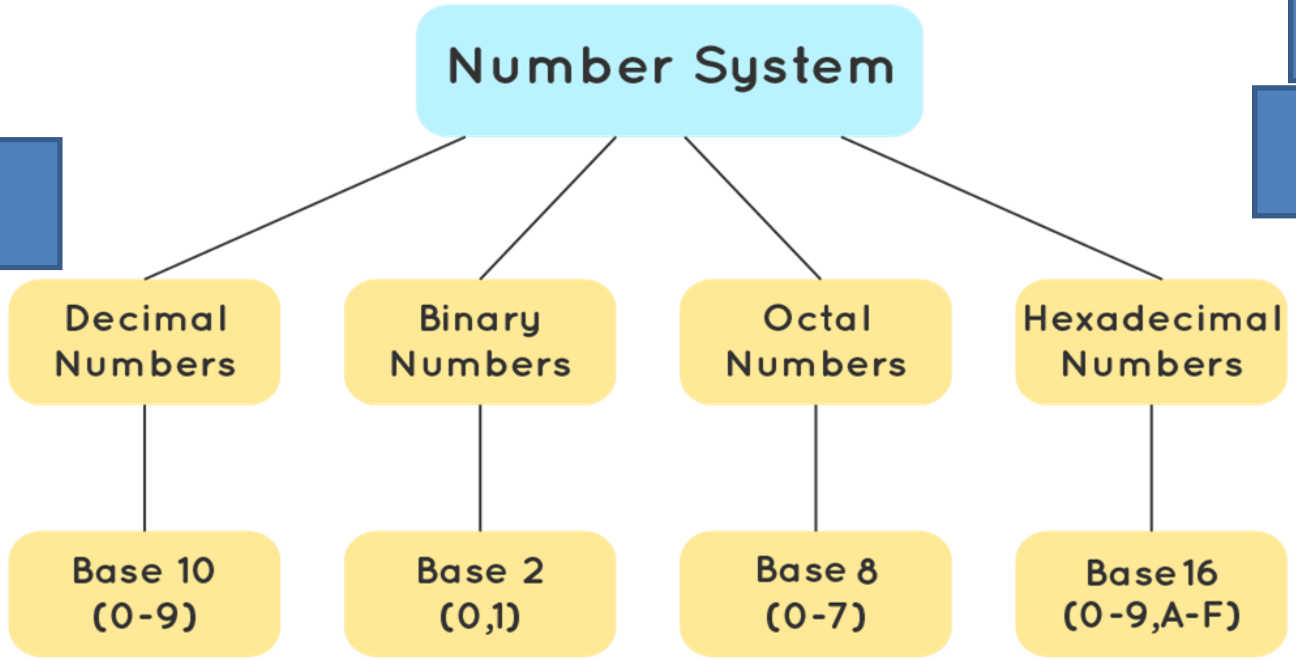
- Base or Radix:

The radix or base is **the number of unique digits, including the digit zero, used to represent numbers**.

For example, for the decimal/denary system (the most common system in use today) the radix (base number) is ten, because it uses the ten digits from 0 through 9.

1.2 Number System

- A numeral system (or system of numeration) is **a writing system for expressing numbers.**



Different types of number systems

Numbering Systems		
System	Base	Digits
Binary	2	0 1
Octal	8	0 1 2 3 4 5 6 7
Decimal	10	0 1 2 3 4 5 6 7 8 9
Hexadecimal	16	0 1 2 3 4 5 6 7 8 9 A B C D E F

Counting

Decimal	Binary	Octal	Hexadecimal
0	0000	0	0
1	0001	1	1
2	0010	2	2
3	0011	3	3
4	0100	4	4
5	0101	5	5
6	0110	6	6
7	0111	7	7
8	1000	10	8
9	1001	11	9
10	1010	12	A
11	1011	13	B
12	1100	14	C
13	1101	15	D
14	1110	16	E
15	1111	17	F
16	10000	20	10
17	10001	21	11
18	10010	22	12

Common Number Systems

System	Base	Symbols	Used by humans?	Used in computers?
Decimal	10	0, 1, … 9	Yes	No
Binary	2	0, 1	No	Yes
Octal	8	0, 1, … 7	No	No
Hexa-decimal	16	0, 1, … 9, A, B, … F	No	No

Then why Octal and Hexadecimal systems used?????

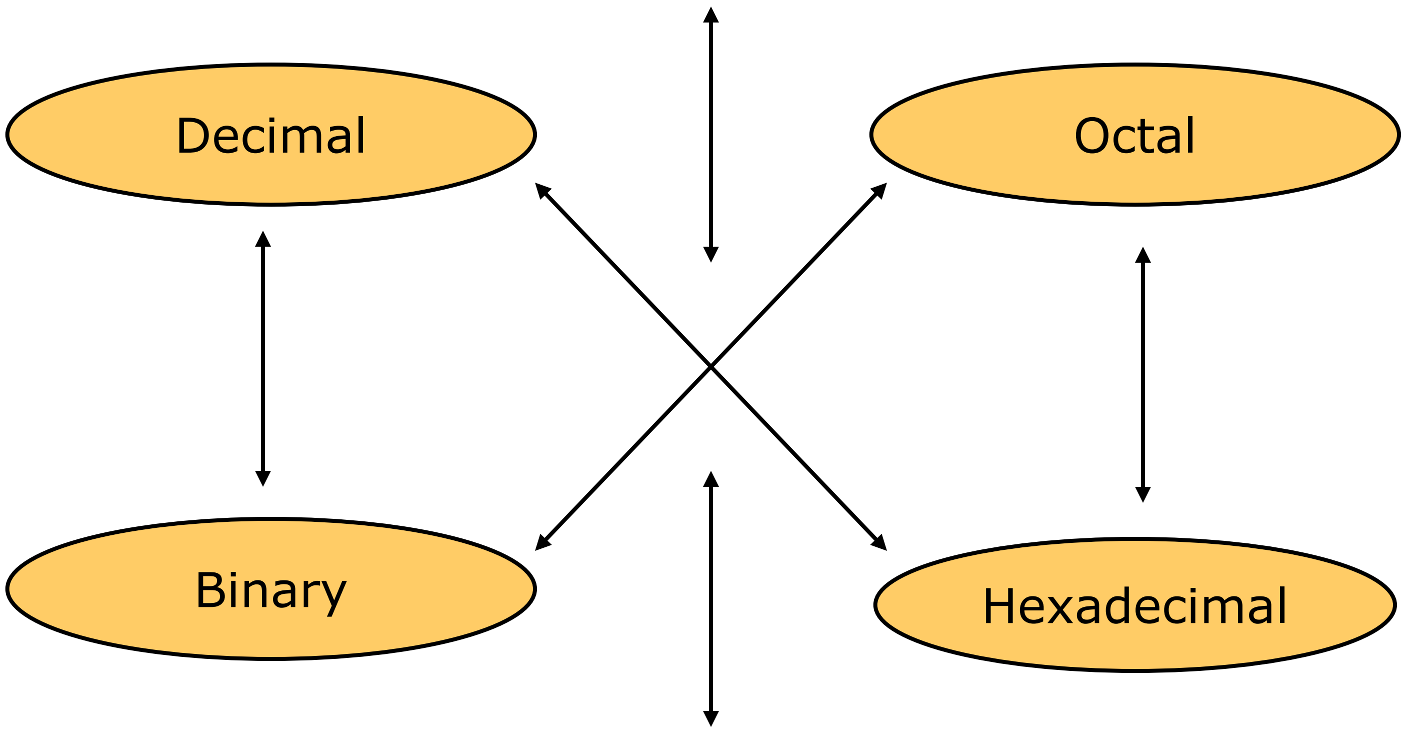
Quick Example

$$25_{10} = 11001_2 = 31_8 = 19_{16}$$

Base

Conversion Among Bases

- The possibilities:



Decimal to Decimal

Decimal

Octal

Binary

Hexadecimal

Next slide…

Conversion Decimal to Decimal

Decimal number	**3**	**4**	**9**	**.**	**2**	**5**
Position of Digits	10^{2}	10^{1}	10^{0}		10^{-1}	10^{-2}
	3×10^{2}	4×10^{1}	9×10^{0}		2×10^{-1}	5×10^{-2}

$$(3 \times 10^{2}) + (4 \times 10^{1}) + (9 \times 10^{0}) + (2 \times 10^{-1}) + (5 \times 10^{-2})$$

$$(300) + (40) + (9) + (0.2) + (0.05)$$

349.25

Other System into Decimal

(into means Multiplication)

Steps to be followed:

1.Note down the given number

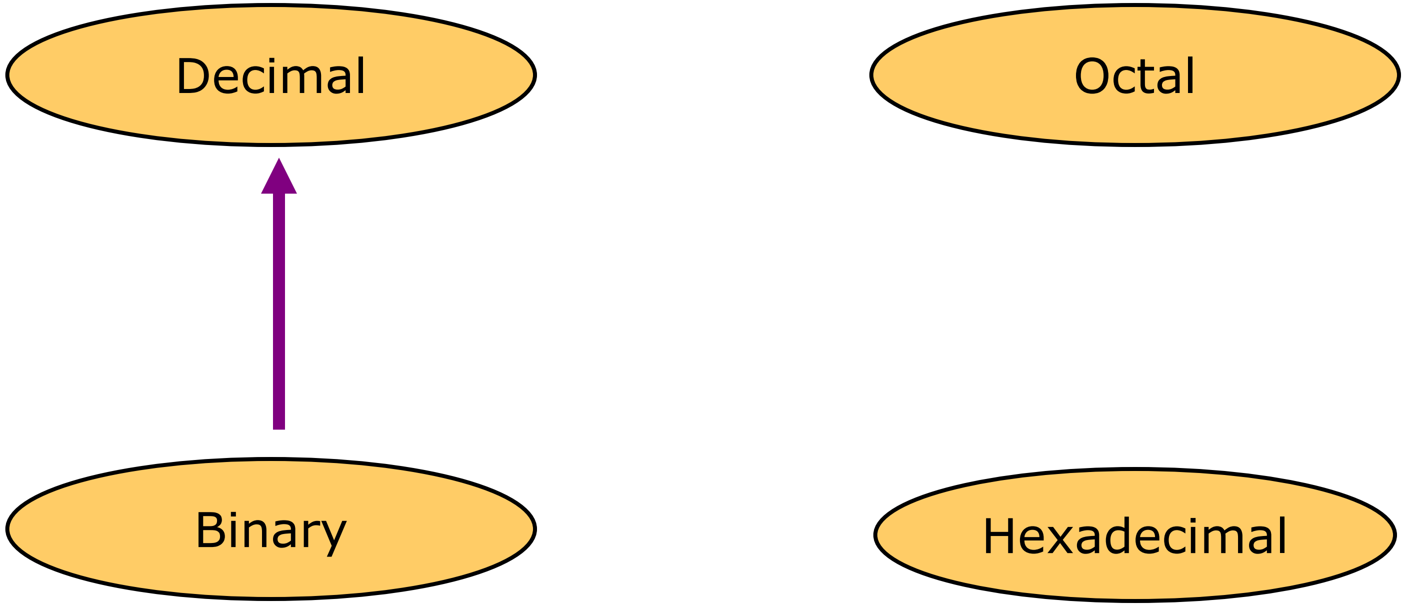
2.Write down the weights corresponding to different positions

3.Multiply each digit in the given number with the corresponding weight

4.Obtain product number

5.Add all the product number to get the decimal equivalent.

Binary to Decimal



Conversion Binary to Decimal

MSB

LSB

Binary number	1	0	1	0	.	0	1
Position of Digits	2^3	2^2	2^1	2^0		2^{-1}	2^{-2}

$$(1 \times 2^3) + (0 \times 2^2) + (1 \times 2^1) + (0 \times 2^0) + (0 \times 2^{-1}) + (1 \times 2^{-2})$$

$$(8) + (0) + (2) + (0) + (0) + (0.25)$$

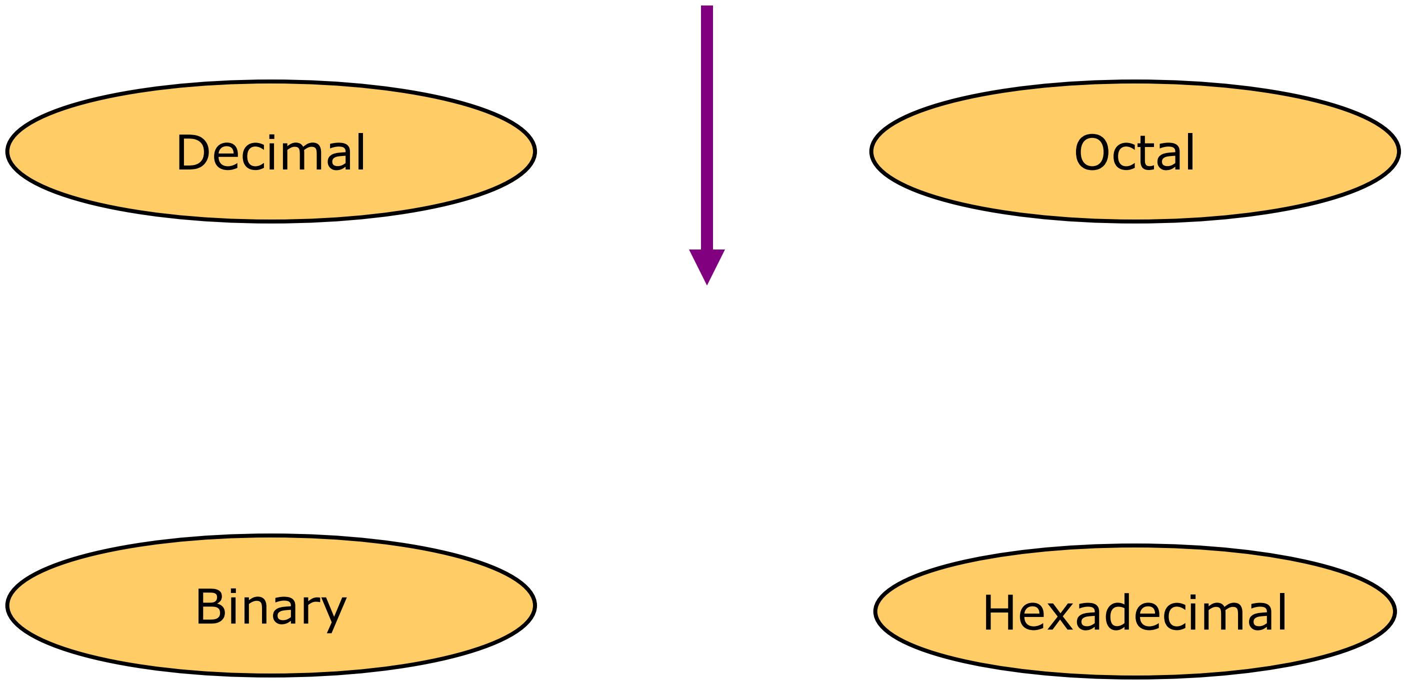
$$(10.25)_{10}$$

Solve

- Convert following Binary numbers into its Decimal equivalent.

1. $(1100010.0101)_2$
2. $(1011.01)_2$

Octal to Decimal



Conversion Octal to Decimal

1. Given Octal Number

Octal number	7	2	3	1	.	4	5
Position of Digits	8^3	8^2	8^1	8^0		8^{-1}	8^{-2}

2.Multiply each digit by its positional weight

$(7 \times 8^3) + (2 \times 8^2) + (3 \times 8^1) + (1 \times 8^0) + (4 \times 8^{-1}) + (5 \times 8^{-2})$

$(7 \times 512) + (2 \times 64) + (3 \times 8) + (1 \times 1) + (4 \times 0.125) + (5 \times 0.0156525)$

3.Final answer

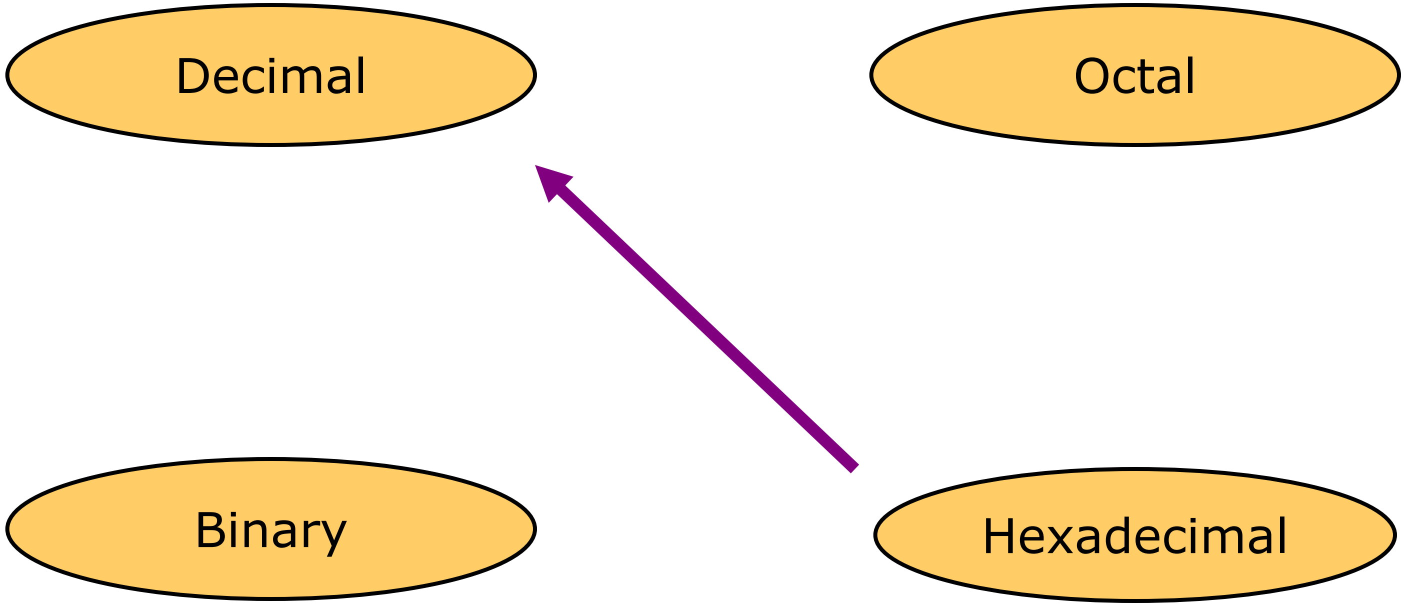
$(7231.45)_8 = (3737.5782625)_{10}$

Solve

- Convert following Octal numbers into its Decimal equivalent

1. **$(753.41)_8$**
2. **$(123.76)_8$**

Hexadecimal to Decimal



Conversion Hexadecimal to Decimal

Hexadecimal number	**7**	**A**	**3**	**D**	**.**	**4**	**C**
Position of Digits	16^3	16^2	16^1	16^0		16^{-1}	16^{-2}

$$(7 \times 16^3) + (A \times 16^2) + (3 \times 16^1) + (D \times 16^0) + (4 \times 16^{-1}) + (C \times 16^{-2})$$

$$(7 \times 4096) + (A \times 256) + (3 \times 16) + (D \times 1) + (4 \times 0.0625) + (C \times 0.0039)$$

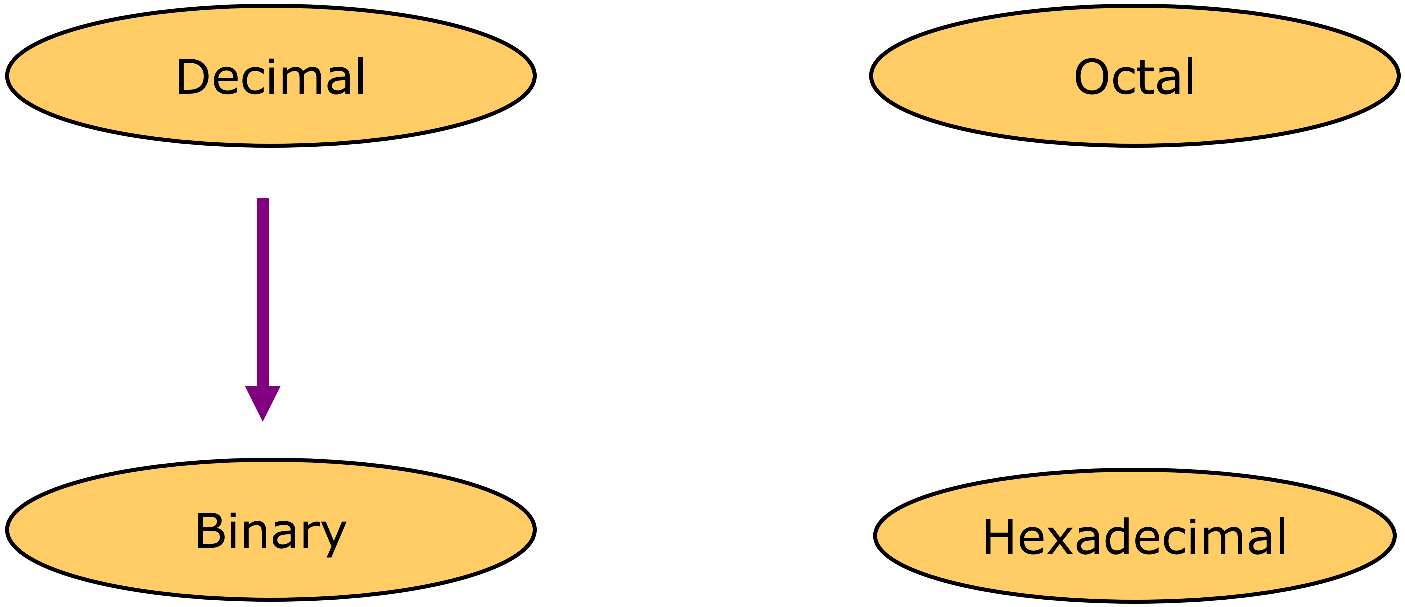
$$(31293.25)_{10}$$

Solve

- Convert following Hexadecimal numbers to Decimal

1. **$(298.41)_{16}$**
2. **$(A234.CD)_{16}$**

Decimal to Binary



Conversion from Decimal into other system

D=Divide

- Technique

1) Divide the decimal number by the base of other system
2) Continue to divide the quotient by the base until there is nothing left
3) List the remainder values in reverse order from bottom to top to find the equivalent.

Decimal to Binary

Divide by two, keep track of the remainder

$125_{10} = ?_2$

2	125	
2	62	1
2	31	0
2	15	1
2	7	1
2	3	1
2	1	1
	0	1

$125_{10} = 1111101_2$

Fractional decimal to binary

$(0.42)_{10} = (?)_2$

$0.42 \times 2 = 0.84$	0
$0.84 \times 2 = 1.68$	1
$0.68 \times 2 = 1.36$	1
$0.36 \times 2 = 0.72$	0
$0.72 \times 2 = 1.44$	1

$(0.42)_{10} = (0.01101)_2$

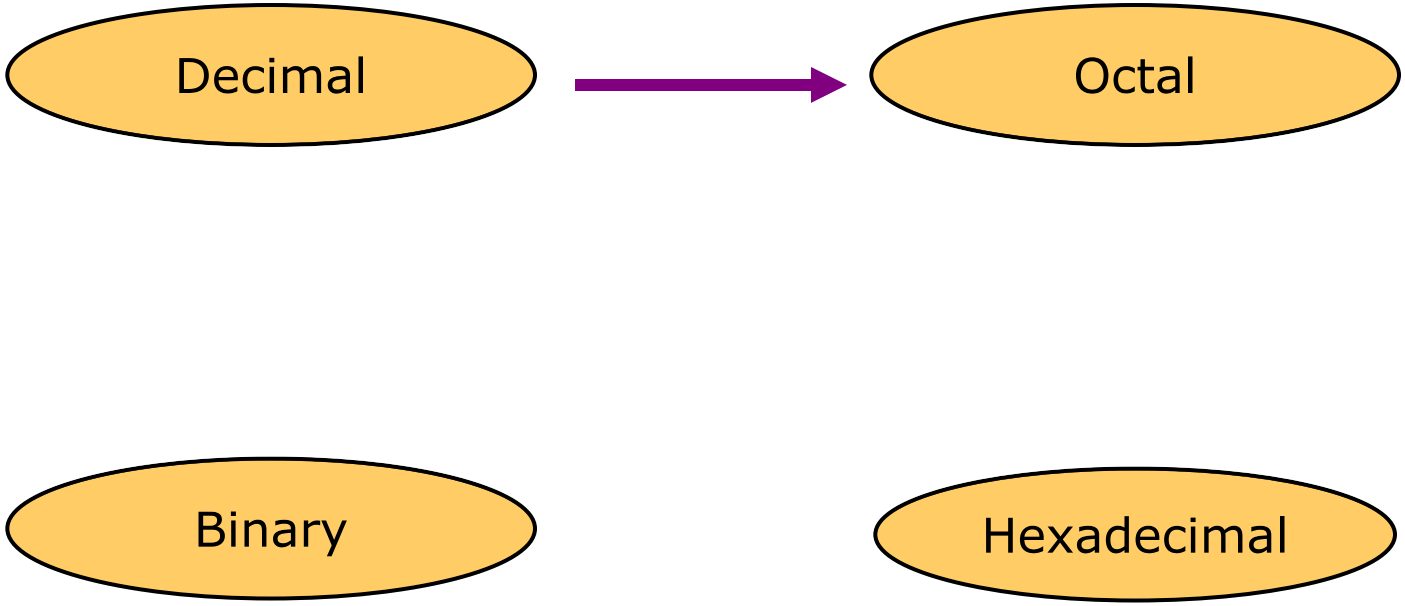
We could have continued further. But the conversion is generally carried out only upto 5 digits.

Solve

- Convert following decimal numbers to binary

1. $(526.42)_{10} = (?)_2$
2. $(1234.78)_{10} = (?)_2$
3. $(9999.99)_{10} = (?)_2$
4. $(4926.25)_{10} = (?)_2$

Decimal to Octal



Technique

Divide by 8

Keep track of the remainder

Example

$1234_{10} = ?_8$

8	1234	
8	154	2
8	19	2
8	2	3
	0	2

$1234_{10} = 2322_8$

Fractional decimal to octal

$(0.6234)_{10} = (?)_8$

$0.6234 \times 8 = 4.9872$	4
$0.9872 \times 8 = 7.8976$	7
$0.8976 \times 8 = 7.1808$	7
$0.1808 \times 8 = 1.4464$	1
$0.4464 \times 8 = 3.5712$	3

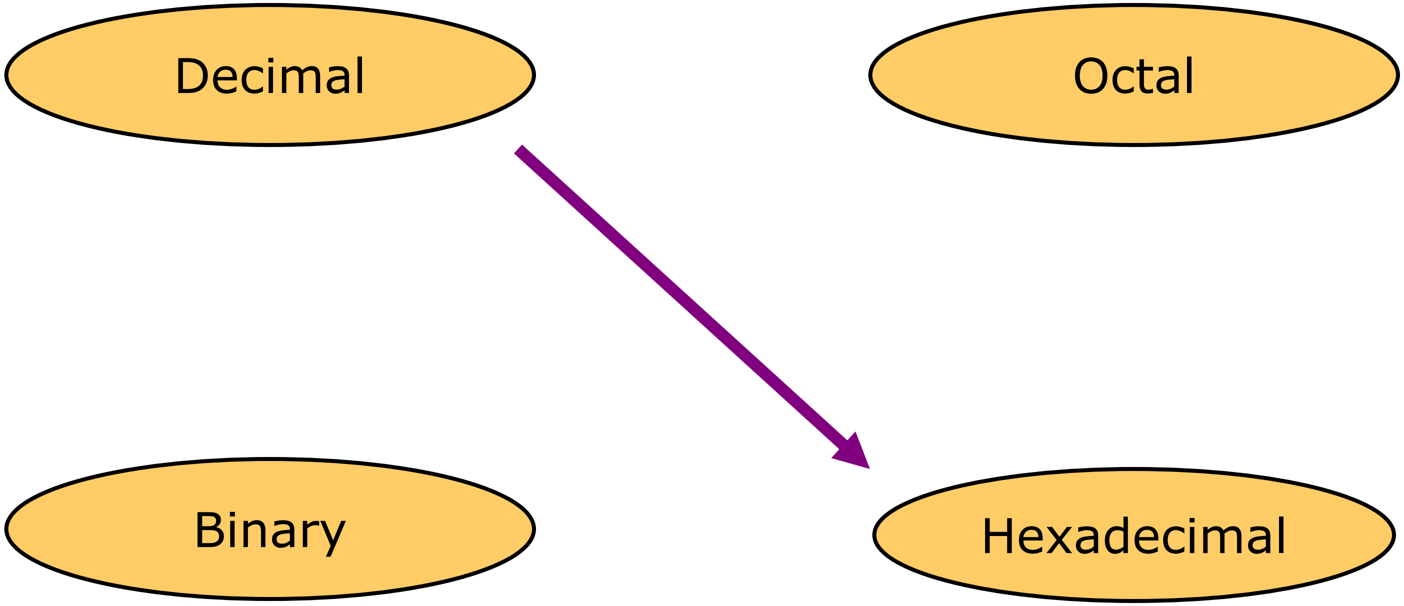
$(0.6234)_{10} = (0.47713)_8$

Solve

- Convert following decimal numbers to octal.

1. $(753.41)_{10} = (?)_8$
2. $(123.76)_{10} = (?)_8$
3. $(1234.78)_{10} = (?)_8$
4. $(9999.99)_{10} = (?)_8$
5. $(4926.25)_{10} = (?)_8$

Decimal to Hexadecimal



Technique

- Divide by 16
- Keep track of the remainder

Example

$1234_{10} = ?_{16}$

16	1234	
16	77	2
16	4	13 = D
	0	4

$1234_{10} = 4D2_{16}$

Fractional decimal to hexadecimal

$(0.31)_{10} = (?)_2$

$0.31 \times 16 = 4.96$	4
$0.96 \times 16 = 15.36$	F
$0.36 \times 16 = 5.76$	5
$0.76 \times 16 = 12.16$	C
$0.16 \times 16 = 2.56$	2

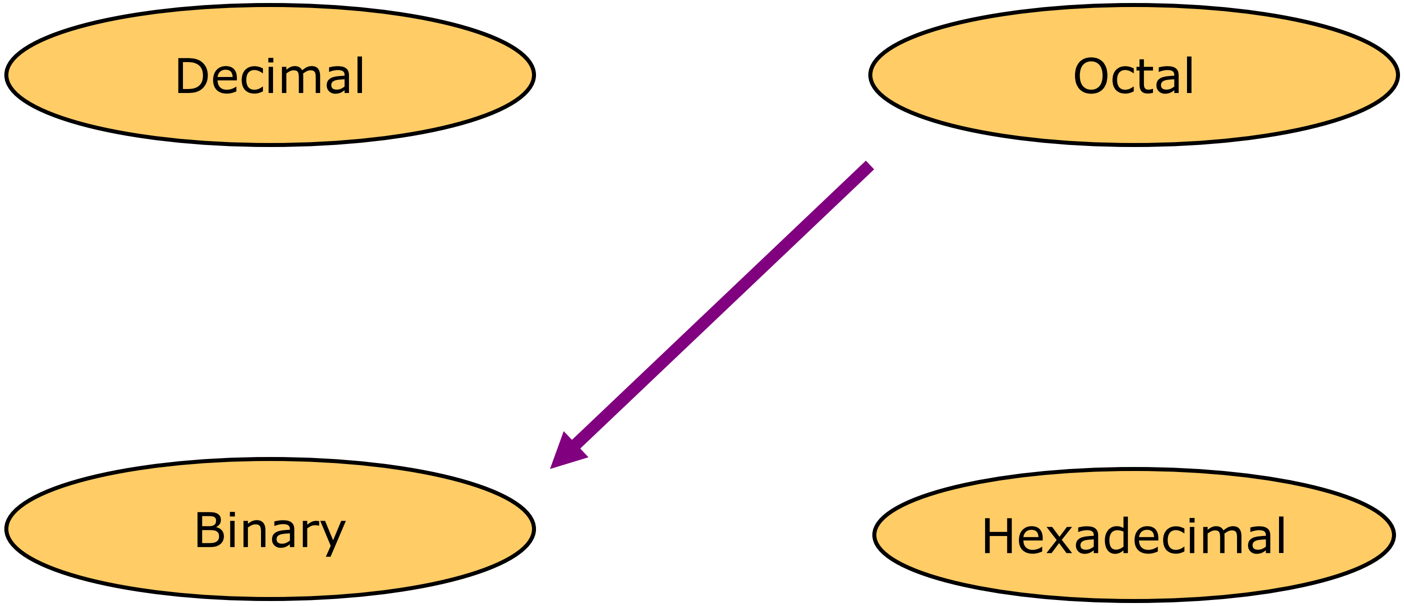
$(0.31)_{10} = (0.4F5C2)_2$

Solve

- Convert following decimal numbers to hexadecimal numbers.

1. $(753.41)_{10} = (?)_{16}$
2. $(123.76)_{10} = (?)_{16}$
3. $(1234.78)_{10} = (?)_{16}$
4. $(9999.99)_{10} = (?)_{16}$
5. $(4926.25)_{10} = (?)_{16}$

Octal to Binary



Octal to Binary

- Technique
  - Convert each octal digit to a 3-bit equivalent binary representation

Example

$705.62_8 = ?_2$

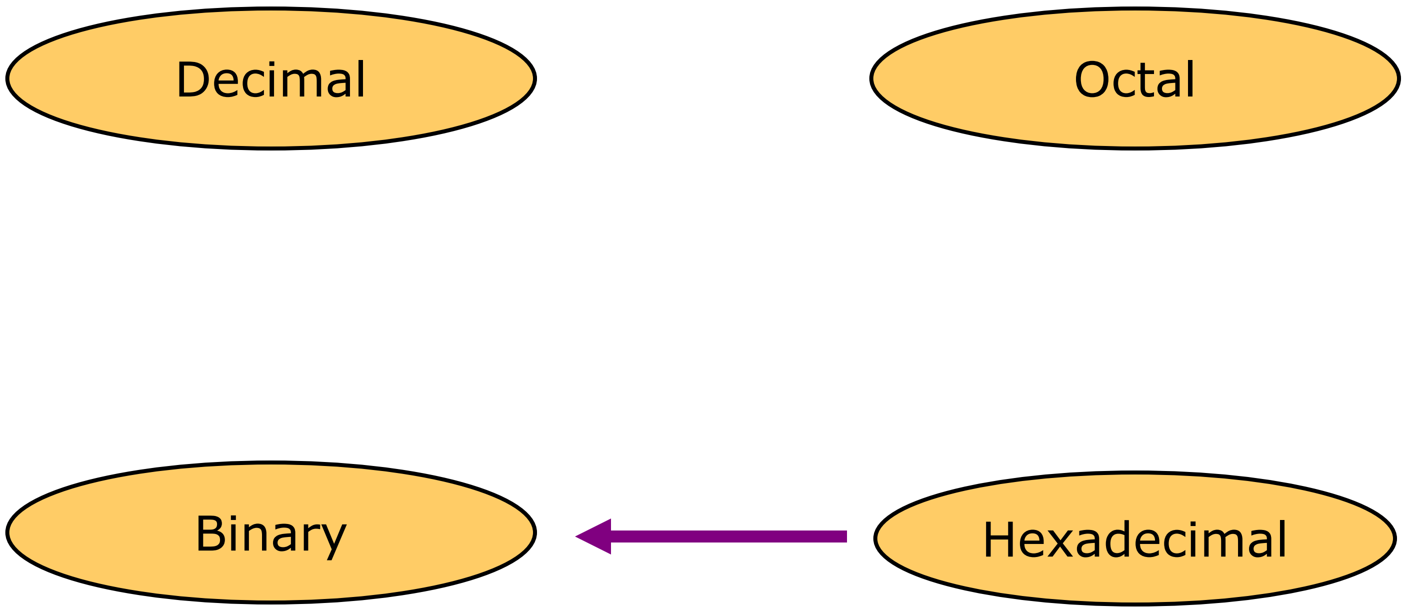
7	0	5	.	6	2
111	000	101		110	010

$705_8 = 111000101.110010_2$

Solve

- $456.12_8 = ?_2$
- $125.37_8 = ?_2$

Hexadecimal to Binary



Hexadecimal to Binary

- Technique
  - Convert each hexadecimal digit to a 4-bit equivalent binary representation

Example

$10AF.C4_{16} = ?_2$

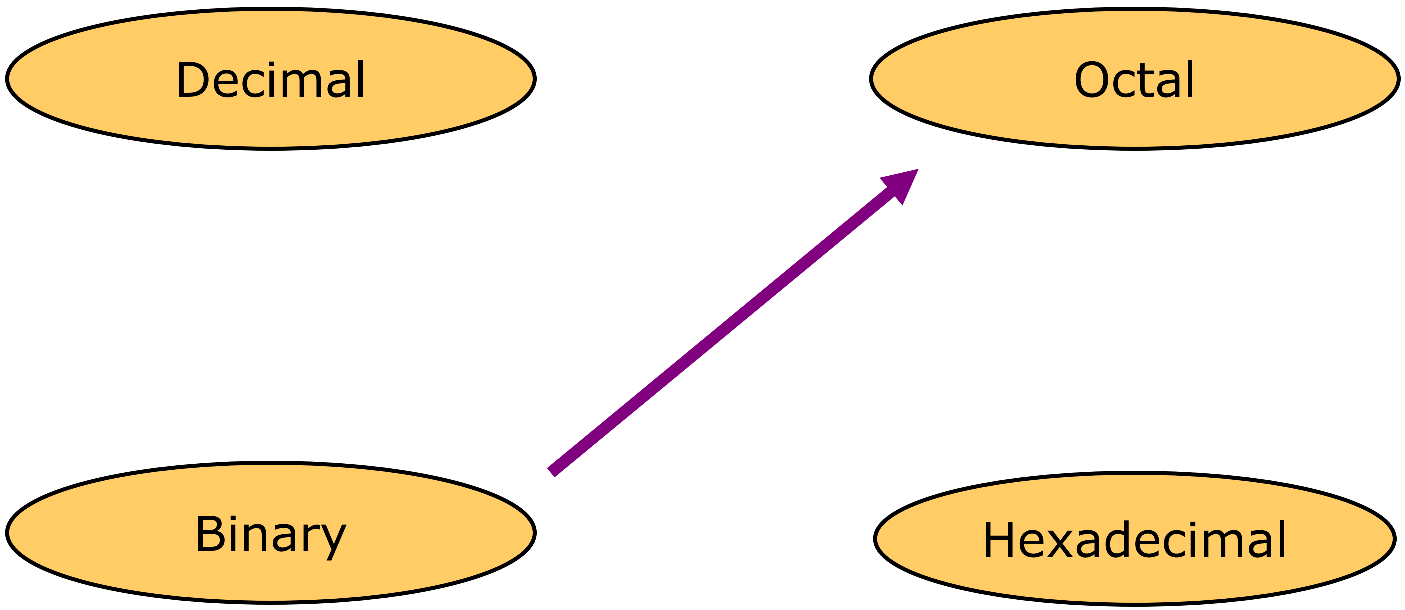
1	0	A	F	.	C	4
0001	0000	1010	1111		1100	0100

$10AF.C4_{16} = 0001000010101111.11000100_2$

Solve

- $\mathbf{45A6.F2_{16} = ?_2}$
- $\mathbf{1CD5.A7_{16} = ?_2}$

Binary to Octal



Binary to Octal

- Technique
  - Group bits in threes, starting on right
  - Convert to octal digits

Example

$1011010111.011111_2 = ?_8$

1 011 010 111 . 011 111

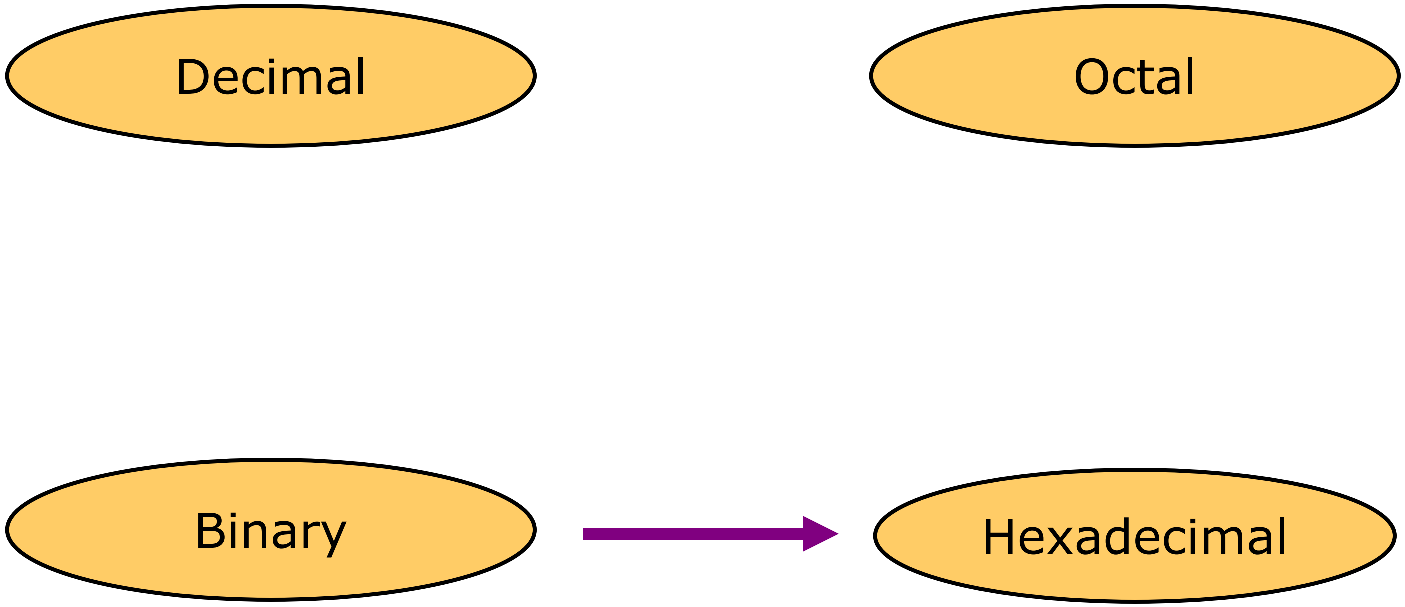
1 3 2 7 3 7

$1011010111.011111_2 = 1327.37_8$

Solve

1. $(1101)_2 = (?)_8$
2. $(10111)_2 = (?)_8$
3. $(101101)_2 = (?)_8$
4. $(1011111)_2 = (?)_8$
5. $(101111101)_2 = (?)_8$

Binary to Hexadecimal



Binary to Hexadecimal

- Technique
  - Group bits in fours, starting on right
  - Convert to hexadecimal digits

Example

$10101110110111.0_2 = ?_{16}$

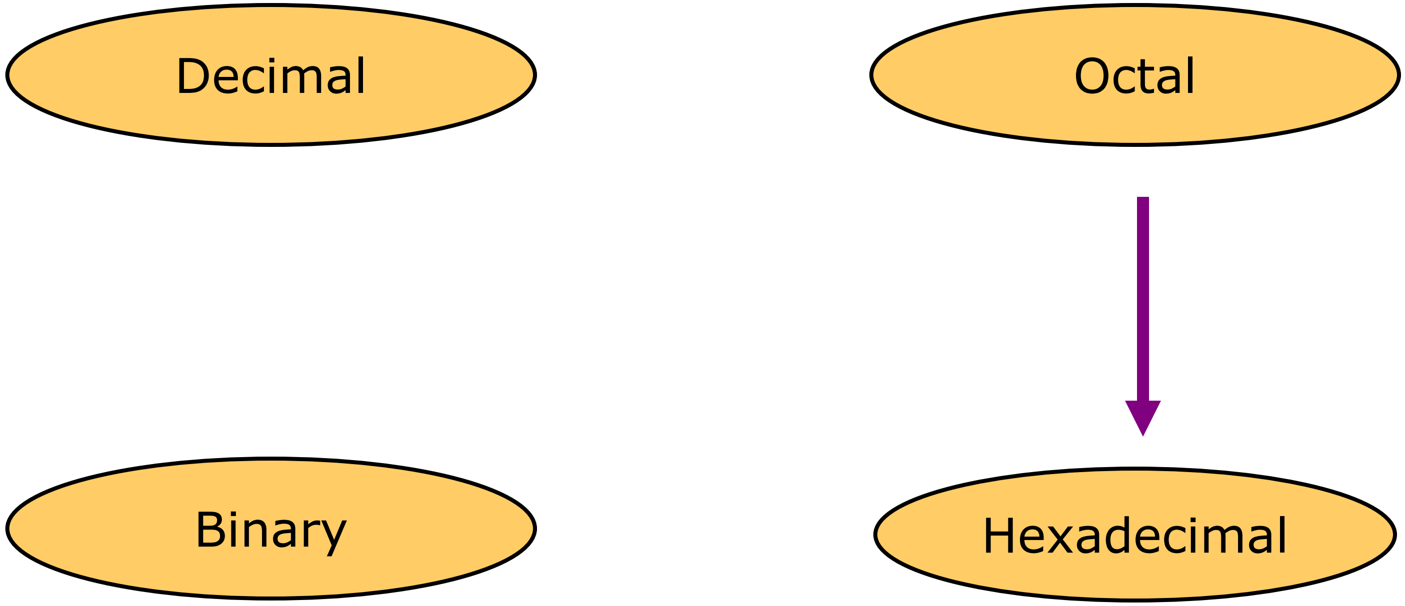
10	1011	1011	.	0011	1000
2	B	B		3	8

$1010111011.00111_2 = 2BB\ .\ 38_{16}$

Solve

1. $(1101)_2 = (?)_{16}$
2. $(10111)_2 = (?)_{16}$
3. $(101101)_2 = (?)_{16}$
4. $(1011111)_2 = (?)_{16}$
5. $(101111101)_2 = (?)_{16}$

Octal to Hexadecimal



Octal to Hexadecimal

- Technique
  - Use binary as an intermediary

Example

$1076.4_8 = ?_{16}$

1 0 7 6 . 4

1 0 00 11 1 110 100 0

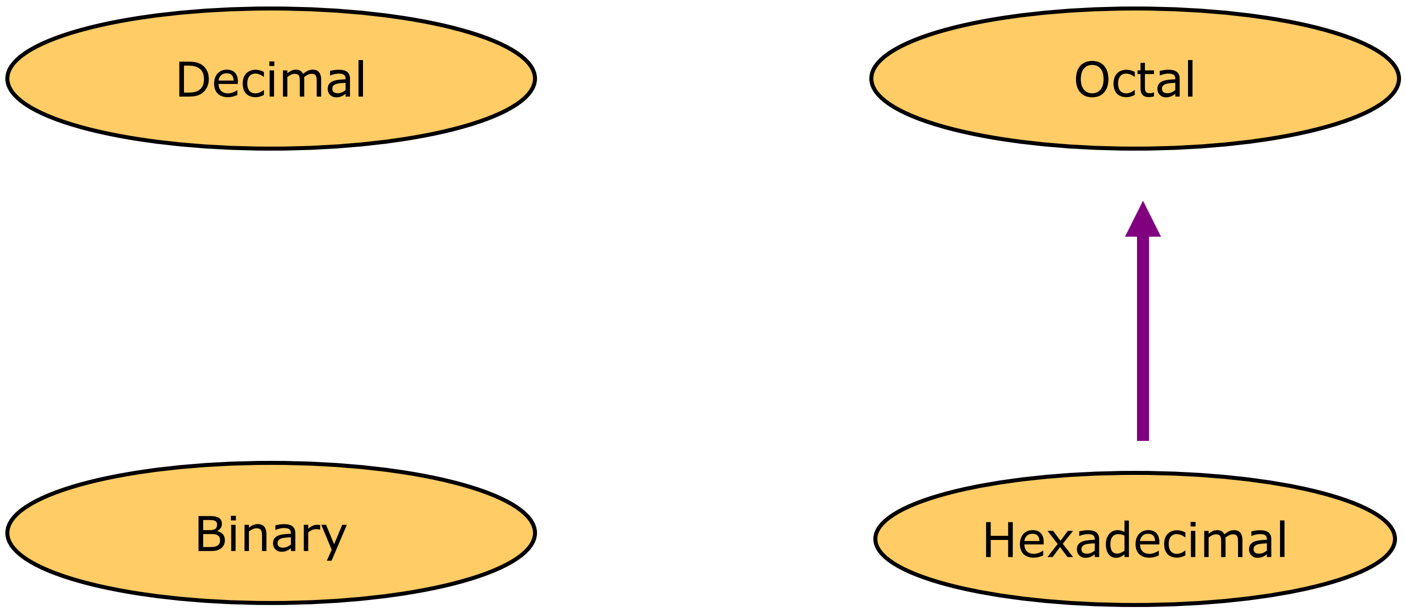
2 3 E 8

$1076.4_8 = 23E.8_{16}$

Solve

1. $(765.43)_8 = (?)_{16}$
2. $(1241.567)_8 = (?)_{16}$
3. $(2354.63)_8 = (?)_{16}$
4. $(1726.54)_8 = (?)_{16}$
5. $(3465.7)_8 = (?)_{16}$

Hexadecimal to Octal



Hexadecimal to Octal

- Technique
  - Use binary as an intermediary

Example

$1F0C.2_{16} = ?_8$

1 F 0 C . 2

0 001 111 1 00 00 1 100 001 0

1 7 4 1 4 1

$1F0C.2_{16} = 17414.1_8$

Solve

1. $(A25.45)_{16} = (?)_8$
2. $(1241.CD)_{16} = (?)_8$
3. $(2F4.6A)_{16} = (?)_8$
4. $(1E.54)_{16} = (?)_8$
5. $(346B.7)_{16} = (?)_8$

Exercise – Convert ...

Decimal	Binary	Octal	Hexa-decimal
33	100001	41	21
117	1110101	165	75
451	111000011	703	1C3
			1AF

Exercise – Convert …

Decimal	Binary	Octal	Hexa-decimal
33	100001	41	21
117	1110101	165	75
451	111000011	703	1C3
431	110101111	657	1AF

Exercise – Convert ...

Decimal	Binary	Octal	Hexa-decimal
29.8			
	101.1101		
		3.07	
			C.82

Exercise – Convert …

Decimal	Binary	Octal	Hexa-decimal
29.8	11101.110011…	35.63…	1D.CC…
5.8125	101.1101	5.64	5.D
3.109375	11.000111	3.07	3.1C
12.5078125	1100.10000010	14.404	C.82

Binary Arithmetic:
Binary Addition

- Four cases of binary addition:

Case	A + B	Sum	Carry
1	0 + 0	0	0
2	0 + 1	1	0
3	1 + 0	1	0
4	1 + 1	0	1

Addition of binary numbers

	1	0	0	1	0
+		1	0	0	1
	1	1	0	1	1

$$\begin{array}{rr} {\scriptstyle 0\,1\,1\,1} & \\ 00111 & 7 \\ 10101 & 21 \\ \hline 11100 & = 28 \end{array}$$

- Ex. 1: Add the following binary numbers.

  $A = (10111)_2$ and $B = (11001)_2$

- Answer:

  Step1: add LSBs (Least significant bits)

				1	Column of LSBs	
A	1	0	1	1	1	LSB
B	1	1	0	0	1	LSB
					0	

  Step 2: complete the addition

	1	1	1	1	1	
A		1	0	1	1	1
B		1	1	0	0	1
	1	1	0	0	0	0

Therefore the answer is $(110000)_2$

Examples

					Carry
	1	0	0	1	
+	0	1	1	0	
	1	1	1	1	Sum
	1	1	1		Carry
	1	1	0	1	
+	0	0	1	1	
1	0	0	0	0	Sum
	1	1			Carry
	0	1	1	0	
+	0	1	1	1	
	1	1	0	1	Sum

Solve

Add following numbers into binary numbering system.

1.$(12)_{10} + (8)_{10}$

2.$(15)_{10} + (10)_{10}$

3.$(35)_{10} + (48)_{10}$

4.$(10101)_2 + (10110)_2$

5.$(10111)_2 + (11000)_2$

Binary Subtraction

Binary subtraction:-

A	B	Difference	Borrow
0	0	0	0
0	1	1	1
1	0	1	0
1	1	0	0

Subtraction of large binary numbers

- Ex. 1: Subtract the following binary numbers.

  A= $(11001)_2$ and B= $(10111)_2$

- Answer:

  Step1: Subtract LSBs (Least significant bits)

					Column of LSBs	
A	1	1	0	0	1	LSB
B	1	0	1	1	1	LSB
					0	

1’s Complement of a number

- 1’s complement of a number is found by changing all 1’s to 0’s and all 0’s to 1’s.
- Ex: 1’s complement of a number 10111 is = 01000
- Solve---- Find 1’s complement of

a. 11010

b. 101101

c. 1010

9’s Complement of a number

- The 9's complement of a decimal number is the number that is obtained by subtracting each digit of the number from 9.

For example, the 9's complement of 546700 is

999999

- 546700

= 453299.

2's Complement of a number

- The 2's complement of a number is obtained by adding 1 to the LSB of 1's complement of that number
- 2's complement = 1's complement + 1
- Ex: obtain 2's complement of a number $\mathbf{(10110010)_2}$

Solution:

Given number =	1 0 1 1 0 0 1 0	
1's complement =	0 1 0 0 1 1 0 1	
Add 1 to LSB of 1's complement =	+ 1	
	1	carry
2's complement =	0 1 0 0 1 1 1 0	

10’s Complement of a number

- 10's complement of a decimal number can be found by adding 1 to the 9's complement of that decimal number.
- For example, let us take a decimal number 456,

9's complement of this number will be

999

\- 456

= 543.

Now 10s complement will be

543+1=544

15’s Complement of a number

- The 15's complement of a number is calculated by subtracting the number from 15 and then subtracting 2 from the result.
- For example, the 15's complement of 12 is 3,
- and the 15's complement of F minus 2 is 13, which is D

16's Complement of a number

- To find the 16's complement of a hexadecimal number, subtract the number from FFFFFFFF and then add 1.
- For example, the 16's complement of C3DF is 3C20

FFFF

\- C3DF

= 3C20

Solve

- Find 2's complement of following numbers.

a. $(1101)_2$
b. $(10111)_2$
c. $(101101)_2$
d. $(1011111)_2$
e. $(101111101)_2$

Binary subtraction using 1's complement method

- **To perform subtraction $(A)_2$ - $(B)_2$**

❖ Step 1: Convert number to be subtracted $(B)_2$ to its 1's complement.

❖ Step 2: Add first number $(A)_2$ and 1's complement of $(B)_2$ using rules of binary addition.

❖ Step 3: if final carry is 1 then add it to the result of addition obtained in step 2 to get final result.

**If final carry in step 2 is 1 then result obtained in step 2 is Positive and in its true form no conversion required.

❖ Step 4: if final carry in step 2 is 0 then result obtained in step 2 is negative and in 1's complement form. So convert it to its true form.

Perform $(9)_{10} - (4)_{10}$ using 1's complement method

$A = (9)_{10} = (1001)_2$ $B = (4)_{10} = (0100)_2$

1's complement of $B = (1011)_2$

Step 1: take 1's complement of $(4)_{10}$

Step 2: Add $(9)_{10}$ and 1's complement of $(4)_{10}$

$(9)_{10}$		1	0	0	1
1's complement of $(4)_{10}$		1	0	1	1
carry	1		1	1	
Result	1	0	1	0	0

Final carry generated

Step 2 result:	0	1	0	0
Final carry				1
Final result	0	1	0	1

Answer is positive and in its true form.

Step 3: add final carry to the result obtained in step 2.

$(A)_{10} - (B)_{10} = (9)_{10} - (4)_{10} = (0101)_2 = (5)_{10}$

Perform $(4)_{10} - (9)_{10}$ using 1's complement method

$A = (4)_{10} = (0100)_2$ $B = (9)_{10} = (1001)_2$

1's complement of B = $(0110)_2$

Step 1: take 1's complement of $(9)_{10}$

$(4)_{10}$		0	1	0	0
1's complement of $(9)_{10}$		0	1	1	0
carry		1			
Result	0	1	0	1	0

Final carry not generated

Step 2: Add $(4)_{10}$ and 1's complement of $(9)_{10}$

Step 2 result:	1	0	1	0
1's complement of step 2 result	0	1	0	1

Final result - $(0 \quad 1 \quad 0 \quad 1)_2$

Answer is Negative

Step 3: as final carry is 0 answer is negative and in 1's complement form

$(A)_{10} - (B)_{10} = (4)_{10} - (9)_{10} = -(0101)_2 = -(5)_{10}$

Solve

- Perform following subtraction using 1's complement method.
- $\mathbf{(11011)_2 - (1010)_2}$ **ans** $\mathbf{(10001)_2}$
- $\mathbf{(10111)_2 - (11000)_2}$ **ans** $\mathbf{-(0001)_2}$

1. $\mathbf{(54)_{10} - (33)_{10}}$ **ans** $\mathbf{-(010101)_2}$

- $\mathbf{(33)_{10} - (54)_{10}}$
- $\mathbf{(99)_{10} - (22)_{10}}$

Binary subtraction using 2’s complement method

- To perform subtraction $(\mathbf{A})_2$ - $(\mathbf{B})_2$

❖ Step 1: convert number to be subtracted $(B)_2$ to its 2’s complement.

❖ Step 2: Add first number $(A)_2$ and 2’s complement of $(B)_2$ using rules of binary addition.

❖ Step 3: if final carry is 1 then the result is Positive and in its true form no conversion required.ult

❖ Step 4: if final carry in step 2 is 0 then result obtained in step 2 is negative and in 2’s complement form. So convert it to its true form.

**** Carry always be discarded.**

Perform $(9)_{10} - (4)_{10}$ using 2's complement method

$A = (9)_{10} = (1001)_2$ $B = (4)_{10} = (0100)_2$

Step 1: take 2's complement of $(4)_{10}$

1's complement of B =	$(1011)_2$
+	1
2's complement of B =	1100

Step 2: Add $(9)_{10}$ and 2's complement of $(4)_{10}$

$(9)_{10}$		1	0	0	1
2's complement of $(4)_{10}$		1	1	0	0
carry	1				
Result		0	1	0	1

Final carry Generated

Discard carry

Final result $(0\ 1\ 0\ 1)_2$

Step 3: as final carry is 1 answer is positive and in true form.

$(A)_{10} - (B)_{10} =$
$(9)_{10} - (4)_{10} =$
$(0101)_2 =$
$(5)_{10}$

Perform $(\mathbf{4})_{10} - (\mathbf{9})_{10}$ using 2's complement method

A= $(4)_{10} = (0100)_2$ B = $(9)_{10} = (1001)_2$

1's complement of B = $(0110)_2$
+ 1

2's complement of B = 0 1 1 1

Step 1: take 2's complement of $(9)_{10}$

$(4)_{10}$		0	1	0	0
2's complement of $(9)_{10}$		0	1	1	1
carry		1			
Result	0	1	0	1	1

Final carry not generated

Step 2: Add $(4)_{10}$ and 2's complement of $(9)_{10}$

Step 2 result:	1	0	1	1
1's complement of step 2 result:	0	1	0	0
2's complement of step 2 result:	0	1	0	1

Step 3: as final carry is 0 answer is negative and in 2's complement form

$(A)_{10} - (B)_{10} = (4)_{10} - (9)_{10} = -(0101)_2 = -(5)_{10}$

Final result - $(0\ 1\ 0\ 1)_2$

Answer is Negative

Solve

- Perform following subtraction using 2's complement method.

1. $(\mathbf{48})_{10} - (\mathbf{61})_{10}$
2. $(\mathbf{33})_{10} - (\mathbf{54})_{10}$
3. $(\mathbf{99})_{10} - (\mathbf{22})_{10}$
4. $(\mathbf{1010})_{2} - (\mathbf{101})_{2}$

Multiplication

- Binary, two 1-bit values

A	B	A × B
0	0	0
0	1	0
1	0	0
1	1	1

Multiplication (3 of 3)

- Binary, two *n*-bit values
  - As with decimal values
  - E.g.,

```
      1110
    x 1011
  --------
      1110
     1110
    0000
   1110
  --------
  10011010
```

Solve

- **$(205)_{10}$ x $(3)_{10}$**
- **$(1110101)_2$ x $(1001)_2$**
- **$(110)_2$ x $(10)_2$**
- **$(1111101)_2$ x $(101)_2$**
- **$(15)_{10}$ x $(8)_{10}$**

Binary division

Ex: $\mathbf{(25)_{10} \div (5)_{10}}$

$(25)_{10} = (11001)_2$

$(5)_{10} = (101)_2$

```
            1  0  1
        ____________
1 0 1 | 1  1  0  0  1
        1  0  1
             1
        ________
        0  0  1  0  1
              1  0  1
              _______
              0  0  0
```

Solve

- $(205)_{10} \div (3)_{10}$
- $(1110101)_2 \div (1001)_2$
- $(110)_2 \div (10)_2$
- $(1111101)_2 \div (101)_2$

Octal Arithmetic
Octal Addition

Example#01: $167)_8+765)_8$

$$
\begin{array}{r}
{}^{1}\,{}^{1} \\
167 \\
+765 \\
\hline
1154
\end{array}
$$

Remember the first step. Think each number as a decimal number and add them as a decimal numbers.

1st column (units column) addition 7+5=12

$$
\begin{array}{l}
\quad {}_{1} \leftarrow \text{quotient as carry} \\
8 \overline{)\,12} \\
\quad\;\; 8 \\
\quad\;\; 4 \leftarrow \text{remainder as sub-sum}
\end{array}
$$

2nd column (tens column) addition 1+6+6=13

$$
\begin{array}{l}
\quad {}_{1} \leftarrow \text{quotient as carry} \\
8 \overline{)\,13} \\
\quad\;\; 8 \\
\quad\;\; 5 \leftarrow \text{remainder as sub-sum}
\end{array}
$$

3rd column (hundreds column) addition 1+1+7=9

$$
\begin{array}{l}
\quad {}_{1} \leftarrow \text{quotient as carry} \\
8 \overline{)\,9} \\
\quad\;\; 8 \\
\quad\;\; 1 \leftarrow \text{remainder as sub-sum}
\end{array}
$$

Answer: $1154)_8$

Example#02: $123)_8+7651)_8$

```
   1 2 3
 +7 6 5 1
  7 7 7 4
```

In this example after adding each column no sub-sum exceeds to 7. So there is no need of converting equivalent octal value.

Answer: $7774)_8$

Example:03:$246.57)_8+357.1)_8$

```
  1 1
  2 4 6 . 5 7
 +3 5 7 . 1
  6 2 5 . 6 7
```

The columns after octal points don't need extra calculation.

1st column (units column) addition 6+7=13

```
 1 ← quotient as carry
8)‾13
    8
    5 ← remainder as sub-sum
```

Example:03:246.57)$_8$+357.1)$_8$

$$
\begin{array}{r}
{\scriptstyle 1\ 1} \\
246.57 \\
+357.1 \\
625.67
\end{array}
$$

The columns after octal points don't need extra calculation.

1st column (units column) addition 6+7=13

$$
\begin{array}{l}
\ \ 1 \leftarrow \text{quotient as carry} \\
8\overline{)\,13} \\
\quad\ \ 8 \\
\quad\ \ 5 \leftarrow \text{remainder as sub-sum}
\end{array}
$$

2nd column (tens column) addition 4+5+1=9

$$
\begin{array}{l}
\ \ 1 \leftarrow \text{quotient as carry} \\
8\overline{)\,10} \\
\quad\ 8 \\
\quad\ 2 \leftarrow \text{remainder as sub-sum}
\end{array}
$$

3rd column (hundreds column) addition don't exceeds to 7. No need of extra calculation.

Answer:625.67)$_8$

ddition of $(321)_8$ and $(47)_8$

Solution:

Solution is

$$\begin{array}{cccc} & & 1 & \\ & 3 & 2 & 1 \\ + & 0 & 4 & 7 \\ \hline & 3 & 7 & 0 \end{array}$$

+ Step by step solution :

Find addition of $(4023)_8$ and $(4162)_8$

Solution:

Solution is

$$\begin{array}{ccccc} 1 & & 1 & & \\ & 4 & 0 & 2 & 3 \\ + & 4 & 1 & 6 & 2 \\ \hline 1 & 0 & 2 & 0 & 5 \end{array}$$

Evaluate:

(i) $(162)_8 + (537)_8$

Solution:

$$
\begin{array}{rrrl}
 & 1 & 1 & \quad \text{<---- carry} \\
1 & 6 & 2 & \\
\underline{5} & \underline{3} & \underline{7} & \\
7 & 2 & 1 &
\end{array}
$$

Therefore, sum = 721_8

(ii) $(136)_8 + (636)_8$

Solution:

$$
\begin{array}{rrrl}
 & 1 & & \quad \text{<---- carry} \\
1 & 3 & 6 & \\
\underline{6} & \underline{3} & \underline{6} & \\
7 & 7 & 4 &
\end{array}
$$

Therefore, sum = 774_8

(iii) $(25.27)_8 + (13.2)_8$

Solution:

$$
\begin{array}{r l}
1 & \quad \leftarrow\text{---- carry} \\
25.27 & \\
\underline{13.2} & \\
40.47 &
\end{array}
$$

Therefore, sum = $(40.47)_8$

(iv) $(67.5)_8 + (45.6)_8$

Solution:

$$
\begin{array}{r l}
11 & \quad \leftarrow\text{---- carry} \\
67.5 & \\
\underline{45.6} & \\
135.3 &
\end{array}
$$

Therefore, sum = $(135.3)_8$

Octal Number System Subtraction Examples:

Example#01: $345)_8$-$146)_8$

```
     8
 2   3   8
 3   4   5
-1   4   6
 1   7   7
```

1st column (units column) subtraction. 5-6

This is not possible. You have to borrow from tens column. The number you borrow is 8. So units column subtraction is 5+8-6

2nd column (tens column) subtraction. 4 becomes 3 after borrow. 3-4. To proceed this subtraction you have to borrow from hundreds column. So tens column subtraction is 3+8-4

Answer:$177)_8$

1. Find subtraction of $(630)_8$ and $(205)_8$

Solution:

Solution is

$$
\begin{array}{rrrr}
 & & 2 & 8 \\
 & 6 & \cancel{3} & \cancel{0} \\
- & 2 & 0 & 5 \\
\hline
 & 4 & 2 & 3
\end{array}
$$

2. Find subtraction of $(321)_8$ and $(47)_8$

Solution:

Solution is

$$
\begin{array}{cccc}
 & & 9 & \\
 & 2 & \cancel{1} & 9 \\
 & \cancel{3} & \cancel{2} & \cancel{1} \\
- & 0 & 4 & 7 \\
\hline
 & 2 & 5 & 2
\end{array}
$$

+ Step by step solution :

---

3. Find subtraction of $(25430)_8$ and $(4077)_8$

Solution:

Solution is

$$
\begin{array}{cccccc}
 & & & & 10 & \\
 & & & 3 & \cancel{2} & 8 \\
 & 2 & 5 & \cancel{4} & \cancel{3} & \cancel{0} \\
- & 0 & 4 & 0 & 7 & 7 \\
\hline
 & 2 & 1 & 3 & 3 & 1
\end{array}
$$

Hexadecimal Addition

Use the following steps to perform hexadecimal addition:
1. Add one column at a time.
2. Convert to decimal and add the numbers.
3.(a) If the result of step two is 16 or larger subtract the result from 16 and carry 1 to the next column.
(b) If the result of step two is less than 16, convert the number to hexadecimal

$$\begin{array}{cccccc} & \color{red}{1} & \color{red}{1} & & & \\ & & 7 & f & b & 3 \\ + & 1 & b & 6 & 2 & 1 \\ \hline & 2 & 3 & 5 & d & 4 \end{array}$$

Addition of Hexadecimal Numbers

15's complement

- **Method : 15's complement of a number is obtained by subtracting all bits from F.**

Method : 15's complement of a number is obtained by subtracting all bits from F.

---

1. Find 15's complement of 1B06

Note : 15's complement of a number is obtained by subtracting all bits from FFFF.
15's complement of 1B06 is

	F	F	F	F
-	1	B	0	6
	E	4	F	9

1. Find Subtraction of 1B06 and 77C using 16's complement method

Here A = 1B06, B = 077C.
Find A - B = ? using 16's complement
First find 16's complement of B = 077C

Note : 16's complement of a number is 1 added to it's 15's complement number.
15's complement of 077C is

$$
\begin{array}{ccccc}
 & F & F & F & F \\
- & 0 & 7 & 7 & C \\
\hline
 & F & 8 & 8 & 3
\end{array}
$$

Now add 1 : F883 + 1 = F884

Now Add this 16's complement of B to A

$$
\begin{array}{ccccc}
1 & 1 & & & \\
 & 1 & B & 0 & 6 \\
+ & F & 8 & 8 & 4 \\
\hline
1 & 1 & 3 & 8 & A
\end{array}
$$

+ Hints : (Move mouse over the steps for detail calculation highlight)

The left most bit of the result is called carry and it is ignored.
So answer is 138A

16's complement

- Add 1 to 15’s Complement

Practice Questions

All 12 conversions

Binary subtraction using 1’s and 2’s complement

Octal Addition

Hexadecimal Addition, Subtraction.