# Problem 1

Set there are 1,2,3 three doors

Set C as the door which the car was hidden behind

Set M as the door which I firstly chose by myself

Set H as the door which the host opened

Suppose I firstly chose the door 1, the host opened the door 3

I need to calculate conditional probability of two situation: switch or do not switch

And from the given information, we can get that C event and M event are independent, so

$P(C=1|M=1)=P(C=1)=\frac{1}{3}=P(C=2|M=1)=P(C=3|M=1)$

According Bayesian Rule:

$P(C=1|M=1,H=3)$

$=\frac{P(H=3|M=1,C=1)P(C=1|M=1)}{P(H=3|M=1,C=1)P(C=1|M=1)+P(H=3|M=1,C=2)P(C=2|M=1)+P(H=3|M=1,C=3)P(C=3|M=1)}$

$=\frac{\frac{1}{2}\times\frac{1}{3}}{\frac{1}{2}\times\frac{1}{3}+1\times\frac{1}{3}+0\times\frac{1}{3}}$

$=\frac{1}{3}$

$P(C=2|M=1,H=3)=1-P(C=1|M=1,H=3)=1-\frac{1}{3}=\frac{2}{3}$

$\because \frac{1}{3}=P(C=1|M=1,H=3)<P(C=2|M=1,H=3)=\frac{2}{3}$

$\therefore$ I will be more likely to get the car if I switch my choice to door 2

# Problem 2

From the image given, we can get that

$x_1$ has no root of arrow, $x_2$ has 1 root of arrow $x_1$, $x_3$ has no root of arrow,

$x_4$ has 2 roots of arrow $x_1$ and $x_3$, $x_5$ has 2 roots of arrow $x_2$ and $x_4$.

According to the fomula $p(x_1,\cdots,x_n)=\coprod_{i=1}^{n}p(x_i\mid pa_i)$,

where $pa_i$ represents the parents of child i = root of arrow

So the factorization of the probability distribution is

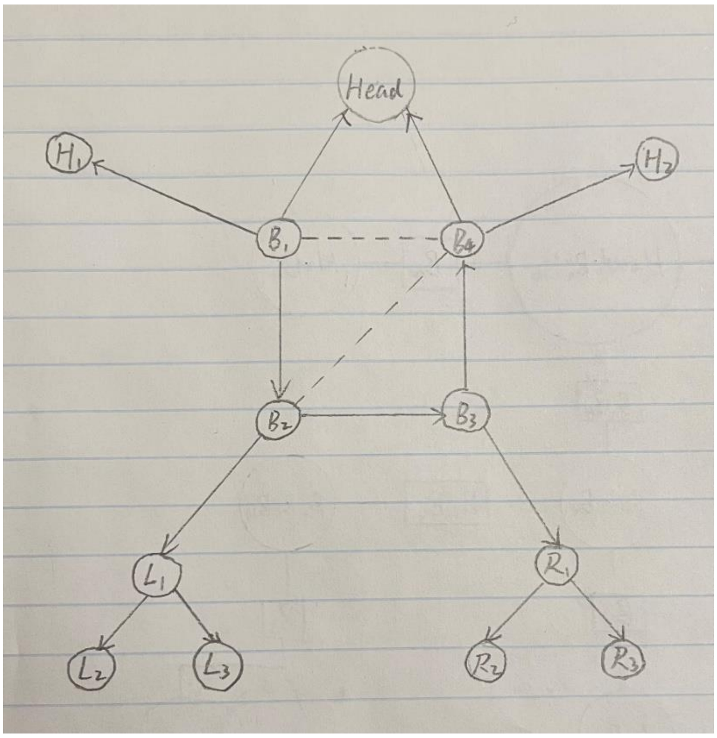
$p(x_1,\cdots,x_5)=\coprod_{i=1}^{5}p(x_i\mid pa_i)=p(x_1)p(x_2\mid x_1)p(x_3)p(x_4\mid x_1,x_3)p(x_5\mid x_2,x_4)$

Use the Bayes ball algorithm

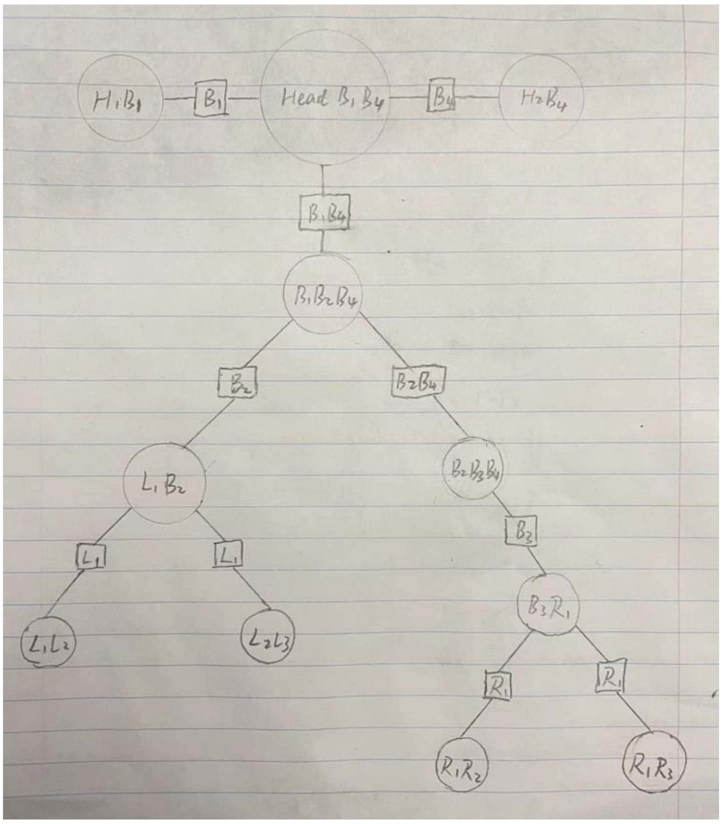
1. False None is given, ball can go through $x_2, x_1, x_4$
2. False $x_5$ is given, ball can go through $x_2, x_5, x_4$
3. True $x_5$ is not given, $x_1$ is given, ball can not arrive $x_4$ from $x_2$
4. True According to Markov Chain condition, ball can not reach $x_5$ from $x_3$
5. True $x_4$ is given, $x_3$ and $x_5$ can not reach each other
6. True $x_4$ is not given, ball from $x_3$ can not reach anywhere and ball can not reach $x_3$ through $x_4$
7. True $x_4$ is not given, ball from $x_3$ can not reach anywhere and ball can not reach $x_3$ through $x_4$
8. True $x_2$ can reach $x_4$ but not $x_3$
9. False $x_3\to x_4\to x_5\to x_2$
10. False $x_3\to x_4\to x_1\to x_2$

**Problem 3**

After Moralization and Triangulation, the graph is like

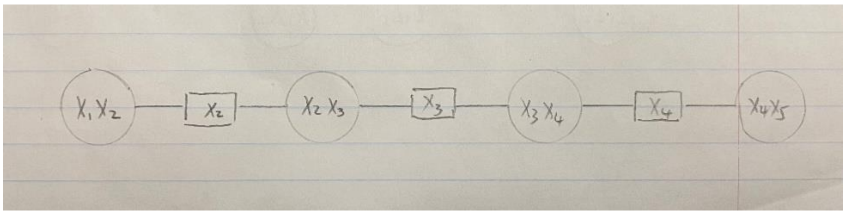



The constructed junction tree is like



## Problem 4

The constructed junction tree is like




We picked clique $x_{n-1}, x_n$ as root. In this problem, it is $x_4, x_5$

First send message from 1 to $n$-1, and then $n$-1 to 1

Our seperator values ends up being equal to those marginals as expected

The Matlab code is

```
n = 5;
psis = cell(n-1, 1);
for i = 1:(n-1)
psis{i} = rand(2,2);
end
[ ma ] = JCT4MarkovChain( psis );
ptest = cell(4,1);
ptest{1} = [0.1, 0.7; 0.8, 0.3];
ptest{2} = [0.5, 0.1; 0.1, 0.5];
ptest{3} = [0.1, 0.5; 0.5, 0.1];
ptest{4} = [0.9, 0.3; 0.1, 0.3];
[ mtest] = JCT4MarkovChain(ptest);
function[ ma ] = JCT4MarkovChain( po )
ma = po;
n = size(ma,1);
se = ones(n-1,2);
for i = 1:n-1
se(i,:) = sum(ma{i});
ma{i+1} = ma{i+1}.*(se(i,:)'*[1,1]);
end
for i = 1:n-1
sold = se(n-i,:);
se(n-i,:) = sum(ma{n-i+1},2)';
ma{n-i} = ma{n-i}.*([1;1]*(se(n-i,:)./sold));
end
for i = 1:n
ma{i} = ma{i}/sum(sum(ma{i}));
end
end
```

Where po = potentials = cell of potentials
ma = marginals = output marginals
se = separators

| $p(x_1, x_2)$ | $x_2 = 0$ | $x_2 = 1$ | $p(x_1)$ |
|---|---|---|---|
| $x_1 = 0$ | 0.0405 | 0.4451 | 0.4856 |
| $x_1 = 1$ | 0.3237 | 0.1908 | 0.5145 |
| $p(x_2)$ | 0.3642 | 0.6358 | |

| $p(x_1, x_2)$ | $x_3 = 0$ | $x_3 = 1$ | $p(x_2)$ |
|---|---|---|---|
| $x_2 = 0$ | 0.2601 | 0.1040 | 0.3642 |
| $x_2 = 1$ | 0.0578 | 0.5780 | 0.6358 |
| $p(x_3)$ | 0.3179 | 0.6821 | |

| $p(x_1, x_2)$ | $x_4 = 0$ | $x_4 = 1$ | $p(x_3)$ |
|---|---|---|---|
| $x_3 = 0$ | 0.1192 | 0.1987 | 0.3179 |
| $x_3 = 1$ | 0.6395 | 0.0426 | 0.6821 |
| $p(x_4)$ | 0.7587 | 0.2413 | |

| $p(x_1, x_2)$ | $x_5 = 0$ | $x_5 = 1$ | $p(x_4)$ |
|---|---|---|---|
| $x_4 = 0$ | 0.5690 | 0.1897 | 0.7587 |
| $x_4 = 1$ | 0.0603 | 0.1810 | 0.2413 |
| $p(x_5)$ | 0.6293 | 0.3707 | |

## Problem 5

The Matlab code is

```
function[ H ] = argMaxInfer( T, E, O, I )
t = size(T, 1);
n = size(O, 2);
psi = zeros(t, t, n);
phi = zeros(t, n);
phi(:, 1) = I;
for i = 2 : n
k = O(1, i);
psi(:, :, i) = diag(phi(:, i - 1)) *T *diag(E(:,k));
phi(:, i) = max(psi(:, :, i));
end
for i = n - 1 : -1 : 1
phinew = max(psi(:, :, i + 1), [], 2);
psi(:, :, i) = psi(:, :, i) *diag(phinew ./ phi(:, i));
phi(:, i) = phinew;
end
[neg,H] = max(phi);
end
```

Where the input T = transition probabilities

E = emission probabilities

O = observed states

I = initial probabilities

the output H = the most likely hidden states

And the most likely sequence of Mario's emotional states for the first five days is

| Day1 | Day2 | Day3 | Day4 | Day5 |
|---|---|---|---|---|
| Happy | Angry | Angry | Angry | Angry |