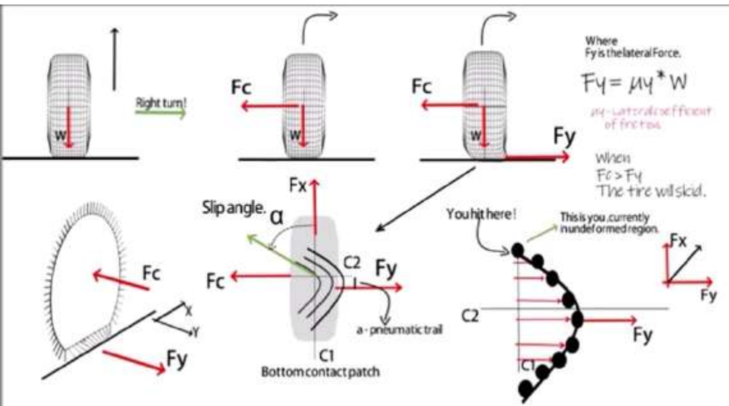
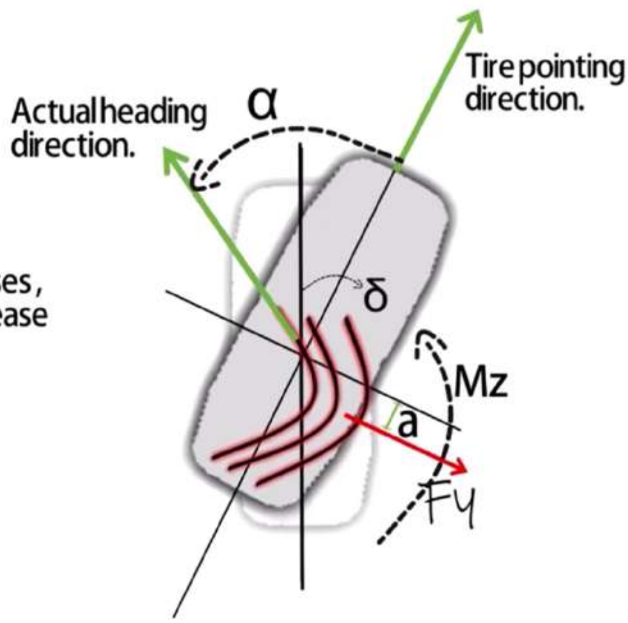
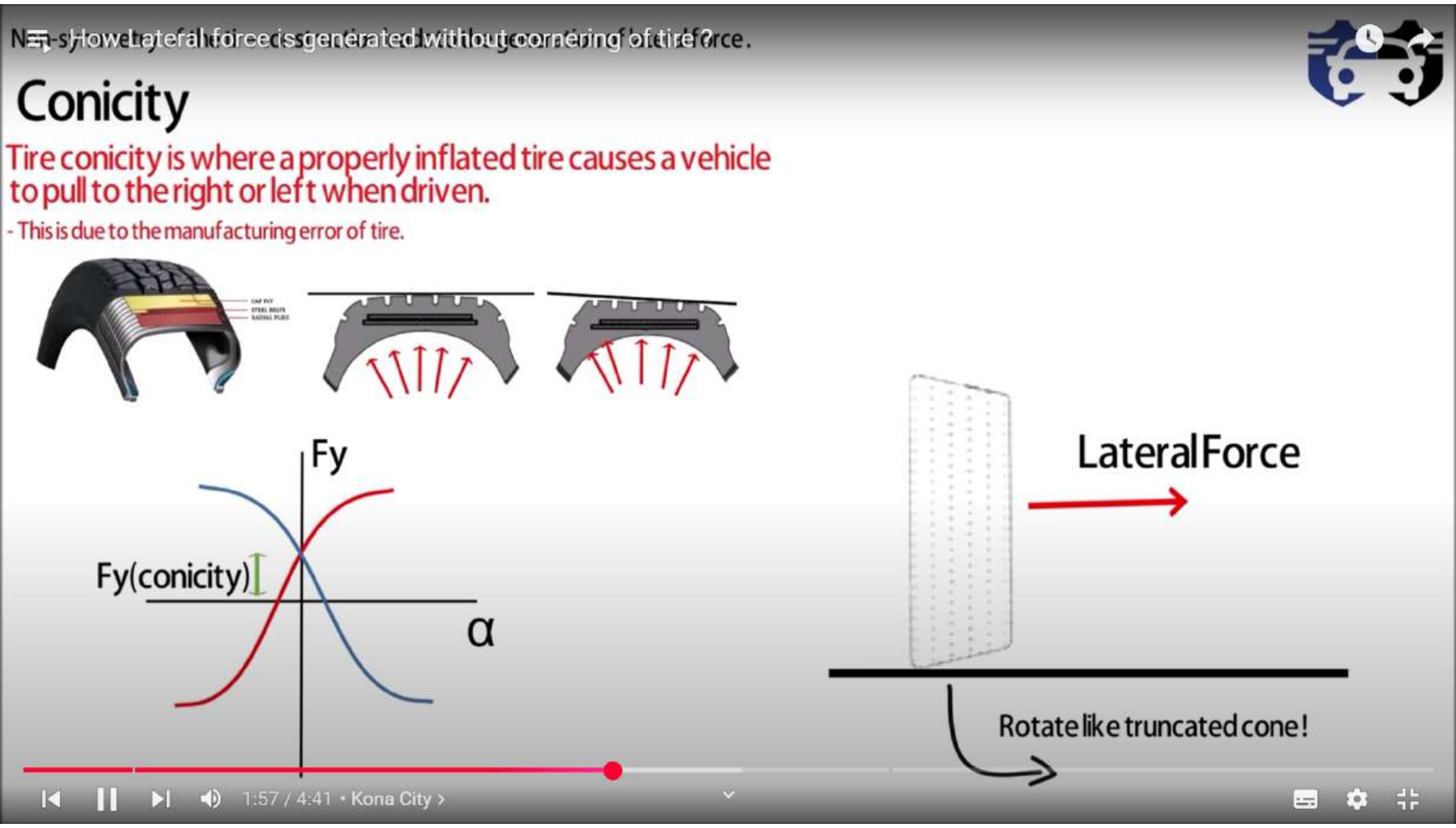


Slip angle is the angle between a rolling wheel's actual direction of travel and the direction towards which it is pointing.



Note :- This torque will reduce , when lateral force decreases , slip angle decreases , as decrease in them will lead to decrease in pneumatic trail .

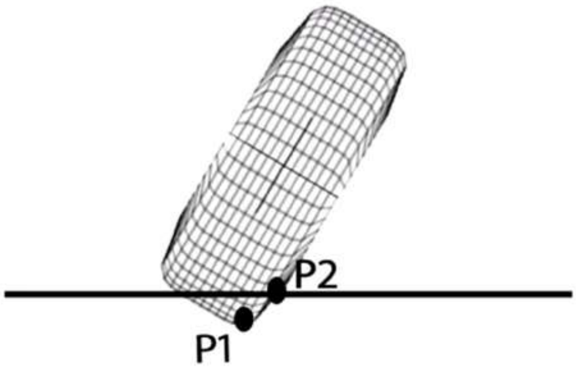
Mz - Self aligning torque.

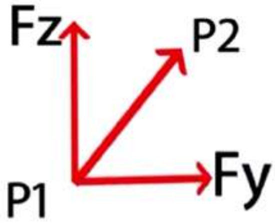

How Lateral force is generated without cornering of tire ?
Conicity
Tire conicity is where a properly inflated tire causes a vehicle to pull to the right or left when driven.
- This is due to the manufacturing error of tire.
Fy
Fy(conicity)
α
Lateral Force
Rotate like truncated cone!
1:57 / 4:41 • Kona City ›

Camber Thrust!

Force generated perpendicular to the direction of travel of a rolling tire due to its camber angle and finite contact patch.

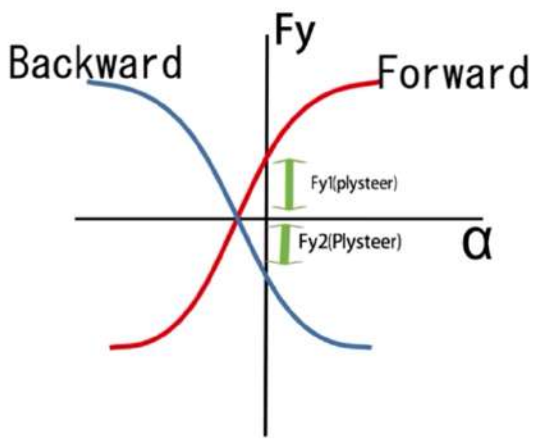
- This is mainly important for bike tires.

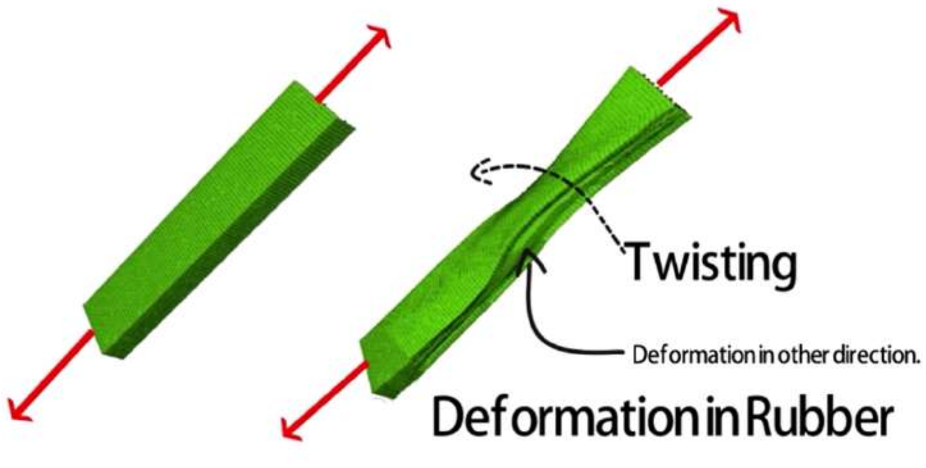


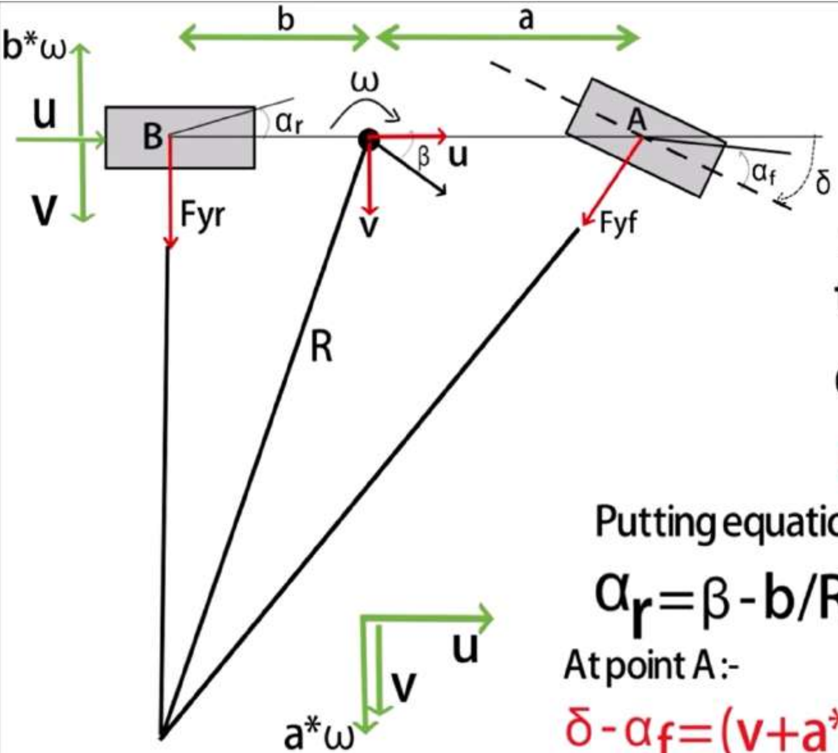


Ply-Steer

Due to coupling between the different planes of tire , i.e forces in one plane effects the deformation in other plane ,this effect is called ply-steer and it develops Lateral force.







$$\tan\beta = v/u$$

$$\beta = v/u \quad (1)$$

At point B

$$\tan\alpha_r = (b^*\omega - v)/u$$

$$\alpha_r = b^*\omega/u - v/u \quad (2)$$

$$u = R^*\omega \quad (3)$$

Putting equation (3) & (1) in (2):-

$$\alpha_r = \beta - b/R \quad (4)$$

At point A :-

$$\delta - \alpha_f = (v + a^*\omega)/u$$

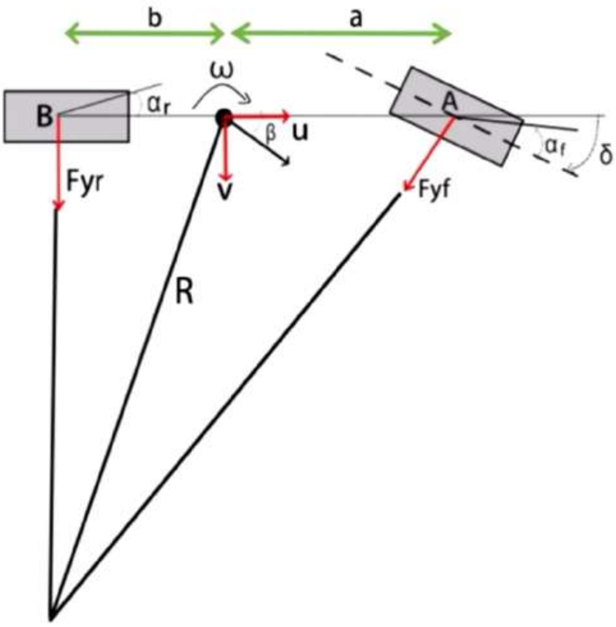
From equation (3) & (1) we get :-

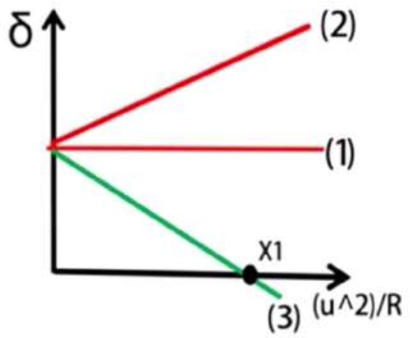
$$\alpha_f = \delta - a/R - \beta \quad (5)$$

From equation (4) & (5) we get :-

$$\delta = (\alpha_f - \alpha_r + L/R$$

How to find that car will Understeer or Oversteer?





$b^* C\alpha r > a^* C\alpha f$

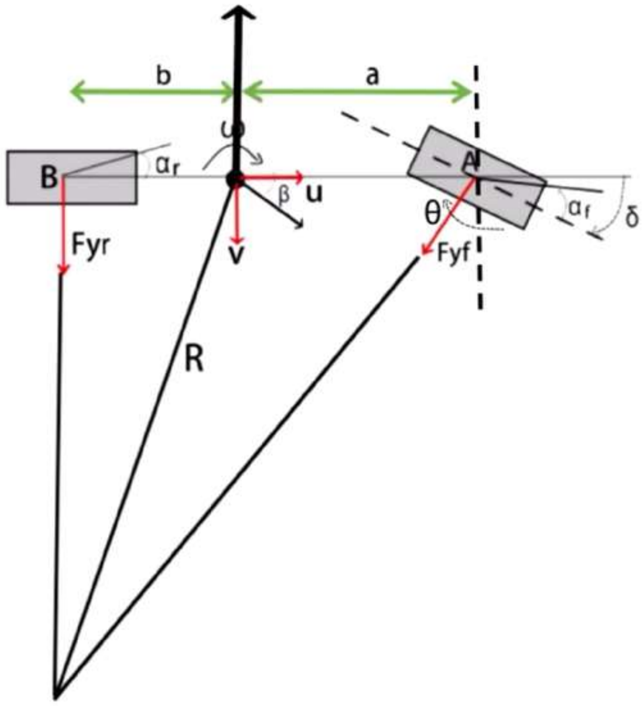
(UNDERSTEER)

$b^* C\alpha r < a^* C\alpha f$

(OVERSTEER)

$$\delta = \left[\frac{m^*(b^* C\alpha r - a^* C\alpha f)}{L^* C\alpha f^* C\alpha r} \right] {}^* \frac{u^2}{R} + \frac{L}{R}$$

Bicycle Model



δ - Small steering angle

αf & αr - Front & rear slip angles

Fyf & Fyr - Front and rear lateral forces

u - Forward velocity

v - Lateral velocity

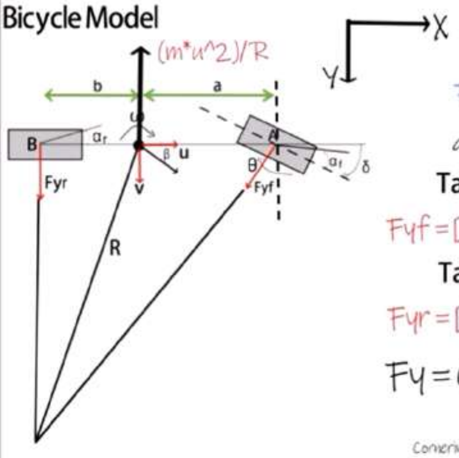
a,b - Position of COG

ω - Yaw rate (yaw angular velocity)

R - turning radius

m - Mass of car

Bicycle Model



Balancing forces along Y axis

$$F_{yf} + F_{yr} = (m*u^2)/R$$

$a+b = L$ (Wheel Base)

Taking moment about B we get :-

$$F_{yf} = [(m*u^2)/R]*b/L \quad (1)$$

Taking moment about A we get :-

$$F_{yr} = [(m*u^2)/R]*a/L \quad (2)$$

$$F_y = \underbrace{C_\alpha}_{\text{Cornering stiffness}} * \underbrace{\alpha}_{\text{Slip angle}}$$

$$F_{yf} = C_{\alpha f} * \alpha f \quad (3)$$

$$F_{yr} = C_{\alpha r} * \alpha r \quad (4)$$

From (3) & (4) we can write :-

$$\alpha f = F_{yf}/C\alpha f \quad (5)$$

$$\alpha r = F_{yr}/C\alpha r \quad (6)$$

$$\delta = \alpha f - \alpha r + L/R \quad (7)$$

Putting (1),(2),(5) & (6) in equation (7) we get :-

$$\delta = \left[\frac{F_{yf}}{C\alpha f} - \frac{F_{yr}}{C\alpha r} \right] + \frac{L}{R}$$

$$\delta = \left[\frac{m*(b*C\alpha r - a*C\alpha f)}{L*C\alpha f*C\alpha r} \right] * \frac{u^2}{R} + \frac{L}{R}$$

$$\delta = \left[\frac{m*(b*C\alpha r - a*C\alpha f)}{L*C\alpha f*C\alpha r} \right] * \frac{u^2}{R} + \frac{L}{R}$$

K1

Case 1:-

$b*C\alpha r = a*C\alpha f$

$\delta = L/R$

At any speed of car , for same turning radius we have to give same steering input.

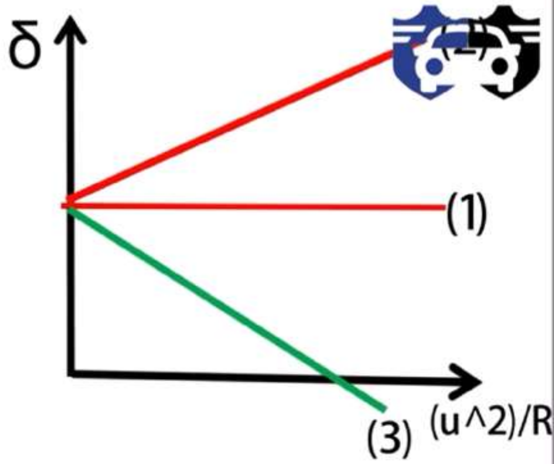
Case 2:-

$b*C\alpha r > a*C\alpha f$

$\delta = K1*(u^2)/R + L/R$

$K1 > 0$ (UNDERSTEER)

If we increase the speed of vehicle for same turning radius R ,then we have to give the higher steering wheel input as value of δ will increase on higher speeds.



Case 3:-

$b*C\alpha r < a*C\alpha f$

$\delta = K1*(u^2)/R + L/R$

$K1 < 0$ (OVERSTEER)

If we increase the speed of vehicle for same turning radius R ,then we have to give the Lower steering wheel input as value of δ will decrease at higher speed.

$$\delta = \left[\frac{m*(b*C_{\alpha r} - a*C_{\alpha f})}{L*C_{\alpha f}*C_{\alpha r}} \right] * \frac{u^2}{R} + \frac{L}{R}$$

K1

Case 1:-

$b*C_{\alpha r} = a*C_{\alpha f}$

$\delta = L/R$

At any speed of car , for same turning radius we have to give same steering input.

Case 2:-

$b*C_{\alpha r} > a*C_{\alpha f}$

$\delta = K1*(u^2)/R + L/R$

$K1 > 0$ (UNDERSTEER)

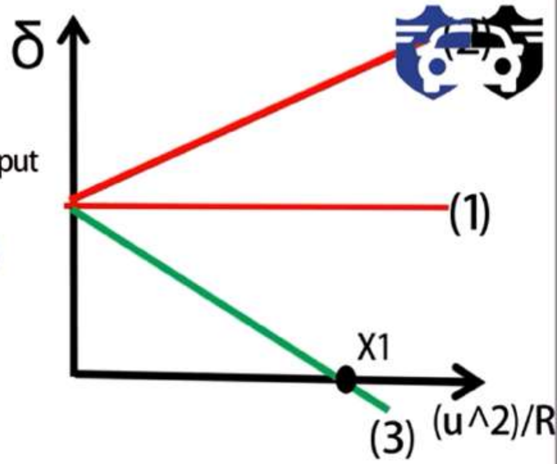
If we increase the speed of vehicle for same turning radius R ,then we have to give the higher steering wheel input as value of δ will increase on higher speeds.

At point X1

$\delta = 0$

No. steering wheel input required for turning.

Velocity at this point is called critical velo



Case 3:-

$b*C_{\alpha r} < a*C_{\alpha f}$

$\delta = K1*(u^2)/R + L/R$

$K1 < 0$ (OVERSTEER)

If we increase the speed of vehicle for same turning radius R ,then we have to give the Lower steering wheel input as value of δ will decrease at higher speed.

$$\delta = \left[\frac{m*(b*C\alpha r - a*C\alpha f)}{L*C\alpha f*C\alpha r}\right]*\frac{u^2}{R}+\frac{L}{R}$$

Beyond point X1

$\delta < 0$

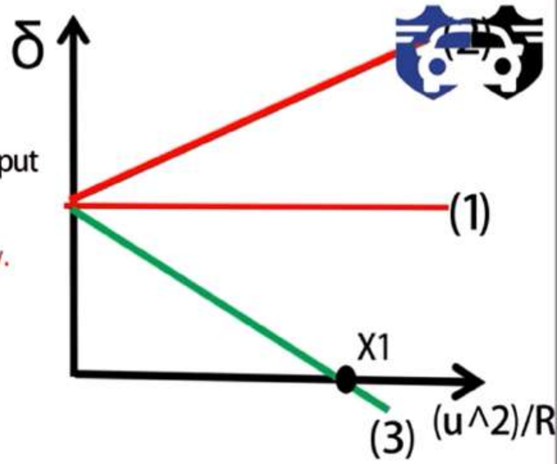
So, beyond critical velocity
If we want to take left turn, we have to turn right and visce versa.

At point X1

$\delta = 0$

No. steering wheel input required for turning.

Velocity at this point is called critical velocity.



Case 3 :-

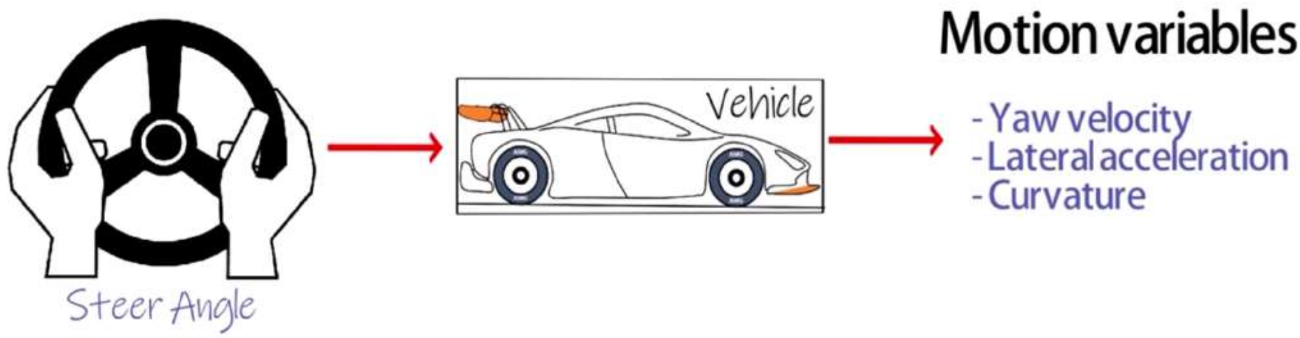
$b*C\alpha r < a*C\alpha f$

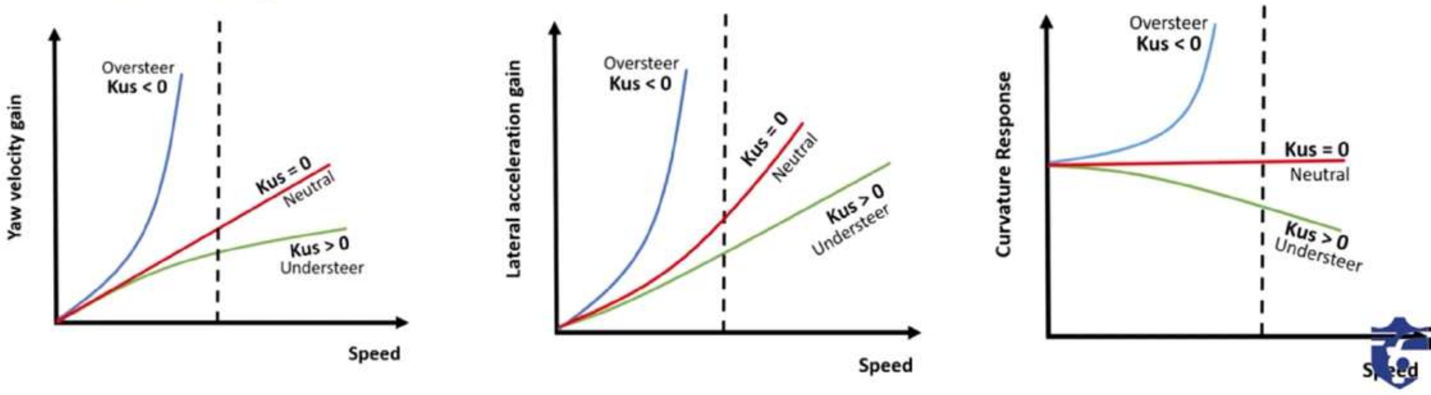
$\delta = K1*(u^2)/R + L/R$

$K1 < 0$ (OVERSTEER)

If we increase the speed of vehicle for same turning radius R, then we have to give the Lower steering wheel input as value of δ will decrease at higher speed.

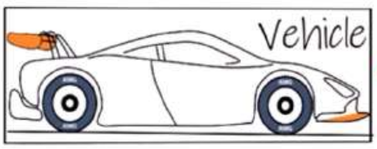
What are the Steady-state response to Steering input ?





- A vehicle may be regarded as a control system upon which various inputs are imposed.

Steer Angle



Motion variables

- Yaw velocity
- Lateral acceleration
- Curvature

$Fyf = Wf * V^2 / R * g$

$Fyr = Wr * V^2 / R * g$

$$\delta = \left[\frac{Fyf}{C\alpha f} - \frac{Fyr}{C\alpha r} \right] + \frac{L}{R}$$

$$\delta = \left[\frac{Wf}{C\alpha f} - \frac{Wr}{C\alpha r} \right] * \frac{V^2}{R*g} + \frac{L}{R}$$

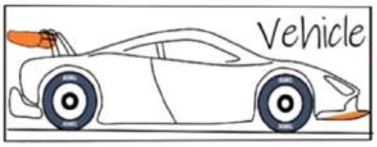
Where, $C\alpha f$ - Front tire cornering stiffness

$C\alpha r$ - Rear tire cornering stiffness

- A vehicle may be regarded as a control system upon which various inputs are imposed.

Steer Angle

→



→

Motion variables

- Yaw velocity
- Lateral acceleration
- Curvature

$K_{us} = 0$ - Neutral
$K_{us} > 0$ - Understeer
$K_{us} < 0$ - Oversteer

$F_{yf} = W_f * V^2 / R*g$

$F_{yr} = W_r * V^2 / R*g$

$$\delta = \left[\frac{F_{yf}}{C_{\alpha f}} - \frac{F_{yr}}{C_{\alpha r}} \right] + \frac{L}{R}$$

$$\delta = \underbrace{\left[\frac{W_f}{C_{\alpha f}} - \frac{W_r}{C_{\alpha r}} \right]}_{= K_{us}} * \frac{V^2}{R*g} + \frac{L}{R}$$

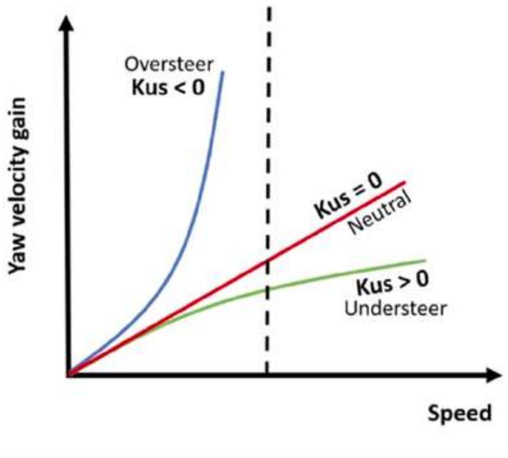
Understeer coefficient

Yaw Velocity Response

$$\Omega_z = V/R$$

$$G_{yaw} = \Omega_z / \delta$$

$$G_{yaw} = V/(L + K_{us} * V^2/g)$$

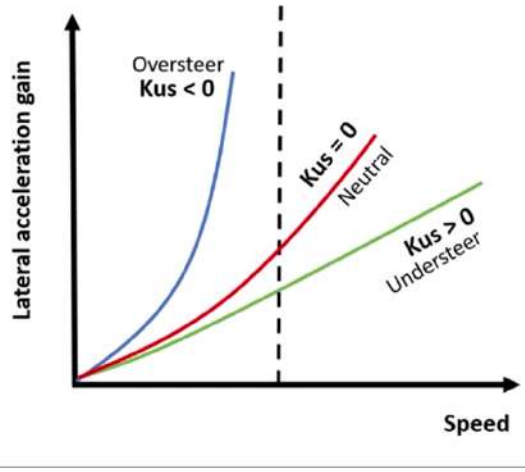


Lateral Acceleration Response

$$a_y = V^2/R$$

$$G_{acc} = (a_y/g)/\delta$$

$$G_{acc} = V^2/(g*L + K_{us} * V^2)$$

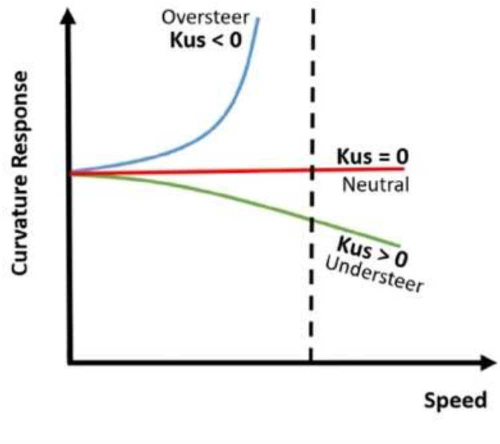


Curvature Response

Steady-state curvature:- $1/R$

$$C(res) = (1/R)/\delta$$

$$C(res) = 1/(L + K_{us} * V^2/g)$$

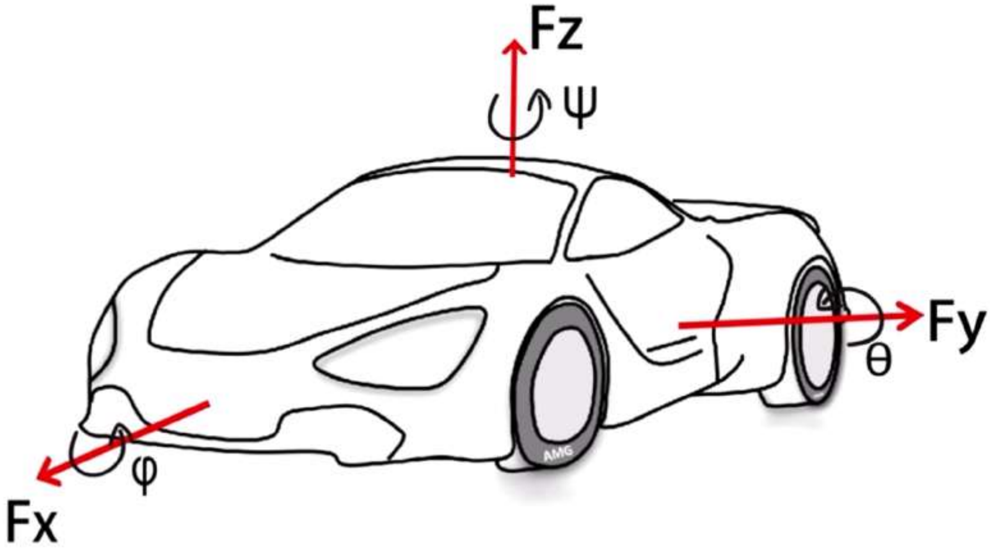


Vehicle coordinate system

Roll angle :- φ
Rollrate :- p
$\Big\} \; p = \dot{\varphi}$

Pitch angle :- θ
Pitchrate :- q
$\Big\} \; q = \dot{\theta}$

Yaw angle :- ψ
Yawrate :- r
$\Big\} \; r = \dot{\psi}$



Note :- In vehicle Roll dynamics vertical movement z and pitch motion θ are neglected.

Degree of freedom :- 4

2 - x, y (Translational)
2 - x, z (Rotational)

Euler's Equation of Motion

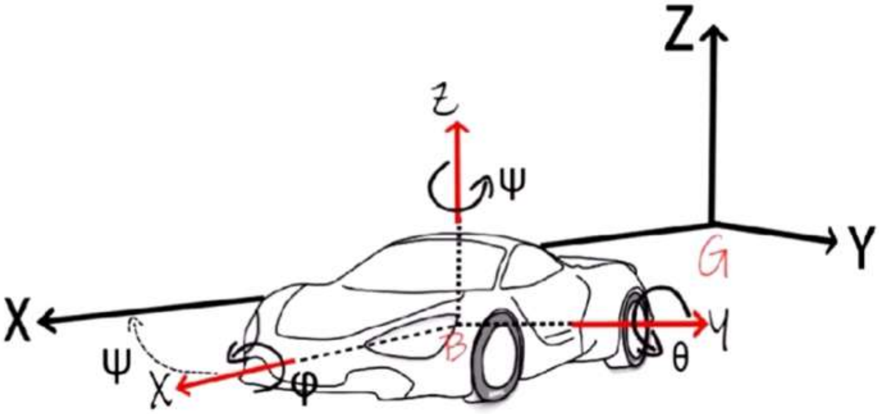
- Force
- Moment

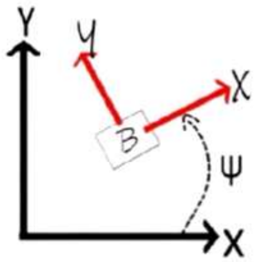
6 DOF Model
4 DOF Model

$$\begin{vmatrix} L \\ M \\ N \end{vmatrix} = \begin{vmatrix} \dot{p}*Ixx - q*r*Iyy + q*r*Izz \\ \dot{q}*Ixx + p*r*Iyy - p*r*Izz \\ \dot{r}*Ixx - p*q*Iyy + p*q*Izz \end{vmatrix}$$

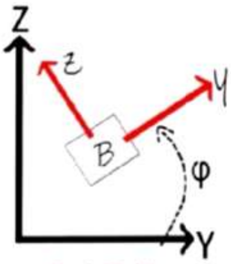
$$\begin{vmatrix} Fx \\ Fy \\ Fz \end{vmatrix} = m \begin{vmatrix} \dot{u} + (q*w - r*v) \\ \dot{v} + (r*u - p*w) \\ \dot{w} + (p*v - q*u) \end{vmatrix}$$

Euler equations of motion

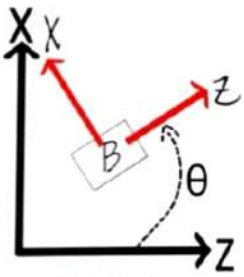




Only Yawing



Only Rolling



Only Pitching

i j k (body centre coordinates)

u v w (Linear velocity)

p q r (Angular velocity)

$F = Fx\,i + Fy\,j + Fz\,k$

$T = L\,i + M\,j + N\,k$

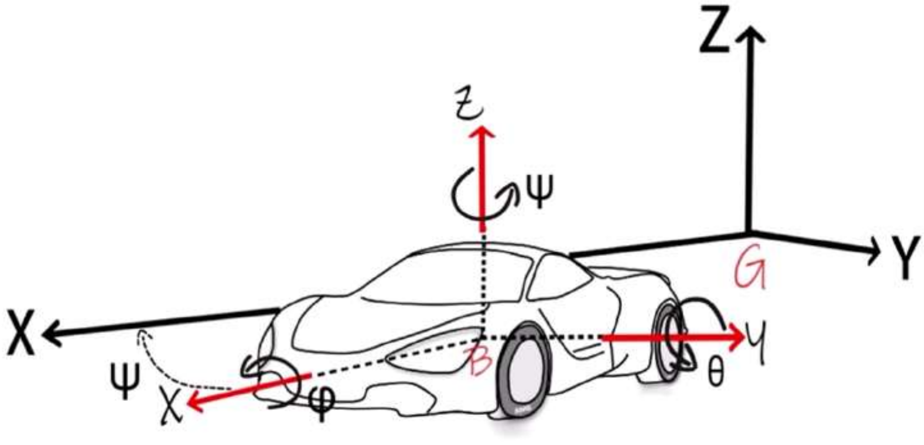
Linear momentum

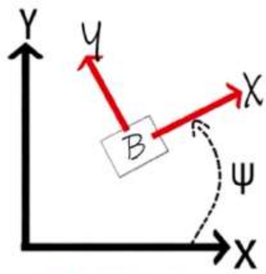
$P = mV$

$V = Vx\,i + Vy\,j + Vz\,k$

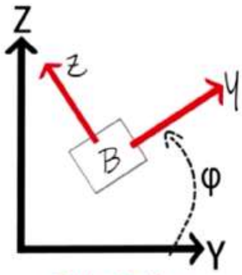
$F = dP/dt$

$F = m * dV/dt$

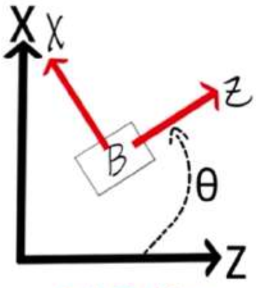




Only Yawing



Only Rolling



Only Pitching

$i \quad j \quad k$ (body centre coordinates)
$u \quad v \quad w$ (Linear velocity)
$p \quad q \quad r$ (Angular velocity)

$F = F_x i + F_y j + F_z k$
$T = Li + Mj + Nk$

Linear momentum

$P = mV$
$V = V_x i + V_y j + V_z k$
$F = dP/dt$
$F = m*dV/dt$

$dv/dt = d/dt(V_x i + V_y j + V_z k)$

$dV/dt = (dV_x/dt)i + (dV_y/dt)j + (dV_z/dt)k + V_x * di/dt + V_y * dj/dt + V_z * dk/dt$

$V = \omega \times r$
$dr/dt = \omega \times r$

$di/dt = \omega \times i$
$dj/dt = \omega \times j$
$dk/dt = \omega \times k$

$\omega = pi + qj + rk$

$$dV/dt = \underbrace{(dV_x/dt)i + (dV_y/dt)j + (dV_z/dt)k}_{\delta V/\delta t} + \underbrace{V_x(\omega \times i) + V_y(\omega \times j) + V_z(\omega \times k)}_{(\omega \times V)}$$

Observed while sitting on body centre

Effect of rotation

$$dv/dt = \dot{u}\, i + \dot{v}\, j + \dot{w}\, k + (\omega \times V)$$

$$\omega \times V = \begin{vmatrix} i & j & k \\ p & q & r \\ u & v & w \end{vmatrix}$$

$$\begin{vmatrix} F_x \\ F_y \\ F_z \end{vmatrix} = m \begin{vmatrix} \dot{u} + (q*w - r*v) \\ \dot{v} + (r*u - p*w) \\ \dot{w} + (p*v - q*u) \end{vmatrix}$$

Angular Momentum

$H = I \times \omega$
$T = dH/dt$ $\qquad T = Li + Mj + Nk$

$$T = (\delta H/\delta t) + \omega \times H$$

$$I = \begin{vmatrix} I_{xx} & I_{xy} & I_{xz} \\ I_{yx} & I_{yy} & I_{yz} \\ I_{zx} & I_{zy} & I_{zz} \end{vmatrix} \qquad \omega = \begin{vmatrix} p \\ q \\ r \end{vmatrix}$$

$$\begin{vmatrix} H_x \\ H_y \\ H_z \end{vmatrix} = m \begin{vmatrix} I_{xx}*p + I_{xy}*q + I_{xz}*r \\ I_{yx}*p + I_{yy}*q + I_{yz}*r \\ I_{zx}*p + I_{zy}*q + I_{zz}*r \end{vmatrix}$$

Assumption:-
- Considering only principle axis

$H = I \times \omega$

$$T = (\delta H/\delta t) + \omega \times H$$

$$I = \begin{vmatrix} Ixx & Ixy & Ixz \\ Iyx & Iyy & Iyz \\ Izx & Izy & Izz \end{vmatrix} \qquad \omega = \begin{vmatrix} p \\ q \\ r \end{vmatrix}$$

$$H = I \times \omega$$

$$\begin{vmatrix} Hx \\ Hy \\ Hz \end{vmatrix} = m \begin{vmatrix} Ixx*p + Ixy*q + Ixz*r \\ Iyx*p + Iyy*q + Iyz*r \\ Izx*p + Izy*q + Izz*r \end{vmatrix}$$

$$\begin{vmatrix} Hx \\ Hy \\ Hz \end{vmatrix} = m \begin{vmatrix} Ixx*p & 0 & 0 \\ 0 & Iyy*q & 0 \\ 0 & 0 & Izz*r \end{vmatrix}$$

Assumption :-

- Considering only principle axis

$$\begin{vmatrix} L \\ M \\ N \end{vmatrix} = \begin{vmatrix} \dot{p}*Ixx - q*r*Iyy + q*r*Izz \\ \dot{q}*Ixx + p*r*Iyy - p*r*Izz \\ \dot{r}*Ixx - p*q*Iyy + p*q*Izz \end{vmatrix}$$

$$\begin{vmatrix} Fx \\ Fy \\ Fz \end{vmatrix} = m \begin{vmatrix} \dot{u} + (q*w - r*v) \\ \dot{v} + (r*u - p*w) \\ \dot{w} + (p*v - q*u) \end{vmatrix}$$

Euler equations of motion

In vehicle Roll dynamics :- 4 DOF Model

Pitching and vertical movement are neglected!

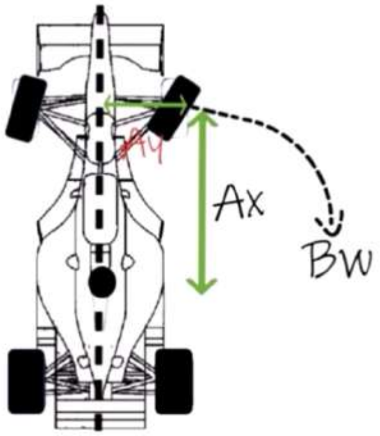
$q = 0$
$w = 0$

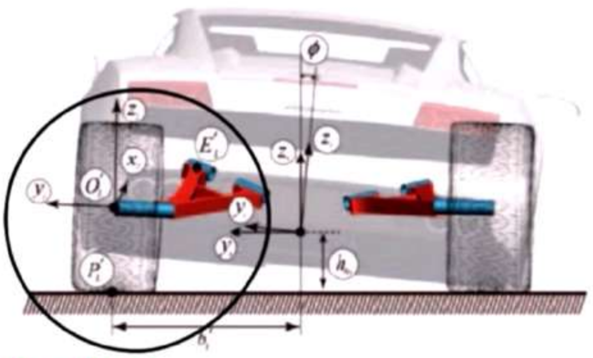
$$Fx = m*\dot{u} - m*r*v$$
$$Fy = m*\dot{v} + m*r*u$$
$$L = Ixx*\dot{p}$$
$$N = Izz*\dot{r}$$

What is the value of slip angle α ?

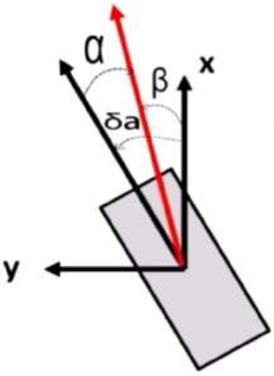
How to find Lateral Force on Tire ?

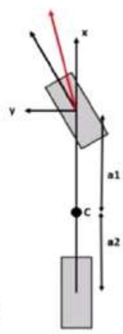
$$\begin{vmatrix} Vx \\ Vy \\ Vz \end{vmatrix} = \begin{vmatrix} Vx - \dot{\psi} * Ay \\ Vy - \dot{\varphi} * Az + \dot{\psi} * Ax \\ \dot{\varphi} * Ay \end{vmatrix}$$





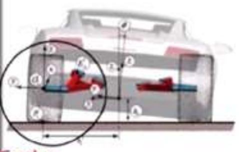
Camber





$$\begin{vmatrix} \omega x \\ \omega y \\ \omega z \end{vmatrix} = \begin{vmatrix} p \\ q \\ r \end{vmatrix} = \begin{vmatrix} \dot{\varphi} \\ \dot{\theta} \\ \dot{\psi} \end{vmatrix}$$

Due to vehicle roll!



Camber

- Camber thrust

(Lateral force without cornering)

$F(y\varphi) = -C\varphi * \varphi$

$C\varphi = dFy / d\varphi$

Tire roll steering angle $\delta\varphi$

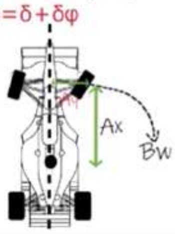
- Suspension Mechanism provides some steering angle $\delta\varphi$, when Vehicle rolls.

$\delta\varphi = C\delta\varphi * \varphi$

$C\delta\varphi = d\delta / d\varphi$

Actual Steering angle :-

$\delta a = \delta + \delta\varphi$



$Bw = Ax\,i + Ay\,j + Az\,k$

$V = dA/dt$

$V = \delta A/\delta t + \omega \times A$

$V = Vx\,i + Vy\,j + Vz\,k + \omega \times A$

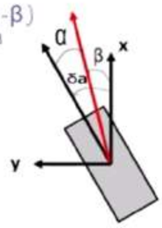
$$\begin{vmatrix} \omega x \\ \omega y \\ \omega z \end{vmatrix} = \begin{vmatrix} p \\ q \\ r \end{vmatrix} = \begin{vmatrix} \dot\varphi \\ \dot\theta \\ \dot\psi \end{vmatrix}$$

$$\begin{vmatrix} Vx \\ Vy \\ Vz \end{vmatrix} = \begin{vmatrix} vx \\ vy \\ 0 \end{vmatrix} + \begin{vmatrix} \dot\varphi \\ 0 \\ \dot\psi \end{vmatrix} \times \begin{vmatrix} Ax \\ Ay \\ Az \end{vmatrix}$$

$$\begin{vmatrix} Vx \\ Vy \\ Vz \end{vmatrix} = \begin{vmatrix} Vx - \dot\psi * Ay \\ Vy - \dot\varphi * Az + \dot\psi * Ax \\ \dot\varphi * Ay \end{vmatrix}$$

<u>Tire slip angle α</u>

$\alpha = -(\delta a - \beta)$
$= \beta - \delta a$



$$\tan\beta = Vy / Vx$$

$$\beta = \frac{Vy - \dot\varphi * Az + \dot\psi * Ax}{Vx - \dot\psi * Ay}$$

$$\alpha = \beta - \delta a$$

$$\boxed{\alpha = \frac{Vy - \dot\varphi * Az + \dot\psi * Ax}{Vx - \dot\psi * Ay} - \delta - \delta\varphi}$$

Slip angle

$$F(y\alpha) = -C\alpha * \alpha$$

Lateral force

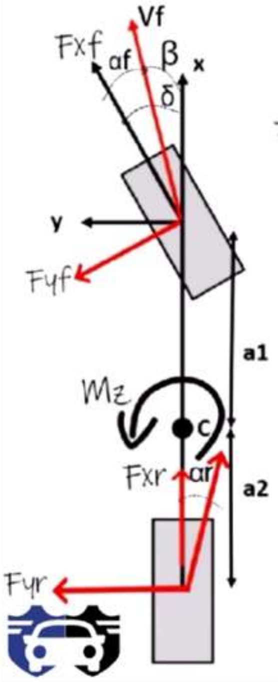
Total Lateral force :-

$$Fy = F(y\alpha) + F(y\varphi)$$

$$Fy = -C\alpha * \alpha - C\varphi * \varphi$$

What is the total value of Roll and Yaw moment on Vehicle ?

What is the values of coefficients involved in Vehicle roll dynamics ?

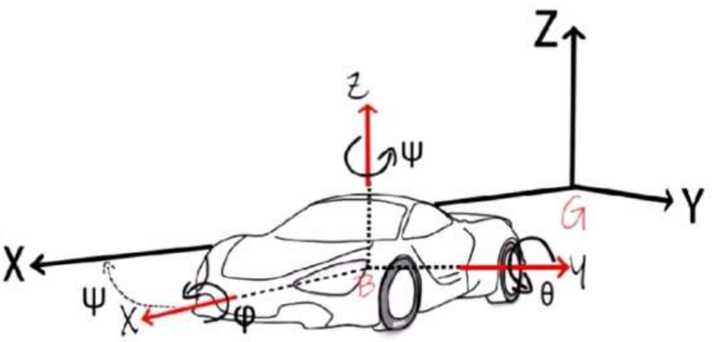


$$\cot\delta = (\cot\delta_o + \cot\delta_i)/2$$

$$F_x = F_{xf}*\cos\delta + F_{xr} - F_{yf}*\sin\delta$$

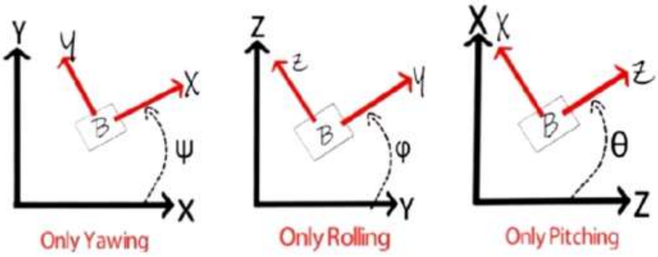
$$F_y = F_{yf}*\cos\delta + F_{yr}$$

$$M_z = F_{yf}*\cos\delta*a_1 - F_{yr}*a_2 \quad (\text{YAW})$$



Vehicle Roll Moment

- Slip Moment
- Camber Moment
- Moment because of change in normal load.



Mxf = C(tf)* Fyf
Mxr = C(tr)* Fyr

Moment due to slip and camber

Where,
C(tf) and C(tr)-
Overall torque coefficient

Roll moment because of change in normal force on left and right tire.

- Force changes in springs and damper
- These imbalance produces roll stiffness moment

Mxk = kφ * φ
Mxc = -Cφ * $\dot{\varphi}$

Where
kφ = T*(kf + kr)
Cφ = T*(Cf + Cr)

T - Track width

Mx = Mxf + Mxr + Mxk + Mxc

Mx = C(tf)*Fyf + C(tr)*Fyr - T*(kf + kr)*φ - T*(Cf + Cr)*$\dot{\varphi}$

For Small steering angle δ:-

Fx = Fxf + Fxr
Fy = Fyf + Fyr

Mx = C(tf)*Fyf + C(tr)*Fyr - Kφ*φ - Cφ*$\dot{\varphi}$
Mz = a1*Fyf - a2*Fyr

$$\boxed{\alpha = \frac{Vy - \dot{\varphi} * Az + \dot{\psi} * Ax}{Vx - \dot{\psi} * Ay} - \delta - \delta\varphi}$$

Slip angle

For bicycle model:-
Ay=0
Ax=a1 (Front tire)
Ax=-a2 (Rear tire)

Az - Not constant

In vehicle roll dynamics
Az = C(β)
C(β) - Tire roll rate coefficient

αf = β + a1*r/vx C(βf)*p/vx - δ - C(δφf)*φ
αr = β - a2*r/vx C(βr)*p/vx - C(δφr)*φ

β = Vy/Vx

Fy = Fyf + Fyr

Fy = -Cαf*αf - Cφf*φ - Cαr*αr - Cφr*φ

$$Fy = \left(\frac{a_2}{v_x}C_{\alpha r} - \frac{a_1}{v_x}C_{\alpha f}\right) r + \left(\frac{C_{\alpha f}C_{\beta_f}}{v_x} + \frac{C_{\alpha r}C_{\beta_r}}{v_x}\right) p$$
$$+ \left(-C_{\alpha f} - C_{\alpha r}\right)\beta + \left(C_{\alpha f}C_{\delta\varphi_f} - C_{\varphi_r} - C_{\varphi_f} + C_{\alpha r}C_{\delta\varphi_r}\right)\varphi$$
$$+ C_{\alpha f}\delta$$

Mx = C(tf)*Fyf + C(tr)*Fyr - Kφ*φ - Cφ*$\dot{\varphi}$

Mz = a1*Fyf - a2*Fyr

$$Mx = \left(\frac{a_2}{v_x}C_{T_r}C_{\alpha r} - \frac{a_1}{v_x}C_{T_f}C_{\alpha f}\right) r$$
$$+ \left(\frac{1}{v_x}C_{\beta_f}C_{T_f}C_{\alpha f} + \frac{1}{v_x}C_{\beta_r}C_{T_r}C_{\alpha r} - c_{\varphi}\right) p$$
$$+ \left(-C_{T_f}C_{\alpha f} - C_{T_r}C_{\alpha r}\right)\beta$$
$$+ \left(-C_{T_f}\left(C_{\varphi_f} - C_{\alpha f}C_{\delta\varphi_f}\right) - C_{T_r}\left(C_{\varphi_r} - C_{\alpha r}C_{\delta\varphi_r}\right) - k_{\varphi}\right)\varphi$$
$$+ C_{T_f}C_{\alpha f}\delta$$

$$Mz = \left(-\frac{a_1^2}{v_x}C_{\alpha f} - \frac{a_2^2}{v_x}C_{\alpha r}\right) r + \left(\frac{a_1}{v_x}C_{\beta_f}C_{\alpha f} - \frac{a_2}{v_x}C_{\beta_r}C_{\alpha r}\right) p$$
$$+ \left(a_2 C_{\alpha r} - a_1 C_{\alpha f}\right)\beta$$
$$+ \left(a_2\left(C_{\varphi_r} - C_{\alpha r}C_{\delta\varphi_r}\right) - a_1\left(C_{\varphi_f} - C_{\alpha f}C_{\delta\varphi_f}\right)\right)\varphi$$
$$+ a_1 C_{\alpha f}\delta$$

$$Fy = \left(\frac{a_2}{v_x}C_{\alpha r} - \frac{a_1}{v_x}C_{\alpha f}\right) r + \left(\frac{C_{\alpha f}C_{\beta_f}}{v_x} + \frac{C_{\alpha r}C_{\beta_r}}{v_x}\right) p$$
$$+\left(-C_{\alpha f} - C_{\alpha r}\right)\beta + \left(C_{\alpha f}C_{\delta\varphi_f} - C_{\varphi_r} - C_{\varphi_f} + C_{\alpha r}C_{\delta\varphi_r}\right)\varphi$$
$$+C_{\alpha f}\delta$$

$$Fy = Fy(r, p, \beta, \varphi, \delta)$$
$$= \frac{\partial F_y}{\partial r}r + \frac{\partial F_y}{\partial p}p + \frac{\partial F_y}{\partial \beta}\beta + \frac{\partial F_y}{\partial \varphi}\varphi + \frac{\partial F_y}{\partial \delta}\delta$$
$$= Cr*r + Cp*p + C\beta*\beta + C\varphi*\varphi + C\delta*\delta$$

$$C_r = \frac{\partial F_y}{\partial r} = -\frac{a_1}{v_x}C_{\alpha f} + \frac{a_2}{v_x}C_{\alpha r}$$
$$C_p = \frac{\partial F_y}{\partial p} = \frac{C_{\alpha f}C_{\beta_f}}{v_x} + \frac{C_{\alpha r}C_{\beta_r}}{v_x}$$
$$C_\beta = \frac{\partial F_y}{\partial \beta} = -(C_{\alpha f} + C_{\alpha r})$$
$$C_\varphi = \frac{\partial F_y}{\partial \varphi} = C_{\alpha r}C_{\delta\varphi_r} + C_{\alpha f}C_{\delta\varphi_f} - C_{\varphi_f} - C_{\varphi_r}$$
$$C_\delta = \frac{\partial F_y}{\partial \delta} = C_{\alpha f}$$

Bumpsteer is effected by four different things:-

1) Tie-rod angle

If the tie-rod angle is not correct, the tie rod can travel in an arc different from that of the upper and lower control arms.

2) Tie-rod length

If the tie rod is too short, it will have a more severe arc. If it is too long, it can have the opposite effect and not arc enough, causing a toe-in condition.

3) Camber

If you lean the tire in or out, you set the spindle at an angle, and its rotation changes with the turn of the steering wheel.Now because of the camber their will be irregular rotation of tie rod and this will cause Bumpsteer.

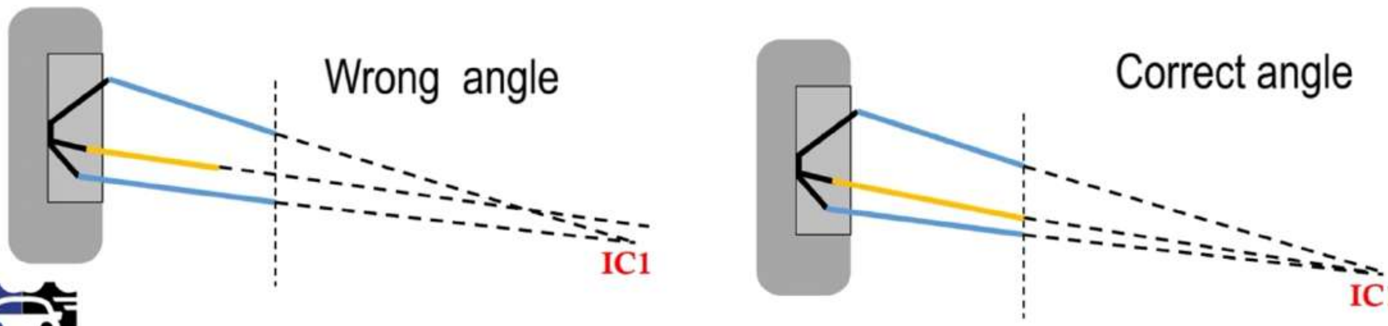
4) Steering-arm pivot

The position of Steering arm pivot point, effects the point of rotation of tie rod attached.

Rule to Eliminate Bump steer

- To eliminate bumpsteer, the tie rod needs to be designed in such a way that the length falls along an imaginary line through the two ball joints and the two control-arm pivot points.

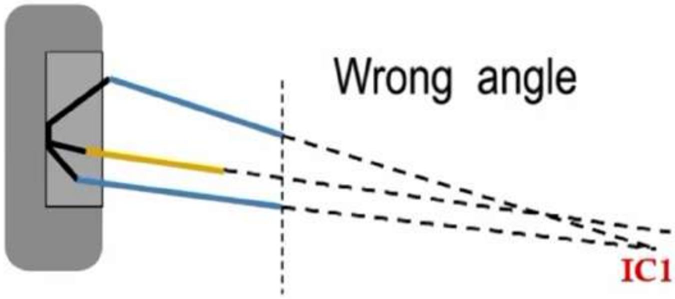
- In addition, the tie-rod angle needs to run its own imaginary line through the instantaneous center created by the two control arms.

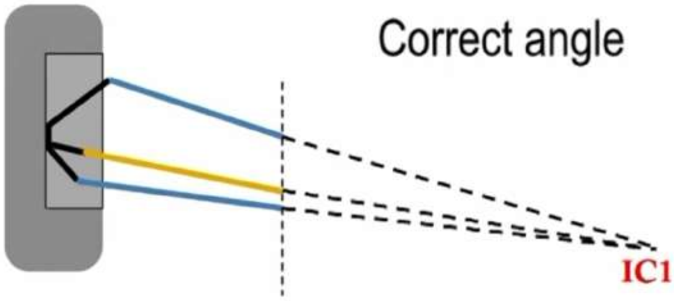


Roll steer

The roll steer is defined as undesirable changes in the steering angle of the steered wheels during the rolling action of the vehicle body due to cornering or at time of asymmetric bumps.

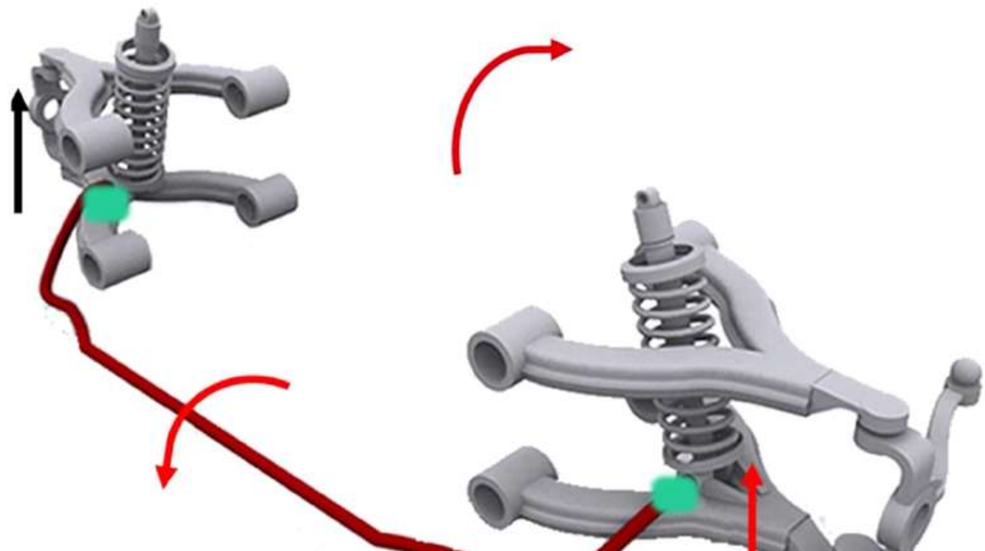
- Understeer i.e when a car steers less than the amount commanded by the driver.
- Oversteer i.e when a car steers by more than the amount commanded by the driver.





Anti-roll bar

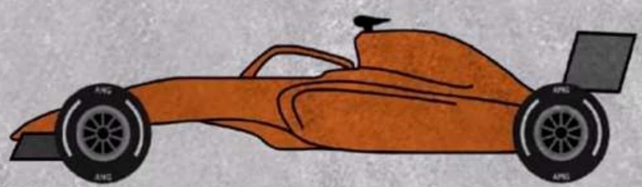
Anti-roll bar is the solid rigid link attached to the suspension link ages and it minimizes the body roll at time of fast cornering or over road irregularities.



Wheel Alignment

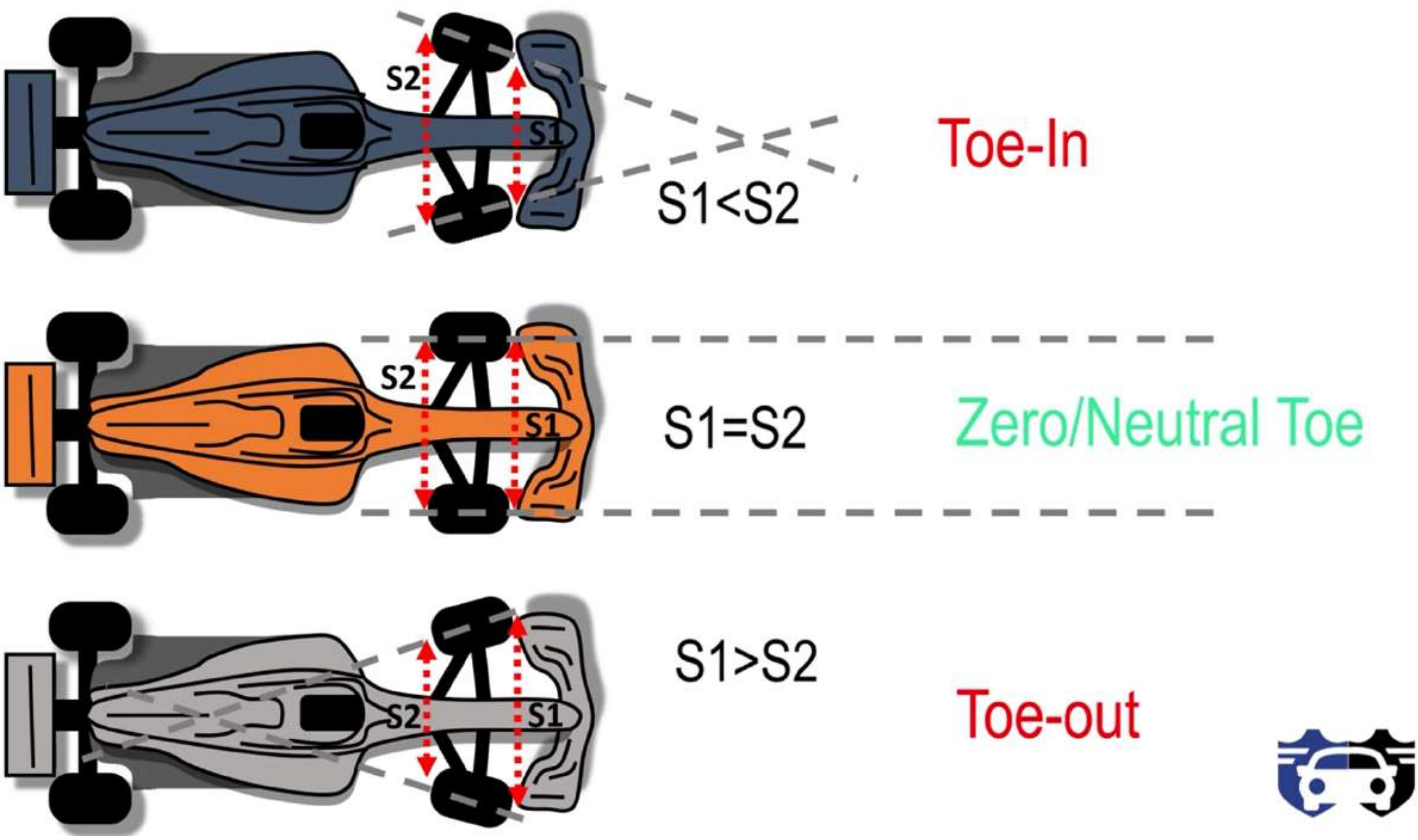
Wheel alignment is the process of adjusting vehicle suspension components to bring the wheels into specific angles, facilitating optimal vehicle handling, tire wear, and performance.

- Alignment angles can also be altered to obtain a specific handling characteristic.



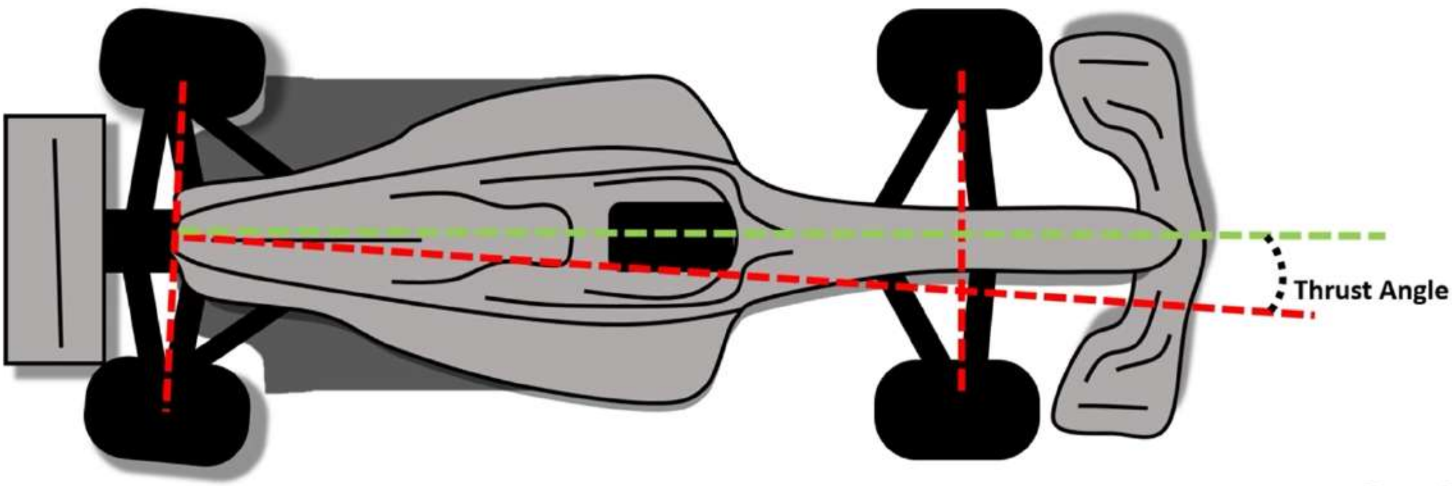
Wheel alignment is required because:-

- To stop the vehicle from pulling to one side.
- To avoid wear on the tyres ,steering & suspension parts
- Ultimately, to ensure an excellent road holding.


S2
S1
S1<S2
Toe-In
S2
S1
S1=S2
Zero/Neutral Toe
S1>S2
S2
S1
Toe-out

Thrust Angle

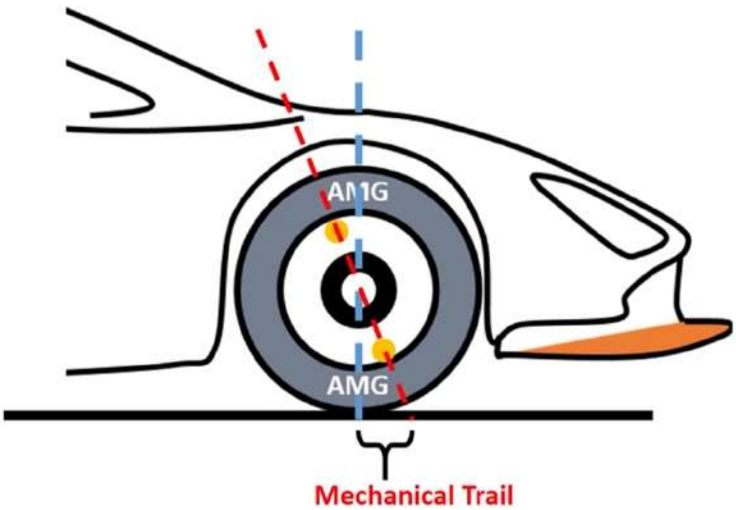
The thrust angle is an imaginary line drawn perpendicular to the rear axles centreline. It compares the direction that the rear axle is aimed with the centreline of the vehicle. It also confirms if the rear axle is parallel to its front axle and that the wheelbase on both sides of the vehicle is the same.

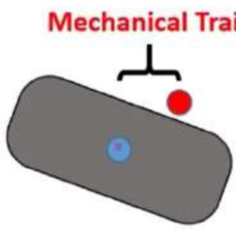


- Generally for independent rear axle geometry, this happens due to improper toe settings.

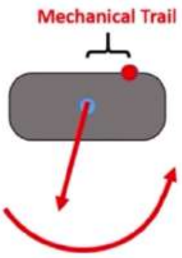
Mechanical Trail

Caster trail is the distance between the steering axis and the centre vertical line of tire at contact patch.





Caster Angles



- What is Caster?
- How caster effects the camber and Handling characteristics of vehicle?
- What is Mechanical Trail ?