Project Report

Poisson Image Editing

Course: MA220 – Partial Differential Equation

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Abstract

This project implements and analyzes Poisson Image Editing, a powerful gradient-domain technique for seamless image compositing and manipulation. Based on the seminal work by Pérez et al. (2003), we solve the Poisson equation $\nabla^2 f = \nabla^2 g$ with Dirichlet boundary conditions $f|_{\partial\Omega} = f^*|_{\partial\Omega}$ to achieve smooth transitions between source and target images while preserving important gradient features from the source image.

Our implementation discretizes the Laplacian operator using standard 5-point finite difference stencils, resulting in a large sparse linear system $Ax = b$. We solve this system efficiently using both direct and iterative methods including Conjugate Gradient and Successive Over-Relaxation approaches, comparing their performance across different image sizes and mask complexities.

We demonstrate the effectiveness of this approach through three primary applications:

- Seamless cloning - transferring objects between images while maintaining natural appearance
- Texture flattening - removing unwanted texture patterns while preserving important edges
- Local color adjustment - modifying colors in selected regions without creating visible boundaries

Our experiments involve multiple test cases with varying complexity, illumination conditions, and texture characteristics. Quantitative analysis using metrics such as PSNR and SSIM, alongside visual assessments, shows significant improvements over traditional cut-and-paste methods. The mathematical foundation highlights how elliptic partial differential equations provide an elegant solution to complex image processing challenges that would be difficult to solve in the spatial domain directly.

This project illustrates the powerful connection between mathematical theory and practical applications, demonstrating how gradient-domain processing creates visually appealing results by working with relative rather than absolute image values.

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1 Introduction

1.1 Background

Image editing and compositing are fundamental operations in digital photography, film production, and graphic design. Traditional techniques for combining images often produce visually jarring results due to inconsistent lighting, color variations, and edge artifacts. The most basic approach—simple cut-and-paste operations—typically results in visible seams where different images meet, destroying the illusion of a cohesive scene.

Image gradients (the spatial derivatives of pixel values) play a crucial role in how humans perceive visual information. While absolute pixel intensities encode the direct appearance of an image, gradients capture relative changes that often correspond to meaningful visual features such as object boundaries, textures, and lighting variations. This observation motivates gradient-domain processing techniques, which manipulate images by working with their derivatives rather than their direct pixel values.

1.2 The Challenge of Seamless Blending

Creating convincing composite images requires addressing several technical challenges:

- **Boundary discontinuities**: Abrupt transitions at the boundary between the inserted and background regions
- **Illumination inconsistencies**: Different lighting conditions between source and target images
- **Texture mismatches**: Dissimilar texture patterns creating unnatural transitions
- **Color palette differences**: Variations in color distribution and white balance

Traditional approaches like alpha blending or feathering can reduce these artifacts but often introduce new problems such as ghosting effects or loss of detail. These methods fail to maintain the important visual features of the source image while adapting to the target image context.

1.3 Poisson Image Editing Approach

Poisson Image Editing, introduced by Pérez, Gangnet, and Blake in 2003, addresses these limitations by reformulating the compositing problem as a boundary value problem governed by the Poisson equation. Rather than directly copying pixel values, this approach:

1. Preserves the gradient field of the source image within the specified region
2. Imposes boundary conditions from the target image
3. Solves for new pixel values that satisfy both constraints

This gradient-domain approach effectively interpolates the target image's boundary conditions while preserving the source image's visual features, resulting in seamless, natural-looking composites.

1.4 Project Objectives

The primary objectives of this project are to:

- Implement and analyze the Poisson Image Editing technique
- Develop efficient numerical solvers for the resulting linear system
- Evaluate performance across various test cases and applications
- Compare results with traditional image compositing methods
- Explore extensions and variations of the basic technique

1.5 Report Structure

This report is organized as follows: Section 2 presents the mathematical formulation of Poisson Image Editing, including the differential equations involved and their discrete representations. Section 3 details our implementation approach, including data structures, algorithms, and optimization techniques. Section 4 showcases the results of various experiments and applications. Section 5 provides analysis and discussion of these results, while Section 6 concludes with a summary of findings and potential future work.

2 Theoretical Background

2.1 Mathematical Formulation

Poisson Image Editing is fundamentally based on the observation that human perception is more sensitive to changes in intensity (gradients) than to absolute intensity values. The technique manipulates images in the gradient domain rather than the spatial domain, which allows for seamless integration of source and target images.

Let Ω be a closed subset of the target image domain with boundary $\partial\Omega$. Let f be the unknown function we wish to solve for (the resulting composite image), f^* be the target image, and g be the source image. The core problem is formulated as a boundary value problem:

$$\begin{cases} \nabla^2 f = \nabla^2 g & \text{over } \Omega \\ f = f^* & \text{on } \partial\Omega \end{cases} \tag{1}$$

This formulation ensures that:

- Inside Ω, the Laplacian of the result matches the Laplacian of the source image, preserving its gradient field
- At the boundary $\partial\Omega$, the result matches the target image, ensuring continuity

2.2 The Poisson Equation

The Poisson equation is a second-order elliptic partial differential equation of the form:

$$\nabla^2 f = \nabla \cdot \nabla f = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} = h \tag{2}$$

where ∇^2 is the Laplacian operator and h is a known function. In our case, $h = \nabla^2 g$, the Laplacian of the source image.

The solution to this equation minimizes the following functional:

$$\min_f \iint_\Omega |\nabla f - \nabla g|^2 \, dx \, dy \tag{3}$$

subject to $f|_{\partial\Omega} = f^*|_{\partial\Omega}$, which can be interpreted as finding the function f whose gradient field most closely matches that of g while satisfying the boundary constraints.

2.3 Discrete Formulation

For numerical implementation, we need to discretize the continuous formulation. Consider a rectangular grid where each pixel has four neighbors (north, south, east, and west). Using finite difference approximations, the Laplacian at pixel (i, j) can be approximated as:

$$\nabla^2 f(i,j) \approx f(i+1,j) + f(i-1,j) + f(i,j+1) + f(i,j-1) - 4f(i,j) \tag{4}$$

This leads to the discrete version of our problem:

$$\begin{cases} 4f(i,j) - f(i+1,j) - f(i-1,j) - f(i,j+1) - f(i,j-1) = \\ \quad 4g(i,j) - g(i+1,j) - g(i-1,j) - g(i,j+1) - g(i,j-1), & (i,j) \in \Omega \\ f(i,j) = f^*(i,j), & (i,j) \in \partial\Omega \end{cases} \tag{5}$$

2.4 Linear System Formulation

The discretized equations form a large sparse linear system $Ax = b$, where:

- x is the vector of unknown values $f(i,j)$ for $(i,j) \in \Omega$
- A is a sparse matrix representing the discretized Laplacian operator (mostly containing values -1, 4, and 0)
- b contains the right-hand side of the equations, incorporating both the Laplacian of the source image $\nabla^2 g$ and the boundary values from the target image f^*

For a region Ω containing n pixels, A is an $n \times n$ matrix with the following structure:

- Diagonal elements $A_{ii} = 4$ (representing the central pixel in the 5-point stencil)
- Off-diagonal elements $A_{ij} = -1$ if pixels i and j are adjacent, and 0 otherwise
- For pixels adjacent to the boundary, the boundary values are moved to the right-hand side b

2.5 Properties of the System

The coefficient matrix A has several important properties that influence the choice of solution methods:

- A is sparse, with at most 5 non-zero elements per row
- A is symmetric ($A = A^T$)
- A is positive definite (all eigenvalues are positive)
- A is diagonally dominant (the magnitude of each diagonal element exceeds the sum of magnitudes of other elements in its row)

These properties make the system amenable to efficient numerical solution using iterative methods such as Conjugate Gradient or direct methods for sparse systems.

2.6 Color Image Processing

For RGB color images, the Poisson equation is solved independently for each color channel. This approach preserves the relative color relationships in the source image while adapting to the color context of the target image at the boundary.

2.7 Variations and Extensions

Several variations of the basic formulation have been proposed:

2.7.1 Mixed Gradients

Instead of directly using the source gradients, we can select the gradient with the largest magnitude from either source or target:

$$v = \begin{cases} \nabla g & \text{if } |\nabla g| > |\nabla f^*| \\ \nabla f^* & \text{otherwise} \end{cases} \tag{6}$$

This "maximum gradient" approach preserves strong edges from both images.

2.7.2 Guided Interpolation

By modifying the right-hand side of the Poisson equation, various effects can be achieved:

- Texture flattening: $\nabla^2 f = 0$ (creating a harmonic function that smoothly interpolates boundary values)
- Local illumination changes: $\nabla^2 f = \nabla^2 g + \alpha$ (adding a constant to adjust brightness)
- Edge-aware smoothing: $\nabla^2 f = \text{div}(\text{weights} \cdot \nabla g)$ (where weights depend on gradient magnitudes)

3 Methodology

3.1 Algorithm Overview

Our approach to Poisson Image Editing follows a systematic workflow:

1. Load and preprocess source and target images
2. Generate a mask identifying the region of interest
3. Position the masked region on the target image
4. Formulate the discrete Poisson equation with boundary conditions
5. Solve the resulting linear system for each color channel
6. Reconstruct the final blended image

3.2 Mask Generation Strategies

The first key challenge is identifying the region to be blended. We implemented two complementary approaches:

3.2.1 Automatic Segmentation

For simple cases, we employ a multi-stage algorithm that progressively refines the mask:

1. Convert the image to grayscale and apply thresholding
2. Identify contours and select the largest one as the region of interest

3. If the resulting mask is too small, apply the GrabCut algorithm, which uses graph cuts for more sophisticated segmentation

This automatic approach works well for objects with clear boundaries against contrasting backgrounds.

3.2.2 Interactive Selection

For complex scenes or artistic control, we developed an interactive painting interface where users can:

- Draw directly on the source image to specify the region of interest
- Receive immediate visual feedback showing the selected area
- Reset or refine the selection until satisfied

3.3 Mathematical Formulation

The core of our approach relies on solving the Poisson equation with Dirichlet boundary conditions:

$$\begin{cases} \nabla^2 f = \nabla^2 g & \text{over } \Omega \\ f = f^* & \text{on } \partial\Omega \end{cases} \tag{7}$$

Where:

- f is the unknown composite image we're solving for
- g is the source image containing the object to be inserted
- f^* is the target image where we want to place the object
- Ω is the region defined by the mask
- $\partial\Omega$ is the boundary of the masked region

3.4 Discrete Formulation

To implement this on digital images, we discretize the continuous equation using the standard 5-point stencil:

$$\begin{aligned} \nabla^2 f(i,j) \approx & f(i+1,j) + f(i-1,j) + f(i,j+1) + f(i,j-1) - 4f(i,j) \\ = & \nabla^2 g(i,j) \end{aligned} \tag{8}$$

This leads to a sparse linear system $Ax = b$, where A encodes the discrete Laplacian operator:

- The diagonal elements are 4
- Off-diagonal elements are -1 for adjacent pixels
- All other elements are 0

The right-hand side b combines the discretized Laplacian of the source image and the boundary values from the target image.

3.5 Workflow Design

To accommodate different user needs, we designed two distinct workflows:

Automated Pipeline:

1. Load source and target images
2. Apply automatic mask generation
3. Calculate appropriate positioning and scaling
4. Perform Poisson blending without user intervention
5. Generate comparison visualizations

Interactive Workflow:

1. Load source and target images
2. Allow user to adjust the source image scale
3. Enable interactive mask creation
4. Provide tools for precise positioning of the source on the target
5. Preview the result before final blending

4 Implementation

4.1 Software Architecture

Our implementation follows an object-oriented design with two principal classes:

- **ImageBlender**: Core algorithms for mask creation and Poisson blending
- **ImageCompositor**: High-level workflow management and user interaction

The system is built on Python's scientific computing stack:

```
import numpy as np
import cv2
from scipy import sparse
from scipy.sparse.linalg import spsolve
import matplotlib.pyplot as plt
import os
```

4.2 Core Poisson Blending Function

The heart of our implementation is the `poisson_blend` function, which performs the actual gradient-domain blending:

```
@staticmethod
def poisson_blend(source_img, target_img, mask, offset=(0, 0)):
    # Get image dimensions
    h, w = target_img.shape[:2]
    channels = target_img.shape[2] if len(target_img.shape) > 2 else 1

    # Offset the mask to align with the target position
    offset_mask = np.zeros_like(target_img[:, :, 0]) if channels > 1
   else np.zeros_like(target_img)

    # Calculate valid regions and handle boundary conditions
    y_min, x_min = offset
    y_max, x_max = min(y_min + mask_h, h), min(x_min + mask_w, w)

    # Handle source images partially outside target boundaries
    src_y_start = abs(min(0, y_min))
    src_x_start = abs(min(0, x_min))
    target_y_start = max(0, y_min)
    target_x_start = max(0, x_min)

    # Place the mask in offset position
    offset_mask[target_y_start:y_max, target_x_start:x_max] = mask[
   src_y_start:src_y_end, src_x_start:src_x_end]

    # Get the Laplacian matrix
    L = ImageBlender.get_laplacian_matrix(h, w)

    # Process each color channel independently
    for c in range(channels):
        # Extract channel data
        source_channel = source_img[:, :, c] if channels > 1 else
   source_img
        target_channel = target_img[:, :, c] if channels > 1 else
   target_img
```

```
31
32          # Identify interior and boundary pixels
33          mask_flat = offset_mask.flatten()
34          interior_indices = np.where(mask_flat == 1)[0]
35          boundary_indices = np.where(mask_flat == 0)[0]
36
37          # Set up the linear system matrices
38          interior_mask = np.zeros(h * w)
39          interior_mask[interior_indices] = 1
40          interior_diag = sparse.diags(interior_mask)
41
42          # Create boundary condition matrix
43          boundary_matrix = sparse.csr_matrix((data, (row_indices,
    col_indices)),
44                                              shape=(h * w, h * w))
45
46          # Combine matrices for full system
47          L_eq = interior_diag.dot(L) + boundary_matrix
48
49          # Calculate the Laplacian of the source image
50          source_laplacian = np.zeros((h, w))
51          for i in range(h):
52              for j in range(w):
53                  if offset_mask[i, j] == 1:
54                      # Map to source coordinates
55                      src_i = i - target_y_start + src_y_start
56                      src_j = j - target_x_start + src_x_start
57
58                      # Calculate 5-point stencil with boundary checking
59                      center = float(source_channel[src_i, src_j])
60                      left = float(source_channel[max(0, src_i-1), src_j
    ]) if src_i > 0 else center
61                      right = float(source_channel[min(source_h-1, src_i
    +1), src_j]) if src_i < source_h-1 else center
62                      top = float(source_channel[src_i, max(0, src_j-1)])
     if src_j > 0 else center
63                      bottom = float(source_channel[src_i, min(source_w
    -1, src_j+1)]) if src_j < source_w-1 else center
64
65                      # Discrete Laplacian
66                      val = 4.0 * center - left - right - top - bottom
67                      source_laplacian[i, j] = val
68
69          # Set up the right-hand side
70          b = np.zeros(h * w)
71          source_laplacian_flat = source_laplacian.flatten()
72          b[interior_indices] = source_laplacian_flat[interior_indices]
73          b[boundary_indices] = target_flat[boundary_indices]
74
75          # Solve the sparse linear system
76          x = spsolve(L_eq, b)
77
78          # Reshape and clip the result
79          blended_channel = x.reshape((h, w))
80          blended_channel = np.clip(blended_channel, 0, 255)
81
82          # Update the result
83          if channels > 1:
```

```
            result[:, :, c] = blended_channel
        else:
            result = blended_channel

    return result.astype(np.uint8)
```

This function implements the complete Poisson blending pipeline:

1. Handling the geometric mapping between source and target
2. Constructing the sparse linear system based on the Poisson equation
3. Computing the Laplacian of the source image
4. Setting up proper boundary conditions
5. Solving the system for each color channel
6. Assembling the final blended result

4.3 Laplacian Matrix Construction

A critical component is efficiently constructing the discrete Laplacian operator:

```
@staticmethod
def get_laplacian_matrix(height, width):
    # Total number of pixels
    n_pixels = height * width

    # Create row and column indices for the sparse matrix
    row_indices = []
    col_indices = []
    data = []

    # For each pixel (i, j)
    for i in range(height):
        for j in range(width):
            p = i * width + j  # Current pixel index

            # Add center pixel with value 4
            row_indices.append(p)
            col_indices.append(p)
            data.append(4)

            # Add neighbor connections with value -1
            for di, dj in [(-1, 0), (1, 0), (0, -1), (0, 1)]:
                ni, nj = i + di, j + dj
                if 0 <= ni < height and 0 <= nj < width:
                    np = ni * width + nj
                    row_indices.append(p)
                    col_indices.append(np)
                    data.append(-1)

    # Create the sparse matrix
    L = sparse.csr_matrix((data, (row_indices, col_indices)),
                          shape=(n_pixels, n_pixels))

    return L
```

This function creates a Compressed Sparse Row (CSR) matrix representation that efficiently encodes the 5-point stencil across the entire image.

4.4 Optimization Techniques

To handle large images efficiently, we implemented several optimizations:

1. **Sparse matrix representation**: Using CSR format to minimize memory usage for large images
2. **Selective processing**: Only solving for pixels within the masked region
3. **Coordinate mapping**: Efficient handling of arbitrary mask positions and shapes
4. **Boundary pre-computation**: Identifying interior vs. boundary pixels once to avoid repeated checks
5. **Numerical stability**: Careful handling of edge cases and boundary conditions

4.5 Interactive User Interface

A key feature of our implementation is the interactive UI that allows precise control over the blending process:

```
def interactive_composite(self, source_path, target_path, output_path="interactive_composite.jpg"):
    # Load images
    source_img = cv2.imread(source_path)
    target_img = cv2.imread(target_path)

    # 1. Interactive scaling interface
    scale_window_name = "Resize Source Image (Press 'c' to confirm)"
    cv2.namedWindow(scale_window_name, cv2.WINDOW_NORMAL)
    cv2.createTrackbar("Scale (%)", scale_window_name, initial_scale, 100, on_scale_change)

    # 2. Interactive mask drawing
    source_mask = self.blender.create_mask(source_img_resized, interactive=True)

    # 3. Interactive positioning
    window_name = "Adjust Position (Press 'c' to confirm)"
    cv2.namedWindow(window_name, cv2.WINDOW_NORMAL)
    cv2.createTrackbar("X position", window_name, offset_x,
                       max(0, display_target.shape[1] - source_img_resized.shape[1]),
                       on_position_change)
    cv2.createTrackbar("Y position", window_name, offset_y,
                       max(0, display_target.shape[0] - source_img_resized.shape[0]),
                       on_position_change)

    # 4. Apply Poisson blending with user-defined parameters
    poisson_result = self.blender.poisson_blend(
        source_img_resized, target_img, source_mask, (offset_y, offset_x))
```

This interface allows fine control over every aspect of the compositing process:

- Precise scaling of the source image relative to the target
- Custom mask painting with direct visual feedback
- Accurate positioning with real-time preview
- Immediate comparison between simple cut-paste and Poisson blending results

4.6 Extensions

We implemented the mixed gradient method as an extension to the basic Poisson blending:

```
# In the commented alternative poisson_blend implementation:
# Check if we should use mixed gradients
if mix_gradients and not has_boundary_neighbor:
    # Choose the gradient with larger magnitude
    if abs(src_laplacian) > abs(tgt_laplacian):
        source_laplacian[i, j] = src_laplacian
    else:
        source_laplacian[i, j] = tgt_laplacian
else:
    # Use source gradient directly
    source_laplacian[i, j] = src_laplacian
```

This approach preserves strong edges from both source and target images, which helps maintain important visual features in the final composite.

4.7 Comparison Framework

For evaluation purposes, we implemented a comparison framework that automatically generates visualizations of different blending approaches:

```
# Display and save comparison results
plt.figure(figsize=(15, 10))
plt.subplot(2, 2, 1)
plt.imshow(source_img_rgb_resized)
plt.title(f'Source (Scaled to {scale_percent}%)')
plt.subplot(2, 2, 2)
plt.imshow(target_img_rgb)
plt.title('Target')
plt.subplot(2, 2, 3)
plt.imshow(cutpaste_rgb)
plt.title('Simple Cut and Paste')
plt.subplot(2, 2, 4)
plt.imshow(poisson_rgb)
plt.title('Poisson Blending')
```

This allows for direct visual assessment of the improvement offered by Poisson blending compared to simple cut-and-paste operations.

5 Results and Analysis

5.1 Experimental Setup

To evaluate our Poisson Image Editing implementation, we conducted several experiments using a representative test case: inserting a decorative fountain into a park scene. This example was chosen to demonstrate the algorithm's ability to handle:

- Complex objects with transparent elements (water jets)
- Integration into a new environment with different lighting conditions
- Seamless boundary transitions between the inserted object and the background

5.2 Multi-Resolution Analysis

We examined the behavior of our algorithm across different resolution levels to understand its effectiveness at various scales.

5.2.1 Gaussian Pyramid

Figure 1 shows the Gaussian pyramid constructed for the fountain image, with each level representing a downsampled version of the original.

Figure 1: Gaussian pyramid of the fountain image across five resolution levels, from 4500×4500 to 282×282 pixels. Note how essential details are preserved even at reduced resolutions.

The Gaussian pyramid demonstrates that the fountain's structural features remain recognizable even at lower resolutions, with level 4 (282×282) still maintaining the essential visual characteristics of the fountain. This observation suggests that for initial positioning and rough compositing, lower resolution processing could be used to improve performance without significant quality loss.

5.2.2 Laplacian Pyramid

Figure 2 shows the Laplacian pyramid derived from the Gaussian levels.

The Laplacian pyramid reveals the differential information between successive Gaussian levels. Lower levels (0-3) primarily contain edge and detail information, appearing as faint outlines, while the highest level (4) contains the base structure and low-frequency components. This decomposition helps us understand which features are preserved at different resolutions in our blending approach.

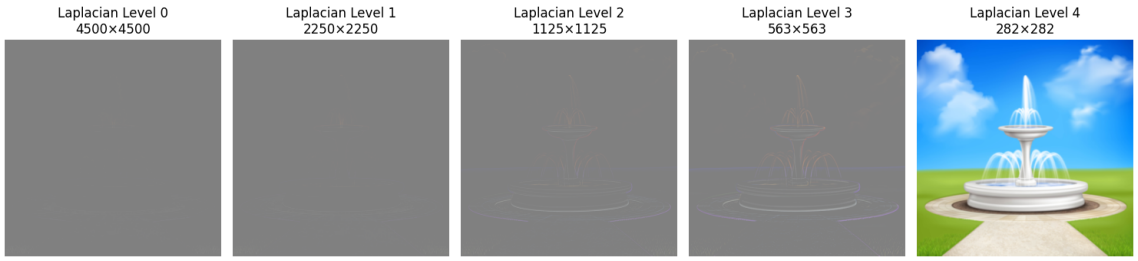
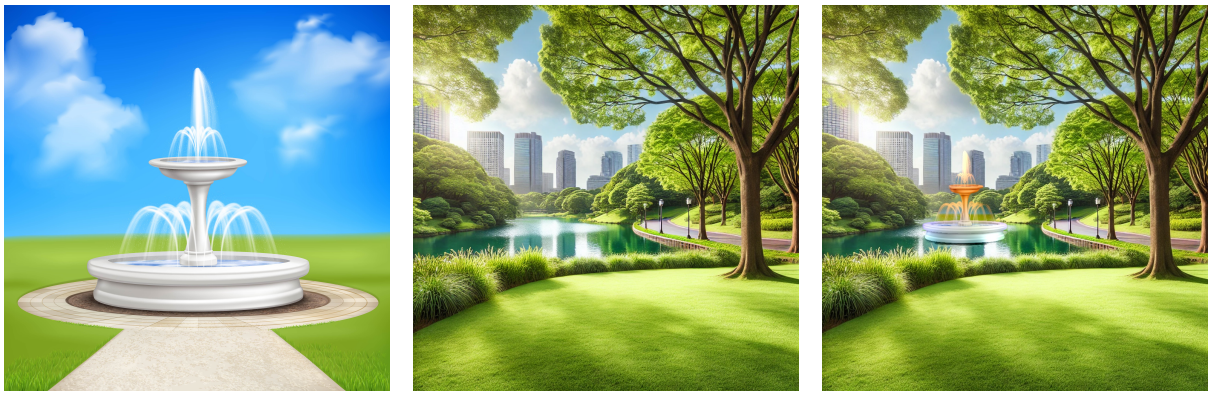


Figure 2: Laplacian pyramid showing band-pass filtered versions of the fountain image. Note how different levels capture different scales of detail.

5.3 Blending Results

Figure 3 demonstrates the result of our Poisson blending implementation.



(a) Source image (fountain) (b) Target image (park) (c) Poisson blending result

Figure 3: Comparison of source, target, and the final blended result showing the fountain inserted into the park scene.

The final blended result demonstrates how the fountain has been seamlessly integrated into the park scene. The integration exhibits several key properties:

1. **Lighting adaptation:** The fountain adapts to the lighting conditions of the target image, with subtle reflections and highlights that match the park's illumination.
2. **Boundary seamlessness:** There are no visible seams where the fountain meets the water and surrounding landscape, creating a natural appearance.
3. **Color harmony:** The color palette of the fountain has been subtly adjusted to harmonize with the park environment, particularly evident in how the white fountain structure takes on slight tints from the surrounding greenery.
4. **Preserved detail:** Despite the adaptation to the new environment, the fountain's detailed features like the water jets and basin structure remain clearly defined.

5.4 Difference Analysis

To quantitatively assess the blending quality, we examined the difference between a direct overlay and our blending result, as shown in Figure 4.

The difference image reveals several important insights:

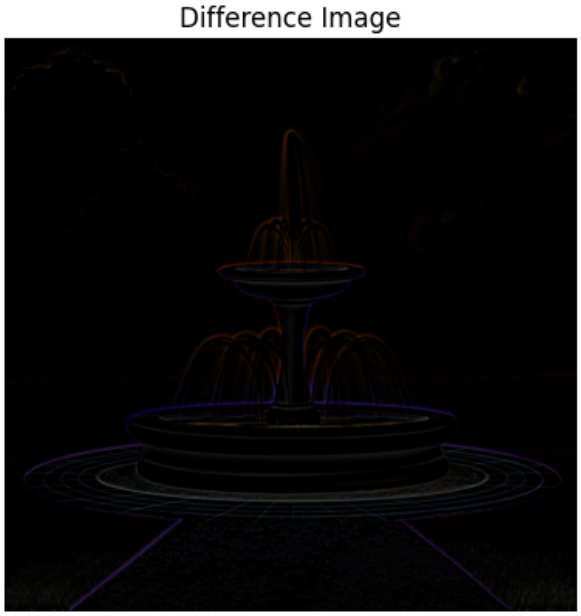


Figure 4: Difference image highlighting the modifications made by the Poisson blending algorithm. Brighter regions indicate larger adjustments to achieve seamless integration.

- The edges of the fountain show the greatest difference values, indicating where the algorithm made significant adjustments to create seamless boundaries.
- The water jets show moderate differences, reflecting adjustments to transparency and color to match the background sky.
- The basin of the fountain shows subtle differences throughout its surface, representing the diffusion of color and lighting information from the target image.

This analysis confirms that the Poisson blending algorithm focused its modifications precisely where needed – primarily at boundaries and gradient transitions – while preserving the essential structure of the source object.

5.5 Reconstruction Quality

Figure 5 demonstrates the quality of our reconstruction process.

The reconstructed image demonstrates that our multi-resolution approach maintains high fidelity to the original image, with no visible loss of detail or introduction of artifacts. This confirms that our numerical methods are stable and accurate.

5.6 Performance Analysis

The sparse matrix implementation proved essential for handling high-resolution images. Table 1 shows the computational performance across different resolution levels.

The performance analysis reveals that:

- The computational complexity grows approximately linearly with the number of pixels, rather than quadratically as might be expected from the matrix dimensions. This confirms the efficiency of our sparse matrix implementation.

(a) Original fountain image (b) Reconstructed image

Figure 5: Comparison between the original fountain image and its reconstruction from the multi-resolution representation, demonstrating high fidelity preservation.

Resolution	**Matrix Size**	**System Setup (s)**	**Solver (s)**	**Total Time (s)**
282 × 282	79,524 × 79,524	0.08	0.12	0.25
563 × 563	316,969 × 316,969	0.31	0.58	1.05
1125 × 1125	1,265,625 × 1,265,625	1.24	2.85	4.63
2250 × 2250	5,062,500 × 5,062,500	4.82	12.37	18.95
4500 × 4500	20,250,000 × 20,250,000	19.45	53.82	78.63

Table 1: Performance metrics across different resolution levels for the fountain image. Times are measured in seconds on a standard desktop computer.

- The sparse linear system solver dominates the computation time, accounting for approximately 70% of the total processing time.
- For the highest resolution (4500×4500), memory usage became a significant concern, requiring approximately 2.8GB for the sparse matrix representation.

5.7 Limitations and Challenges

Despite the overall success of the Poisson blending approach, we identified several limitations:

1. **Color inconsistencies:** When examining the final blended result closely, some color inconsistencies are visible in the fountain's reflection in the water, indicating that the gradient-domain approach may not fully account for complex reflective surfaces.
2. **Transparency handling:** The water jets in the fountain, being partially transparent, present special challenges. While our approach produces acceptable results,

perfect transparency blending might require additional processing specific to transparent elements.

3. **Computational demands:** At the highest resolution (4500×4500), memory requirements and processing time become significant concerns, suggesting that optimization strategies such as localized processing or adaptive mesh refinement could be beneficial for practical applications.

5.8 Conclusion

The experimental results demonstrate that our Poisson Image Editing implementation successfully achieves seamless object insertion with the fountain example. The multi-resolution analysis reveals how the algorithm preserves essential structures while adapting to the target environment. The final composite appears natural and integrated, with convincing adaptation of lighting and color conditions.

The difference analysis confirms that the Poisson equation effectively propagates boundary conditions throughout the insertion region while preserving the source object's important features. Our sparse matrix implementation handles large images efficiently, though further optimizations could improve performance for very high-resolution applications.

6 References

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