Assignment

Graphs & Tress

1. The adjacency matrix of a graph is given below.

$$\begin{bmatrix} 0 & 1 & 1 & 1 & 0 & 1 \\ 1 & 0 & 0 & 1 & 1 & 1 \\ 1 & 0 & 0 & 1 & 1 & 1 \\ 1 & 1 & 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 0 & 1 & 0 \end{bmatrix}$$

(a) Draw the graph defined by this adjacency matrix. Label the vertices of your graph $1, 2..., 6$ so that vertex i corresponds to row and column i of the matrix.

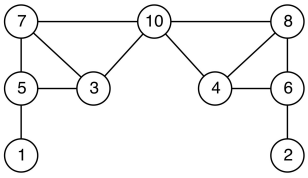
(b) In a graph, we define the *distance* between two vertices to be the length of the shortest path between them. We define the *diameter* of a graph to be the largest distance between any two nodes. What is the diameter of this graph.

(c) Find a cycle in this graph of maximum length.

(d) Give a coloring of the vertices that uses the minimum number of colors.

2. Let G be a connected graph. Prove that every vertex of odd degree is connected to at least one other vertex of odd degree in G.

3. Consider a graph representing the main campus buildings at IIT Mandi.



Is this graph *bipartite*? Please give a brief argument for your answer.

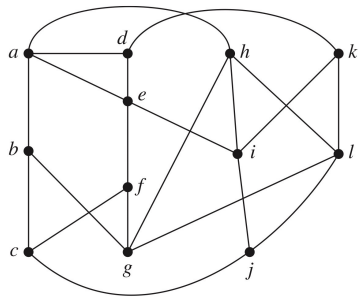
4. Let G be an undirected graph with a loop at every vertex. Show that the relation R on the set of vertices of G such that uRv if and only if there is an edge associated to $\{u, v\}$ is a symmetric, reflexive relation on G.

5. In a round robin tournament the Tigers beat the Blue Jays, the Tigers beat the Cardinals, the Tigers Beat the Orioles, the Blue Jays beat the Cardinals, the Blue Jays beat the Orioles, and the Cardinals beat the Orioles. Model the outcome with a directed graph.

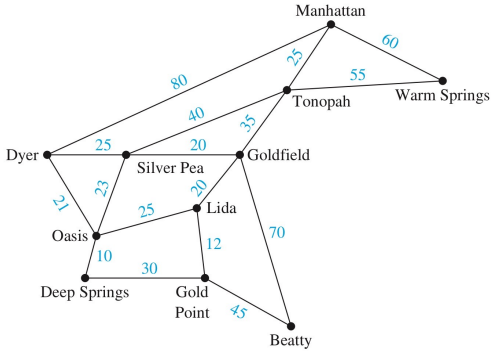
6. The **complimentary graph** $\overline{G}$ of a simple graph G has the same vertices as G. Two vertices are adjacent in $\overline{G}$ if and only if if they are not adjacent in G. Describe complimentary graphs for K_5 and $K_{4,3}$.

7 Show that a simple graph G with n vertices is connected if it has more than $(n-1)(n-2)/2$ edges.

8. How many edges must be removed from the below graph to get a spanning tree.



9. The roads represented by this graph are all unpaved. The length of the roads between pairs of towns are represented by edges weights. Which roads should be paved so that there is a path of paved roads between each pair of towns so that a minimum road length is paved?



10. In a tree that has 100 vertices, how many nodes of degree 10 can be at most?

11. What is the least number of colors needed to color the map of India.

12. A zoo wants to set up natural habitats in which to exhibits its animals. Unfortunately, some animals will eat some of the others when given the opportunity. How can a graph model and a coloring can be used to determine the number of different habitats needed and the placement of the animals in these habitats?

13. A $6-ary$ tree has 54 internal vertices and 254 leaves. How many leaves should be added to this tree to ensure that it becomes a full $6-ary$ tree?

14. Given a graph G with vertex set $V = \{v1, ..., v_n\}$ we define the *degree sequence* of G to be the list $d(v1), ..., d(v_n)$ of degrees in decreasing order. For each of the following lists, give an example of a graph with such a degree sequence or prove that no such graph exists:
(a) 3, 3, 2, 2, 2, 1
(b) 6, 6, 6, 4, 4, 3, 3
(c) 6, 6, 6, 4, 4, 2, 2