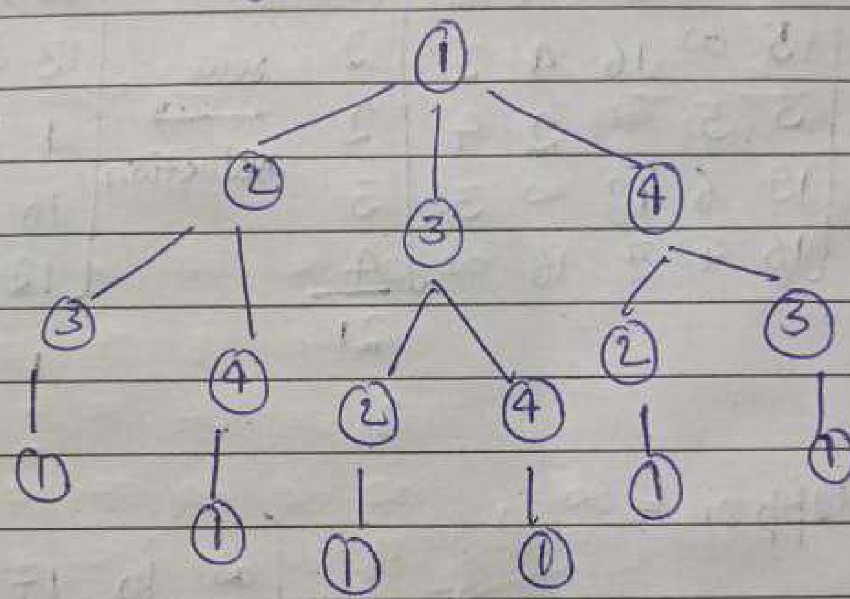
Travelling Salesman Problem :

① **Dynamic Programming Approach - (Back-Tracking)**

$$g(v_1, \{v_2, v_3, v_4 \cdots v_n\}) = \min_{v_k \in V} \{c_{1k} + g(v_k, \{v_2, v_3 \cdots v_n\} - \{v_k\})\}$$

for, $G = (V, E)$

General way : $g(i, S) = \min_{k \in S} \{c_{ik} + g(k, S - \{k\})\}$

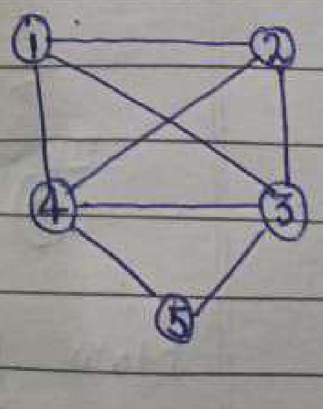


② **Branch & Bound Approach -**

Let us see this approach using an example-

The following adjacency matrix is :

$$\begin{array}{c|c} & \begin{array}{ccccc} 1 & 2 & 3 & 4 & 5 \end{array} \\ \begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{array} & \begin{bmatrix} \infty & 20 & 30 & 10 & 11 \\ 15 & \infty & 16 & 4 & 2 \\ 3 & 5 & \infty & 2 & 4 \\ 14 & 6 & 18 & \infty & 3 \\ 16 & 4 & 7 & 16 & \infty \end{bmatrix} \end{array} \quad \text{--- (i)}$$



Step 1 : <u>Reduction</u> → select the smallest element & then subtract it from each element of that particular row / column

Perform row reduction & column reduction on (i) we get

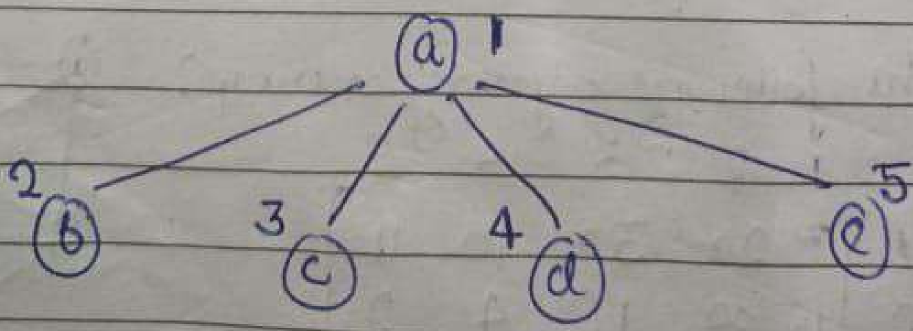
$$\begin{array}{c} \begin{bmatrix} \infty & 20 & 30 & 10 & 11 \\ 15 & \infty & 16 & 4 & 2 \\ 3 & 5 & \infty & 2 & 4 \\ 19 & 6 & 18 & \infty & 3 \\ 16 & 4 & 7 & 16 & \infty \end{bmatrix} \end{array} \begin{array}{c} 10 \\ 2 \\ 2 \\ 3 \\ \underline{4} \\ 21 \end{array} \xrightarrow[\text{reduction}]{\text{row}} \begin{array}{c} \begin{bmatrix} \infty & 10 & 20 & 0 & 1 \\ 13 & \infty & 14 & 2 & 0 \\ 1 & 3 & \infty & 0 & 2 \\ 16 & 3 & 15 & \infty & 0 \\ 12 & 0 & 3 & 12 & \infty \end{bmatrix} \\ \begin{matrix} 1 & 0 & 3 & 0 & 0 \end{matrix} \end{array}$$

↓ column reduction

Bound : Upper = ∞

Cost at (a) →

$$C(a) = 21 + 4 = 25 \qquad (a) - \begin{bmatrix} \infty & 10 & 17 & 0 & 1 \\ 12 & \infty & 11 & 2 & 0 \\ 0 & 3 & \infty & 0 & 2 \\ 15 & 3 & 12 & \infty & 0 \\ 11 & 0 & 0 & 12 & \infty \end{bmatrix}$$



Now for 1 → 2 reduce (a) ← matrix w.r.t 1 respectively

Steps for the reduction $p \to q$

S-1 make p^{th} row & q^{th} column ∞

S-2 make (q,p) as ∞.

S-3 In the trailing path before make any connection b/w 'q' & above node ∞.

S-4 Reduce the matrix like before

we get,

$$(B) \leftarrow \begin{bmatrix} \infty & \infty & \infty & \infty & \infty \\ \infty & \infty & 11 & 2 & 0 \\ 0 & \infty & \infty & 0 & 2 \\ 15 & \infty & 12 & \infty & 0 \\ 11 & \infty & 0 & 12 & \infty \end{bmatrix}$$

lly, we get reduced matrices for c, d, e

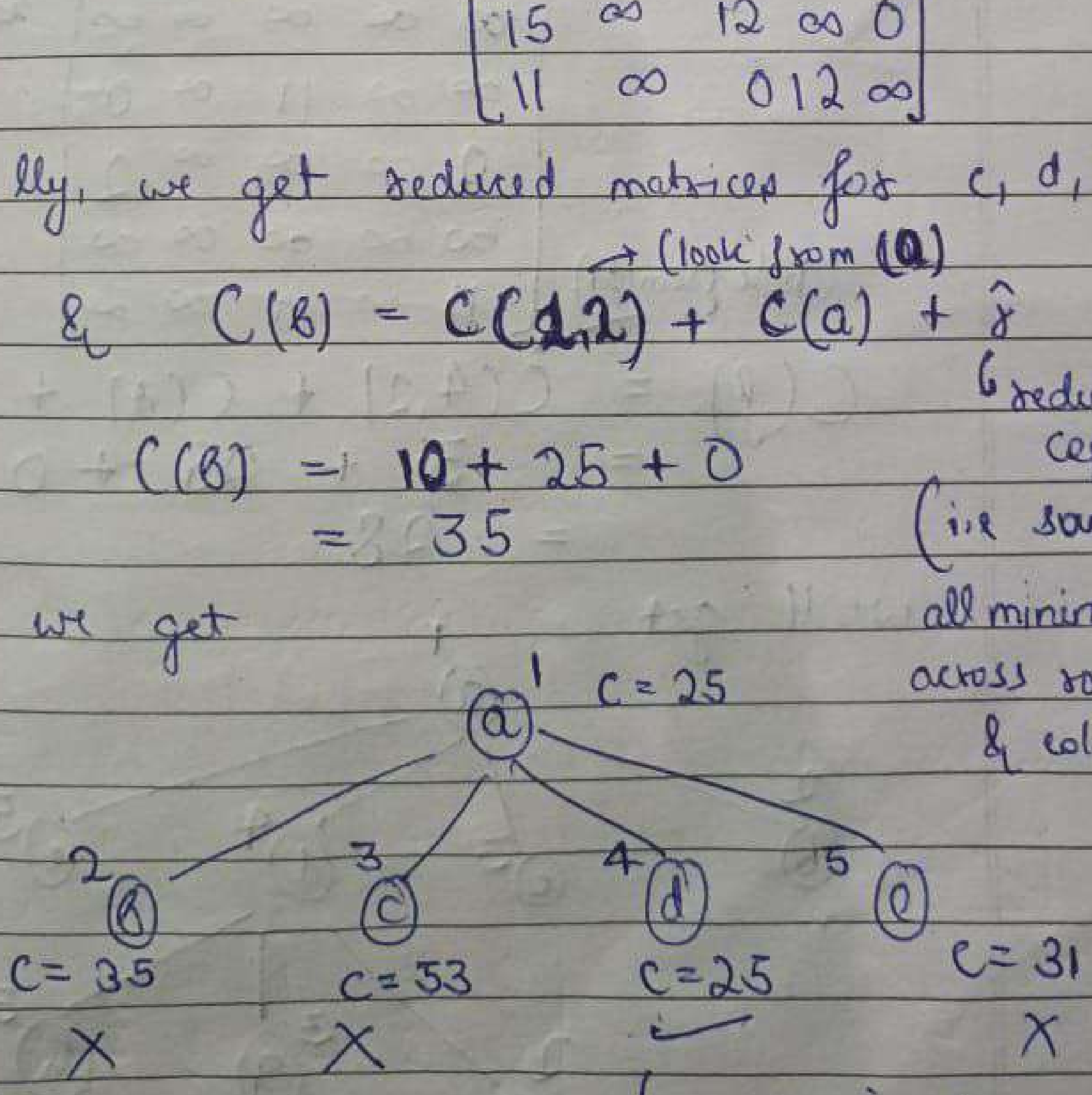
→ (look from (a))

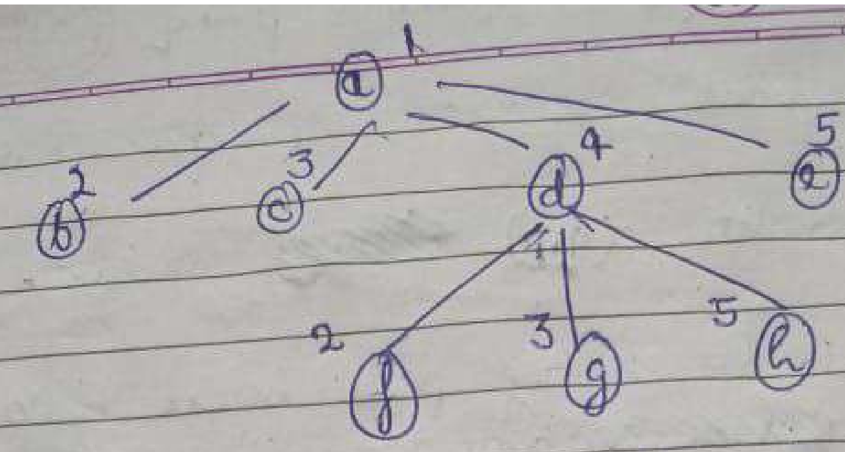
& $C(B) = C(1,2) + C(a) + \hat{r}$

reduction cost (i.e sum of all minimums across rows & columns)

$C(B) = 10 + 25 + 0$
$= 35$

we get





Now calculate the costs for f, g & h node in similar manner

for ex. reduced matrix for $4 \rightarrow 2$ will be

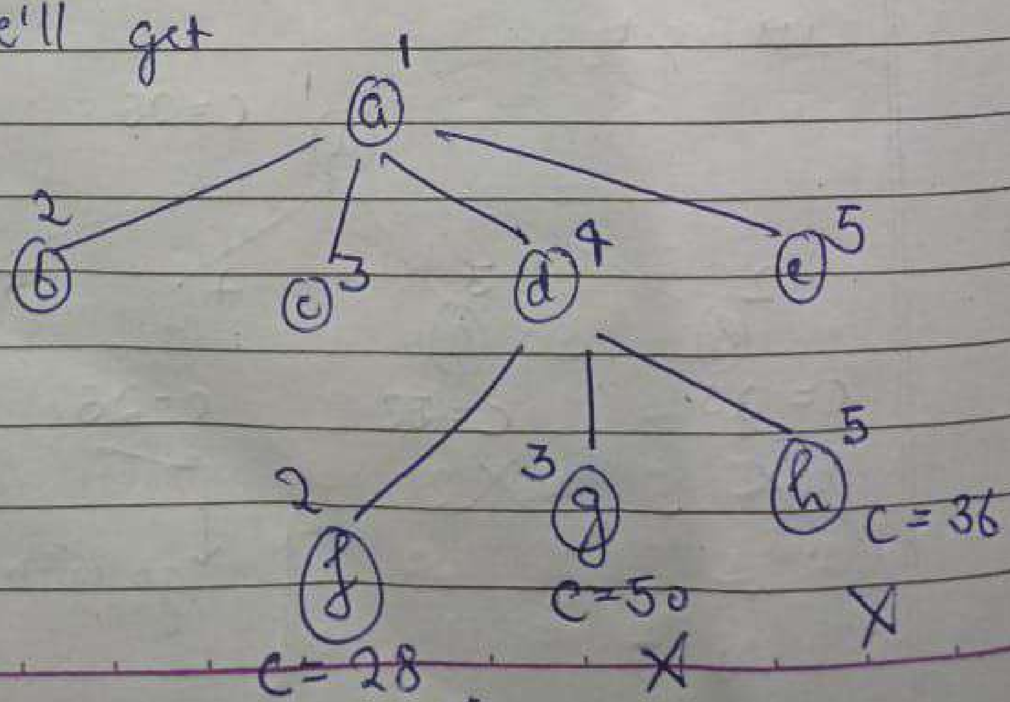
from (d)

$$\begin{bmatrix} \infty & \infty & \infty & \infty & \infty \\ \infty & \infty & 11 & \infty & 0 \\ 0 & \infty & \infty & \infty & 2 \\ \infty & \infty & \infty & \infty & \infty \\ 11 & \infty & 0 & \infty & \infty \end{bmatrix}$$

(look from (d))

$$C(f) = C(4,2) + C(d) + \hat{r}$$
$$= 3 + 25 + 0$$
$$= 28$$

we'll get



And so on till path is completed

Here, the final & optimized path will be

$1 \rightarrow 4 \rightarrow 2 \rightarrow 5 \rightarrow 3$ ans