Worksheet 3 part 3

02561 - Computer Graphics

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The transformation matrices for part 1:

In part 1 we only model transform with a view matrix:

$$V = \begin{bmatrix} \vec{b}_1 & \vec{b}_2 & \vec{b}_3 \end{bmatrix},$$

where $\vec{b}_1 = \dfrac{u \times \vec{b}_3}{\left\| u \times \vec{b}_3 \right\|}$, $\vec{b}_2 = \vec{b}_3 \times \vec{b}_1$ and $\vec{b}_3 = \dfrac{e - a}{\|e - a\|}$,

where e is the eyepoint, a is the look-at point and u is the up-vector.
This matrix is multiplied with the model positions

$$V \cdot m = \begin{bmatrix} \vec{b}_1 & \vec{b}_2 & \vec{b}_3 \end{bmatrix} \cdot \langle m_1, m_2, m_3 \rangle$$

in order to move the model from world space to eye space.

The transformation matrices for part 2:

In part 2 we use the view matrix from above as well as a perspective and a rotation matrix.
The perspective matrix is given as

$$P = \begin{bmatrix} \frac{1}{A} \cdot \cot\left(\frac{\alpha}{2}\right) & 0 & 0 & 0 \\ 0 & \cot\left(\frac{\alpha}{2}\right) & 0 & 0 \\ 0 & 0 & \frac{n+f}{n-f} & \frac{2\,nf}{n-f} \\ 0 & 0 & -1 & 0 \end{bmatrix},$$

where α is the vertical field of view, A is the aspect ratio (canvas height divided by canvas width), n is the near distance and f is the far distance.
This matrix transforms the view frustum to clip space from which objects between the near and far distance are shown.

The rotation matrix has 3 different variants based on the axis of rotation:

$$R_x(\theta) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos(\theta) & -\sin(\theta) \\ 0 & \sin(\theta) & \cos(\theta) \end{bmatrix},$$

$$R_y(\theta) = \begin{bmatrix} \cos(\theta) & 0 & \sin(\theta) \\ 0 & 1 & 0 \\ -\sin(\theta) & 0 & \cos(\theta) \end{bmatrix},$$

$$R_z(\theta) = \begin{bmatrix} \cos(\theta) & -\sin(\theta) & 0 \\ \sin(\theta) & \cos(\theta) & 0 \\ 0 & 0 & 1 \end{bmatrix}.$$

Rotation on several axes is achieved by multiplying the different rotation matrices before applying them. Rotation matrices transforms the model in world space before the view matrix is applied.

The 3 cubes:

The CTM of each cube is calculated through using these formulae:

$$C_1 = P \times V \times m$$

$$C_2 = P \times V \times R_y(\theta) \times m$$

$$C_3 = P \times V \times R_y(\theta) \times R_x(\theta) \times m$$