WorkSheet 4

Intro - Rendering

Part 1.

The number of photons emitted per second can be calculate using:

$$\frac{\#\text{Photons}}{\text{s}} = \frac{Power_{light} \cdot \varepsilon_{light} \cdot \lambda}{h \cdot c}$$

$$\frac{25\,\frac{\text{J}}{\text{s}} \cdot 0.2 \cdot 500\,\text{nm} \cdot 10^{-9}\,\frac{\text{m}}{\text{nm}}}{6.626 \cdot 10^{-34}\,\frac{\text{J} \cdot \text{s}}{\text{photon}} \cdot 2.9979 \cdot 10^{8}\,\frac{\text{m}}{\text{s}}}$$

$$\frac{1.258552785\;10^{19}\;photon}{s} \tag{1}$$

Part 2.

$$V := 2.4\,\frac{\text{J}}{\text{C}}:$$

$$A := 0.7\,\frac{\text{C}}{\text{s}}:$$

$$\text{Diameter} := 0.01\,\text{m}:$$

$$r := \frac{Diameter}{2}:$$

Radiant Flux is calculated as

$Flux := V \cdot A$

$$Flux := \frac{1.68\;J}{s} \tag{2}$$

Radiant Intensity is calculated as

$$Intensity := \frac{Flux}{4 \cdot \pi \cdot \text{sr}}$$

$$Intensity := \frac{0.1336901522\;J}{s\;sr} \tag{3}$$

Radiant Exitance is calculated as

$$Exitance := \frac{Flux}{4 \cdot \pi \cdot r^2}$$

$$Exitance := \frac{5347.606088\;J}{s\;m^2} \tag{4}$$

The Emitted Energy in 5 minutes is

$Energy = Flux \cdot 300\;s$

$$Energy = 504.00\;J \tag{5}$$

Part 3.

$pupil := 0.006\ \mathrm{m}:$
$distance := 1\ \mathrm{m}:$

We assume that the pupil is a circle.

$A := \left(\frac{pupil}{2}\right)^2 \cdot \pi$

$$A := 0.00002827433389\ m^2 \qquad \textbf{(6)}$$

The steradians of which light is projected into the pupil is

$\omega := \frac{A}{(distance)^2} sr$

$$\omega := 0.00002799369708\ sr \qquad \textbf{(7)}$$

The irradiance is then the intensity times the steradians divided by the area of the pupil

$Irradiance := \frac{Intensity \cdot \omega}{A}$

$$Irradiance := \frac{0.1323632110\ J}{s\ m^2} \qquad \textbf{(8)}$$

Part 4.

First we calculate the flux of the lamp

$Flux := 200\ \frac{\mathrm{J}}{\mathrm{s}} \cdot 0.2$

$$Flux := \frac{40.0\ J}{s} \qquad \textbf{(9)}$$

Then we calculate the intensity. Since it goes in all directions the steradians are $4\ \pi$ (surface of a sphere)

$Intensity := \frac{Flux}{4\ \pi}$

$$Intensity := \frac{0.1336901522\ J}{s} \qquad \textbf{(10)}$$

The irradiance is then the intensity times cosine to the angle, however we assume that we look straight down on the table, divided by the distance to the object (2 meters to the table)

$Irradiance := \frac{Intensity}{(2\ m)^2}$

$$Irradiance := \frac{0.03342253805\ J}{s\ m^2} \qquad \textbf{(11)}$$

We now convert this radiometric irradiance to the photometric illuminance:

$Illuminance := Irradiance \cdot 685 \cdot 0.1$

$$Illuminance := \frac{2.289443856\ J}{s\ m^2} \qquad \textbf{(12)}$$

Part 5.

For the known source (I_s):

$$E_{source} = \frac{I_s}{\left(4 \cdot \pi \cdot (0.35\ m)^2\right)}$$

For the unknown source (I_x):

$$E_{source} = \frac{I_x}{\left(4 \cdot \pi \cdot (0.65\ m)^2\right)}$$

Since the screen is equally illuminated by both sources, E_{source} for the known and unknown sources is the same:

$$\frac{40\ cd}{\left(4 \cdot \pi \cdot (0.35\ m)^2\right)} = \frac{I_x}{\left(4 \cdot \pi \cdot (0.65\ m)^2\right)}$$

We solve for I_x:

$$solve\left(\frac{40\ cd}{\left(4 \cdot \pi \cdot (0.35\ m)^2\right)} = \frac{I_x}{\left(4 \cdot \pi \cdot (0.65\ m)^2\right)}, I_x\right)$$

$$137.9591837\ cd \tag{13}$$

The luminous intensity for the uknown light source is then $I_x = 137.96$ cd.

Part 6.

We use formula

$$B = L \cdot \pi$$

$$B = \frac{5000\ W}{sr \cdot m^2} \cdot \pi = \frac{15708\ W}{m^2}$$

We then use the formula for emitted energy:

$$E = B \cdot A$$

$$E = \frac{15708\ W}{m^2} \cdot (0.1\ m)^2$$

$$E = 157.08\ W \tag{14}$$

The emitted energy is then 157 W.

Part 7.

To find the Radiant Flux we recalculate the integral:

$$\Phi = \int_A \int_{2\pi} L \cdot \cos(\theta)\ \mathrm{d}\omega\ \mathrm{d}A_o$$

L is given as $6000 \cos(\theta) \frac{W}{m^2 \cdot sr}$, so we rewrite to

$$6000 \frac{W}{m^2 \cdot sr} \cdot \int_A \mathrm{d}A_o \cdot \int_0^{2\pi} \mathrm{d}\phi \cdot \int_0^{\frac{\pi}{2}} \cos^2(\theta) \cdot \sin(\theta)\ \mathrm{d}\theta$$

This results in the following formula:

$$Flux = 6000 \frac{W}{m^2 \cdot sr} \cdot A \cdot 2\,\pi \cdot sr \cdot \frac{1}{3} = 6000 \frac{W}{m^2 \cdot sr} \cdot A \cdot \frac{2}{3} \pi \cdot sr$$

The area of the light source is

$A := (0.1\,m)^2 :$

We input the area in the formula and get

$$Flux := 6000 \frac{W}{m^2 \cdot sr} \cdot A \cdot \frac{2}{3} \pi \cdot sr$$

$$Flux := 125.6637062\,W \tag{15}$$

The exitance is then

$$'Exitance' = \frac{Flux}{A}$$

$$Exitance = \frac{12566.37062\,W}{m^2} \tag{16}$$