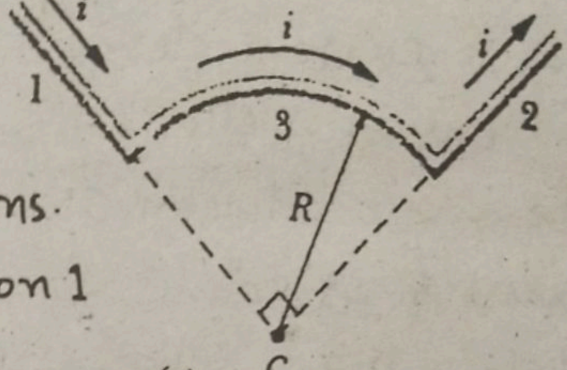
SOLUTION OF PROBLEMS

PROBLEM : 1

The wire in figure carries a current i and consists of a circular arc of radius R and central angle π/2rad. and two straight sections whose extensions intersect the centre Ç of the arc. What magnetic field B does the current produce at C?

SOLUTION :



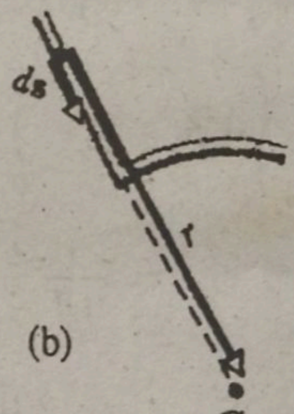
(a)

Let us divide the wire into three sections.

(i) The straight section 1 at left.

(ii) The straight section 2 at right.

(iii) The circular arc.



(b)

For section (1), the angle between any current element $\vec{ds}$ and $\vec{r}$ is zero, as in fig. (b).

The magnetic field dB can be calculated by the following relation.

$$dB = \frac{\mu_0}{4\pi} \times \frac{i\,ds \sin\theta}{r^2} \qquad \text{——— (1)}$$

$$\therefore \quad dB_1 = \frac{\mu_0}{4\pi} \times \frac{i\,ds \sin 0^\circ}{r^2} = 0$$

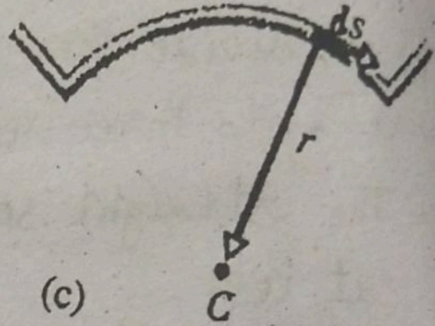
Thus the field along the entire length of section (1) at C is zero.

i.e. $B_1 = 0$ ——— (2).

Similary the field along the entire length of section (2) at C is zero, because the angle between any current element $\vec{ds}$ and $\vec{r}$ is 180°.

$\therefore \quad B_2 = 0$ ——— (3)

For a current element in curved section (3), the angle between $\vec{ds}$ and $\vec{r}$ is 90°.



(c)

Putting $R\,d\theta = dS$, and integrate from $\theta = 0$ to $\theta = \pi/2$ rad.

$$\int dB_3 = \int_0^{\pi/2} \frac{\mu_0 i}{4\pi}\frac{R\,d\theta}{R^2} = \int_0^{\pi/2} \frac{\mu_0 i}{4\pi}\frac{d\theta}{R}$$

or $$B_3 = \frac{\mu_0 i}{4\pi R}\int_0^{\pi/2} d\theta = \frac{\mu_0 i}{4\pi R}\Big|\theta\Big|_0^{\frac{\pi}{2}}$$

$$B_3 = \frac{\mu_0 i}{4\pi R}\left(\frac{\pi}{2} - 0\right) = \frac{\mu_0 i}{8R}$$

Thus the total magnetic field $\vec{B}$ produced at point C by the current in the wire has magnitude $B = B_1 + B_2 + B_3$

$$B = 0 + 0 + \frac{\mu_0 i}{8R} = \frac{\mu_0 i}{8R}$$

It points into the plane of the page.

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PROBLEM 2:

Show that equation $F_{ba} = \frac{\mu_0 L I_a I_b}{2\pi d}$ is consistent with the definition of the ampere. "The ampere is that constant current which, if maintained in two straight parallel conductors of infinite length, of negligible circular cross-section, and placed 1 meter apart in vacuum would produce on each of these conductor s a force equal to 2×10^{-7} newtons per meter of length.

SOLUTION : Let us put $i_a = i_b = 1\,A$

$d = 1\,m$ (from definition)

$\mu_0 = 4\pi \times 10^{-7}\, Wb/A \times m = 4\pi \times 10^{-7}\, T\text{-}m/A$

$$F_{ba} = \frac{\mu_0 L i_a i_b}{2\pi d} \quad (1\,Wb = 1\,T \times m^2)$$

or
$$\frac{F_{ba}}{L} = \frac{\mu_0 i_a i_b}{2\pi d}$$

$$= \frac{4\pi \times 10^{-7} \times 1 \times 1}{2\pi \times 1}$$

$$\frac{F_{ba}}{L} = 2 \times 10^{-7}\, N/m$$

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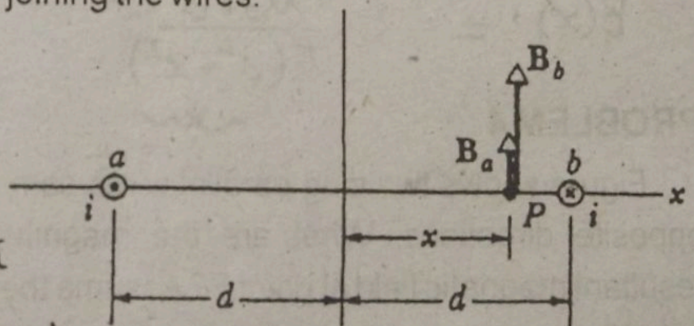
PROBLEM 3:

Two long parallel wires a distance 2d apart carry equal currents i in opposite directions, as shown in figure. Derive an expression for B(x), the magnitude of the resultant magnetic field for points at a distance x from the midpoint of a line joining the wires.

SOLUTION :



The right hand rule shows that the fields B_a and B_b set up by the currents in the wires a and b point in the same direction

for all points between the wires.
By using the relation, $B = \frac{\mu_0 i}{2\pi R}$, we can get $\vec{B}$ at point P between the wires.

$$B_a(x) = \frac{\mu_0 i}{2\pi(d+x)} \quad \text{———— (1)}$$

$$B_b(x) = \frac{\mu_0 i}{2\pi(d-x)} \quad \text{———— (2)}$$

Magnitude of resultant magnetic field is

$$B(x) = B_a(x) + B_b(x)$$

$$B(x) = \frac{\mu_0 i}{2\pi(d+x)} + \frac{\mu_0 i}{2\pi(d-x)}$$

$$= \frac{\mu_0 i}{2\pi}\left[\frac{1}{d+x} + \frac{1}{d-x}\right]$$

$$= \frac{\mu_0 i}{2\pi}\left[\frac{d-x+d+x}{(d+x)(d-x)}\right]$$

$$= \frac{\mu_0 i}{2\pi}\left[\frac{2d}{(d^2-x^2)}\right]$$

$$B(x) = \frac{\mu_0 i d}{\pi(d^2-x^2)}$$

PROBLEM 4:

or $$B = \left(\frac{\mu_0 i_0}{2\pi R^2}\right) r$$

Generally, now dropping the subscript of the current to generalize the result, we can write as

$$B = \left(\frac{\mu_0 i}{2\pi R^2}\right) r$$

where i now represents the full current with in the wire.

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PROBLEM 6:

A long straight wire of radius R=1.5mm carries a steady current 32A.

(a) What is the magnetic field at the surface of the wire?

(b) What is the magnetic field at r=1.2mm?

SOLUTION: (a)

$$R = 1.5\ \text{mm} = 1.5\times10^{-3}\ \text{m}$$

$$i = 32\ \text{A} \qquad B = ?$$

At the surface of the wire, $r = R$

$$\because\ B = \frac{\mu_0 i}{2\pi r} \qquad \mu_0 = 4\pi\times10^{-7}\ \text{T-m}$$

$$\therefore\ B = \frac{\mu_0 i}{2\pi R}$$

$$B = \frac{4\pi\times10^{-7}\times32}{2\pi\times1.5\times10^{-3}} = 4.27\times10^{-3}\ \text{T}$$

or $$B = 4.3\ \text{mT} \qquad (1\,\text{mT} = 10^{-3}\,\text{T})$$

(b) $$r = 1.2\ \text{mm} = 1.2\times10^{-3}\ \text{m}$$

Such points lie inside the wire. Magnetic field inside the wire is given by

$$B = \frac{\mu_0 i r}{2\pi R}$$

$$B = \frac{4\pi \times 10^{-7} \times 32 \times 1.2 \times 10^{-3}}{2\pi \times 1.5 \times 10^{-3}} = 3.41 \times 10^{-3} T$$

or $B = 3.41 \ mT$

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PROBLEM 7

A solenoid has length L=1.23m and inner diameter d=3.55 cm. It has five layers of windings of 850 turns each and carries a current $i_o = 5.57$ A. What is B at its center?

SOLUTION :

$$\mu_o = 4\pi \times 10^{-7} \ T-m/A$$

$L = 1.23 \ m \qquad d = 3.55 \ cm$

$i_o = 5.57 \ A \qquad N = 850 \text{ turns}$

Layers of windings $= 5$

$$n = \frac{\text{no. of turns}}{\text{Length}}$$

$$n = \frac{5 \times 850}{1.23} = \frac{4250}{1.23} = 3455.28$$

$B = ?$

$B = \mu_o i_o n$

$B = 4\pi \times 10^{-7} \times 5.57 \times 3455.28$

$B = 2.42 \times 10^{-2} \ T$

$B = 24.2 \ mT$

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PROBLEM 8 :

In the Bohr model of the hydrogen atom, the electron circulates around the nucleus in a path of radius 5.29×10^{-11} m at a frequency ν of 6.63×10^{15} Hz.

(a) What value of B is set up at the centre of the orbit?

(b) What is the equivalent magnetic dipole moment?

PROBLEM 9 :

A long wire carrying a current of 100 A is placed in a uniform external magnetic field of 5.0 mT. The wire is perpendicular to this magnetic field. Locate the points at which the resultant magnetic field is zero.

SOLUTION :

$$i = 100\,A$$

$$(1\,mT = 10^{-3}\,T)$$

$$B_{ext} = 5.0\,mT = 5.0 \times 10^{-3}\,T$$

$$\mu_o = 1.26 \times 10^{-6}\ T\text{-}m/A \qquad r = ?$$

$$B_{wire} = \frac{\mu_o i}{2\pi r} = B_{ext}$$

or

$$r = \frac{\mu_o i}{2\pi B_{ext}}$$

$$r = \frac{1.26 \times 10^{-6} \times 100}{2 \times 3.14 \times 5.0 \times 10^{-3}} = 4.0 \times 10^{-3}\ m$$

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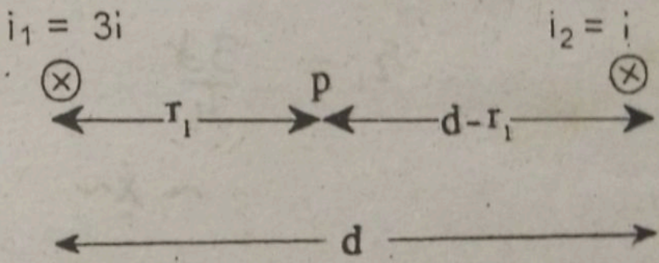
PROBLEM 10 :

Two long parallel wires a distance d apart carrying currents i and 3i in the same direction. Locate the point or points at which their magnetic fields cancel.

SOLUTION:



$$i_1 = 3i$$

$$i_2 = i$$

Separation between the wires $= d$

Let a point P at which total magnetic field $B_P = 0$

Magnetic field due to current i_1 is B_{P_1}

Now $B_{P_1} \propto \frac{i_1}{r_1}$

$$B_{P_1} = \frac{\mu_o}{2\pi} \times \frac{i_1}{r_1} \qquad \text{———— (1)}$$

Magnetic field due to current i_2 is B_{P_2}.

$$B_{P_2} \propto \frac{i_2}{(d-r_1)}$$

$$B_{P_2} = \frac{\mu_0}{2\pi} \times \frac{i_2}{(d-r_1)} \quad \text{———} (2)$$

Let $B_{P_1} = B_{P_2}$

$$\therefore \cancel{\frac{\mu_0}{2\pi}} \frac{i_1}{r_1} = \cancel{\frac{\mu_0}{2\pi}} \frac{i_2}{(d-r_1)}$$

$$i_1(d-r_1) = i_2 r_1$$

$$i_1 d - i_1 r_1 = i_2 r_1$$

$$i_1 d = i_2 r_1 + i_1 r_1$$

$$i_1 d = (i_1 + i_2) r_1$$

$$r_1 = \frac{i_1}{i_1 + i_2}$$

$$r_1 = \frac{3i\,d}{3i+i} = \frac{3i\,d}{4i}$$

$$r_1 = \frac{3d}{4}$$

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