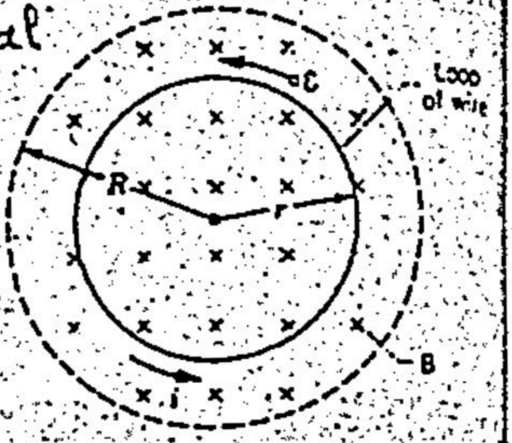
INDUCED ELECTRIC FIELDS :

Suppose we place a loop of wire (copper ring) of radius r in a uniform external magnetic field as shown in fig.(a). The field fills a cylinderical volume of radius R. The field, which we assume to have a uniform strength over the area of the loop, may be established by an external electromagnet.



(a)

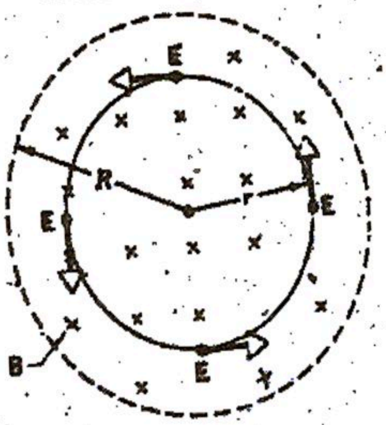
By varying the current in the electromagnet, we can vary the strength of the magnetic field. If the magnetic field increases at a steady rate, a constant current appears as shown in the loop of wire of radius r.

As $\vec{B}$ is varied, the magnetic flux through the loop varies with time and from Faraday's and Lenz's laws we can calculate the magnitude and direction of the induced emf and the induced current in the loop. The direction of induced current is counterclockwise in fig.(a).

If there is a current in the copper ring, an electric field must be present at various points with in the ring and it must have been produced by the changing magnetic flux. This induced electric field $\vec{E}$ is just as real as an electric field produced by static charges, it exerts a force $q_0\vec{E}$ on a test charge. Thus a changing magnetic field produce an electric field.

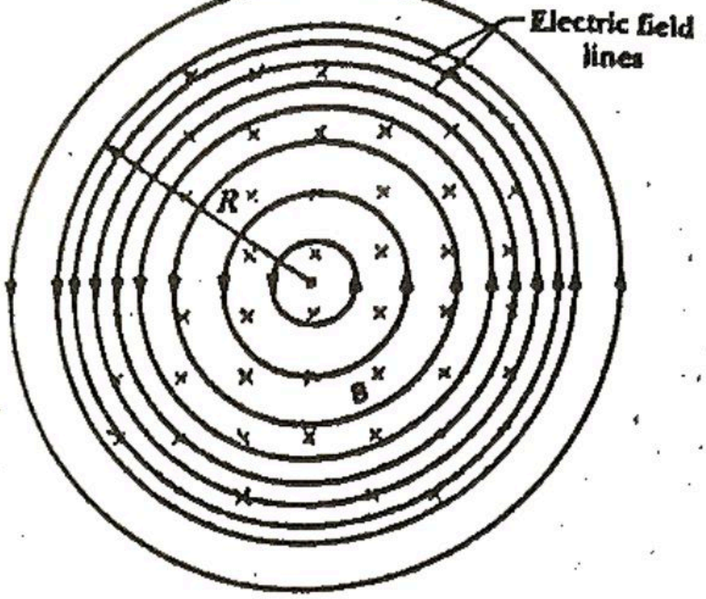
Let us replace the loop of wire with a hypothetical circular path of radius r. The electric fields induced at various points around the circular path must be tangent to the circle as shown in fig.(b).

Thus the electric field lines produced by changing magnetic field are concentric circles as shown in fig.(c)



(b)

If the magnetic field is decreasing with time, the electric field lines will still be concentric circles as in fig.(c) but the direction is opposite



(c)

A REFORMULATION OF FARADAY'S LAW :

Consider a test charge q_0 moving around the circular path of fig. (b). The work 'w' done on it in one revolution by the electric field is given by

$$\mathcal{E} = \frac{w}{q_0}$$

or $$w = \mathcal{E} q_0 \qquad \text{------- (1)}$$

where $\mathcal{E}$ = induced emf.

Equivalently, we can express the work as

$$w = \int \vec{F} \cdot \vec{ds} = \int F\,dS = F \int ds$$

$$w = q_0 E (2\pi r) \qquad \text{------- (2)}$$

where $q_0 E$ = magnitude of electric force acting on the test charge.

$2\pi r$ = distance covered in one revolution.

Comparing equation (1) and (2)

$$\mathcal{E} q_0 = q_0 E (2\pi r)$$

$$\mathcal{E} = E(2\pi r) \qquad \text{——— (3)}$$

The right side of equation (3) can be expressed as a line integral of $\vec{E}$ around the circle, which can be written in more general cases (for instant, when E is not constant or when the path is not a circle) as

$$\mathcal{E} = \oint \vec{E} \cdot \vec{ds} \qquad \text{——— (4)}$$

where $\vec{E}$ = electric field induced by a changing magnetic flux.

$\vec{ds}$ = differential (small) length vector along the path.

According to Faraday's law of induction

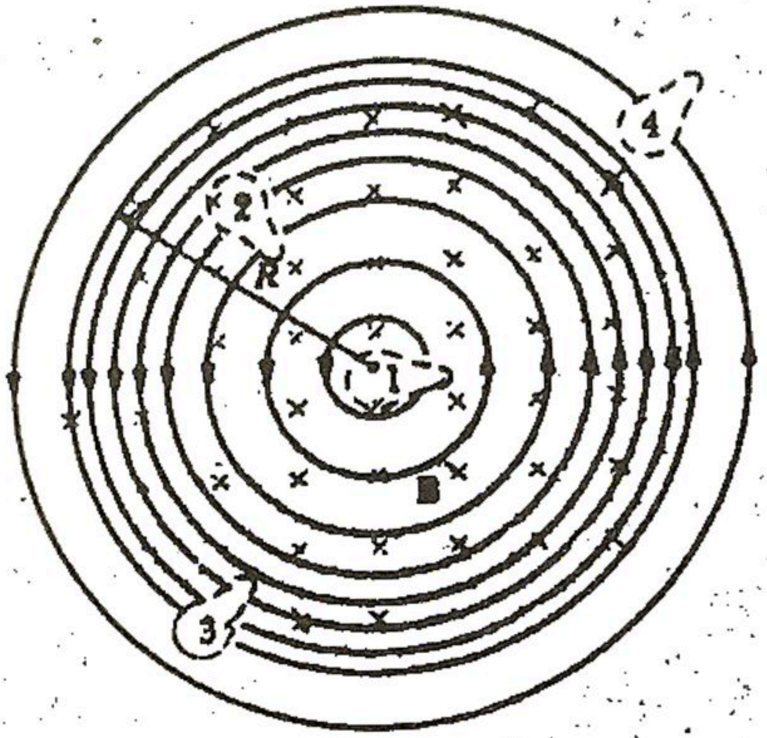
$$\mathcal{E} = -\frac{d\phi_B}{dt} \qquad \text{——— (5)}$$

From equation (4) and (5)

$$\oint \vec{E} \cdot \vec{ds} = -\frac{d\phi_B}{dt}$$

This result says that a changing magnetic field induces an electric field.

Figure (d) shows four similar closed paths that en[illegible] identical areas.



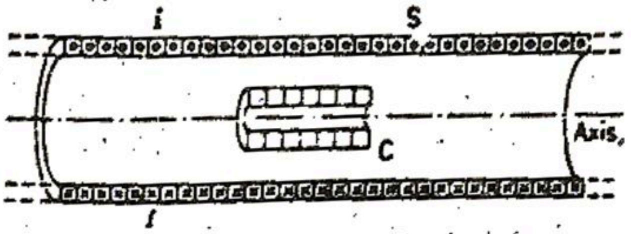
(d)

Equal emfs are induced around paths 1 and 2, which lie entirely with in the region of changing magnetic field. A smaller emf is induced around path 3, which only partially in that region. No emf is induced around path 4, which lies entirely out side the magnetic field.

SOLUTION OF PROBLEMS

PROBLEM : 1

The long solenoid S of figure has 220 turns/cm and carries a current i=1.5A, its diameter d is 3.2 cm. At its center we place a 130 turn closed-packed coil C of diameter dc=2.1cm. The current in the solenoid is increased from zero to 1.5A at a steady rate over a period of 0.16s. what is the absolute value (that is, the magnitude without regard for sign) of the induced emf that appears in the central coil while the current in the solenoid is being changed.



SOLUTION :

The magnitude of the final flux through each turn of this coil is

$$\phi_B = BA \quad \text{——— (1)}$$

The magnetic field B at the center of the solenoid is given by

$$B = \mu_0 i n$$

$\mu_0 = 4\pi \times 10^{-7}\ T\text{-}m/A$ $\quad i = 1.5\ A$

$n = 220$ turns/cm $\quad d_c = 2.1\,cm = 2.1 \times 10^{-2}\ m.$

or $n = 220 \times 100$ turns/m

$$\therefore B = 4\pi \times 10^{-7} \times 1.5 \times 220 \times 100$$

$$B = 4.15 \times 10^{-2}\ T \quad \text{——— (2)}$$

The area of coil C (not of solenoid) = A

$$A = \frac{1}{4}\pi d_c^2$$

$$A = \frac{1}{4} \times 3.14 \times (2.1 \times 10^{-2})^2 = 3.46 \times 10^{-4}\ m^2$$

——— (3)

Using equation (2) and (3) in equation (1).

$$\phi_B = 4.15 \times 10^{-2} \times 3.46 \times 10^{-4}$$

$$= 1.44 \times 10^{-5}\ Wb \quad (1\mu Wb = 10^{-6} Wb)$$

or $\phi_B = 14.4\ \mu Wb$.

we know that $\mathcal{E} = N \dfrac{\Delta \phi_B}{\Delta t}$

N = number of turns in the inner coil C = 130

Note: we ignore the minus sign because we are taking absolute value (magnitude) of the induced emf.

$\Delta \phi_B = 14.4 \times 10^{-6}\ Wb$, $\Delta t = 0.16\ s$

$$\mathcal{E} = \frac{130 \times 14.4 \times 10^{-6}}{0.16} = 1.2 \times 10^{-2}\ V$$

or $\mathcal{E} = 12\ mV$

PROBLEM : 2

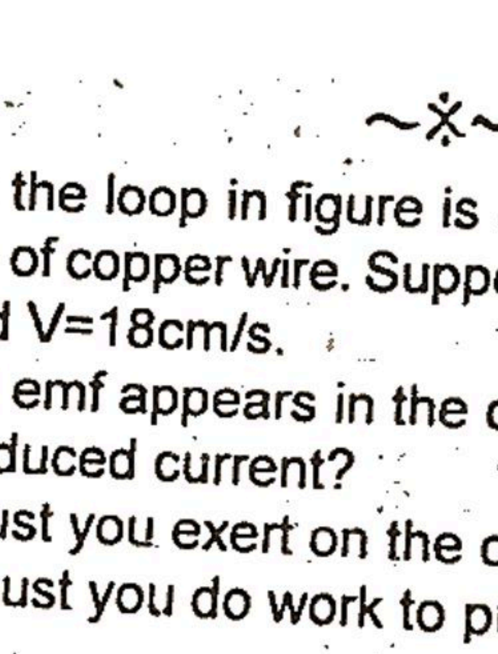
Suppose that the loop in figure is actually a tightly woun coil of 85 turns, made of copper wire. Suppose further that L=13cm B=1.5T, R=6.2 Ω and V=18cm/s.

(a) What induced emf appears in the coil?

(b) What is the induced current?

(c) What force must you exert on the coil to pull it along?

(d) At what rate must you do work to pull the coil along?



SOLUTION: $N = 85$ turns

$L = 13\,cm = 0.13\,m$

$B = 1.5\,T$; $R = 6.2\,\Omega$,

$V = 18\,cm/s = 0.18\,m/s$

(a) Induced emf $= \mathcal{E} = ?$

$$\mathcal{E} = NBLV$$

$$= 85 \times 1.5 \times 0.13 \times 0.18 = 2.98V$$

(b) $i = ?$

$$i = \frac{\mathcal{E}}{R}$$

$$i = \frac{2.98}{6.2} = 0.48\,A$$

(c) $F = ?$

$$F = NiLB$$

$$F = 85 \times 0.48 \times 0.13 \times 1.5 = 8.0\,N$$

(d) Rate of work = Power = $P = ?$

$$P = FV$$

$$P = 8.0 \times 0.18 = 1.4\,W$$

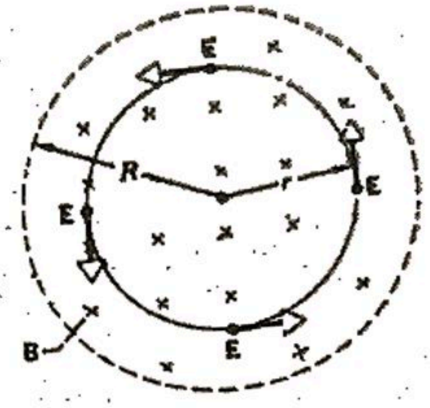
~x~

PROBLEM : 4

In figure, assume that R=8.5cm and that dB/dt=0.13 T/s.

(a) What is the magnitude of the electric field $\vec{E}$ for r=5.2cm?

(b) What is the magnitude of the induced electric field for r=12.5 cm?



SOLUTION: $R = 8.5\ cm$

or $R = 8.5 \times 10^{-2}\ m$

$\frac{dB}{dt} = 0.13\ T/s$

(a) $E = ?$ $r = 5.2\ cm$

or $r = 5.2 \times 10^{-2}\ m$

Faraday's law gives

$$E(2\pi r) = -\frac{d\phi_B}{dt} \quad \text{———— (1)}$$

As $r < R$

The flux ϕ_B through a closed path of radius r is $\phi_B = BA = B(\pi r^2)$ Putting in equation (1)

$$E(2\pi r) = -\frac{d}{dt}(B\pi r^2)$$

$$E(2\pi r) = -\pi r^2 \frac{dB}{dt}$$ Taking magnitude of E

$$\text{or} \quad E = \frac{r}{2}\frac{dB}{dt} \quad \text{———— (2)}$$

$$E = \frac{5.2 \times 10^{-2}}{2} \times 0.13 = 0.0034\ V/m$$

or $E = 3.4\ mV/m$ $(1\ mV = 10^{-3}\ V)$

(b) $r = 12.5\ cm = 12.5 \times 10^{-2}\ m$

$\frac{dB}{dt} = 0.13\ T/s$

In this case, $r > R$, So that the electric flux through the circular path.

Thus $\phi_B = B(\pi R^2)$ ——— (3)

From Faraday's law,

$$E(2\pi r) = -\frac{d\phi_B}{dt} \quad \text{——— (4)}$$

Using equation (3)

$$\therefore \quad E(2\pi r) = -\frac{d}{dt}(B\pi R^2)$$

$$E(2\pi r) = -\pi R^2 \frac{dB}{dt}$$

or $$E = -\frac{1}{2r} R^2 \frac{dB}{dt}$$

Again taking magnitude.

$$E = \frac{1}{2r} R^2 \frac{dB}{dt}$$

$$E = \frac{1}{2 \times 12.5 \times 10^{-2}} \times (8.5 \times 10^{-2})^2 \times 0.13$$

$$E = 3.8 \times 10^{-3} \ V/m \qquad (\because 1mV = 10^{-3} V)$$

or $$E = 3.8 \ mV/m$$