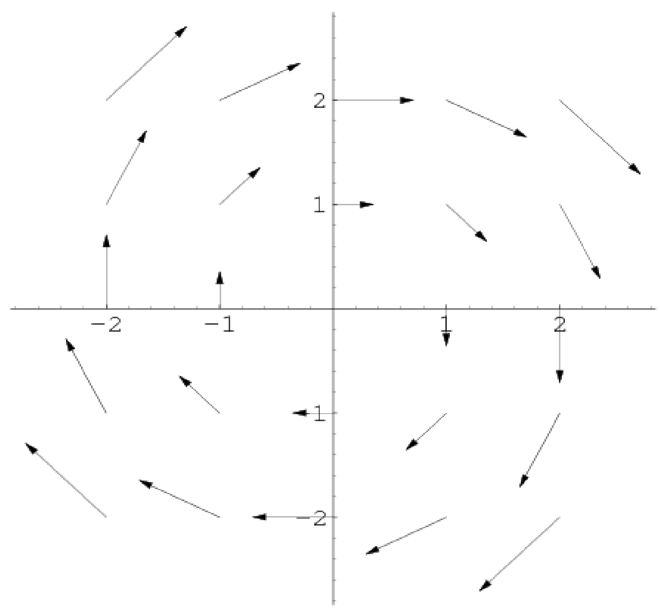
**Scalar Field:**
The field that takes a single scalar value at each point in a space is called scalar field. For example Temperature field is a scalar filed. Each point in the room has a temperature but different points may have different temperatures.
**Vector Field:**

A vector field associates a vector with each point in space. Some common vector fields are given

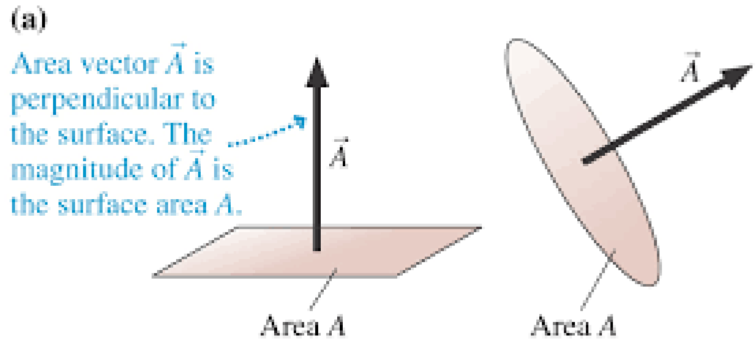
- **Electric Fields**
- **Magnetic Fields**
- **Gravitational Fields**
- **Wind Velocity**
- **Fluid Velocity**






Vector fields as fluid flow

**Vector Area:**
It is an area whose magnitude is equal to the surface area A of the element but its direction is normal to this area.






**Electric Flux**

As you have just seen, we can think of an electrical charge as having a number of electric field lines, either converging on it or diverging from it, that is proportional to the magnitude of its charge. We can now explore the mathematics of enclosing charges with surfaces and seeing how many electric field lines pass through a given surface. Electric flux is defined as a measure of the number of electric field lines passing through a surface.

It should be obvious that the number of field lines passing through a surface depends on how that surface is oriented relative to the lines. The orientation of a small surface of area A is usually defined as a vector which is perpendicular to the surface and has a magnitude equal to the surface area. By convention, the normal vector points away from the outside of the surface.

By convention, if an electric field line passes from theinside to the outside of a surface we say the flux ispositive. If the field line passes from the outside to the inside of a surface, the flux is negative.

**A Mathematical Representation of Flux through a Surface:**

One convenient way to express the relationship between angle and flux for a uniform electric field is to use the dot product so that.

$$\Phi_e = \vec{E}.\vec{A}$$

Flux is a scalar. Its unit is $Nm^2/C$. If the electric field is not uniform or if the surface subtends different angles with respect to the electric field lines, then we must calculate the flux by breaking the surface into infinitely many infinitesimal areas so that

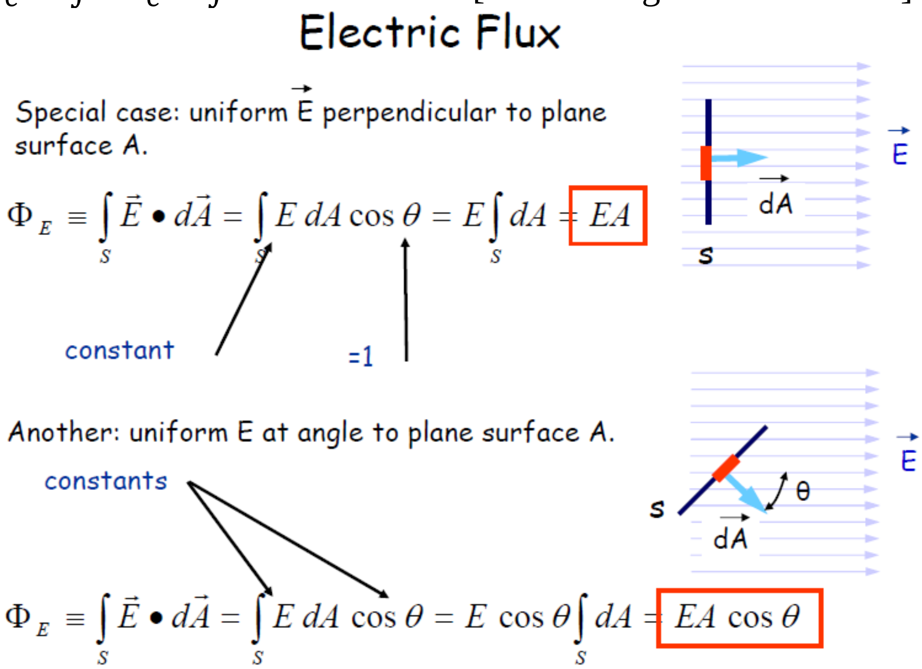
$$d\Phi_e = \vec{E}.d\vec{A}$$

and then taking the integral sum of all of the flux elements. This gives

$\Phi_e = \int d\Phi_e = \int \vec{E}.d\vec{A}$

Some surfaces, like that of a sphere or that representing a rectangular box, are closed surfaces, i.e. they have no holes or breaks in them. Because we want to study the amount of flux passing through closed surfaces, there is a special notation to represent the integral of through a closed surface. It is represented as follows:

$\Phi_e = \oint d\Phi_e = \oint \vec{E}.d\vec{A}$ [Flux through closed surface]




## Fundamental Laws of Electrostatics:

**(i) Coulomb's Law**

Force between two point charges

**(ii) GAUSS'S LAW**

How is the flux passing through a closed surface related to the enclosed charge?

## <u>GAUSS' LAW</u>

The net electric flux through any closed surface is proportional to the charge enclosed by that surface.

$\Phi_e = \oint d\Phi_e = \oint \vec{E}.d\vec{A} = \frac{q_{enclosed}}{\varepsilon_0}$

where $\varepsilon_0$ is the *permittivity of free space* ($\varepsilon_0$= 8.85 x 10-12 C2/(Nm2).

**How do we use this equation??**

•The above equation is ALWAYS TRUE but it may not be easy to use.

•It is very useful in finding **E** when the physical situation exhibits massive SYMMETRY.

By applying Gauss' law you can calculate the electric field intensity **E** from *symmetric* charge distributions. What is a "symmetric" distribution? It's an arrangement of charges that can be rotated about an axis and/or reflected in a mirror and still look the same.

Following steps must be followed for calculating the electric field intensity:

- Draw an imaginary closed surface which passes through the point at which the electric field intensity is to be calculated. This closed surface is known as Gaussian surface.
- Calculate the electric flux enclosed by the Gaussian surface $\Phi_e = \oint d\Phi_e = \oint \vec{E}.d\vec{A}$.

Calculate the electric flux using Gauss's law $\Phi_e = \oint d\Phi_e = \oint \vec{E}.d\vec{A} = \frac{q_{enclosed}}{\varepsilon_0}$.

- Comapre both the electric fluc to calculate the electric intensity **E.**

GAUSSIAN SURFACE:

A Gaussian surface is an imaginary closed surface of arbitrary shape which passes through the point where we want to calculate the electric intensity.

**Faraday's Cage: An Important Application of Gauss' Law**

Gauss' Law has an important practical application. Imagine a solid conducting object which may be in a strong electric field. If all excess charge is on the surface of the conductor and if there is no electric field inside the conductor, then it should be possible to remove any conducting material from the interior without changing the location of the charges or the electric field inside. This will leave a hollow metallic shell which has no electric field inside it. There is no electric field inside even if there is a strong field outside: the conducting shell acts as a "shield" against electric fields.. Such a space surrounded by a conductor is called a "Faraday cage". You can test the Faraday cage with the following

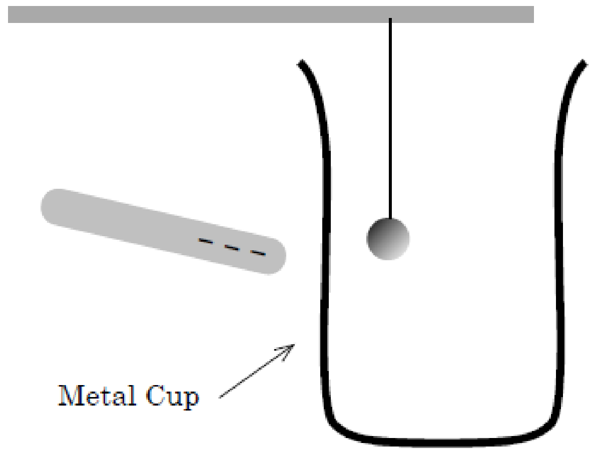
•A metal cup

•A hard plastic rod and fur

•A metallic ball suspended by a thread

**Experimental Verification of Faraday's Cage**

Hold the metal cup with your hand and place around the ball with the ball near but not touching the metal. If the ball is far below the lip of the cup then the metal cup effectively shields the ball from the electric field created by the rod. Try to attract the ball with a charged rod outside the cup. Does the electric field of the rod reach inside the metal cup?

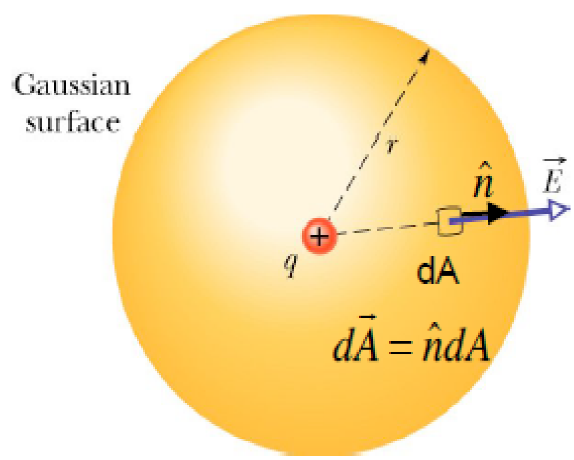



Electric lines of flux and Derivation of Gauss' Law using Coulombs law

- Consider a sphere drawn around a positive point charge. Evaluate the net flux through the closed surface.

Net Flux = $\Phi = \oint \vec{E} \cdot d\vec{A} = \oint E \cos\theta dA = \oint EdA$  $E \parallel n$  $\cos 0^\circ = 1$

For a Point charge $E = \dfrac{kq}{r^2}$




$$\Phi = \oint EdA = \oint \frac{kq}{r^2} dA$$

$$\Phi = \frac{kq}{r^2} \oint dA = \frac{kq}{r^2}(4\pi r^2)$$

$$\Phi = 4\pi kq$$

$4\pi k = \dfrac{1}{\varepsilon_0}$ where $\varepsilon_0 = 8.85\text{x}10^{-12} \dfrac{C^2}{Nm^2}$

$$\boxed{\Phi_{net} = \frac{q_{enc}}{\varepsilon_0}}$$ Gauss' Law

Home work calculate electric flux due to negative charge.