Practice Problems: Applications of Gauss's Law Solutions

1. (easy) Determine the electric flux for a Gaussian surface that contains 100 million electrons.

$\Phi = q/\varepsilon_o$

$\Phi = 100\text{x}10^6(1.6\text{x}10^{-19})/8.85\text{x}10^{-12}$

$\Phi = 1.8\ \text{Nm}^2/\text{C}$

2. (easy) A uniformly charged solid spherical insulator has a radius of 0.23 m. The total charge in the volume is 3.2 pC. Find the E-field at a position of 0.14 m from the center of the sphere.

$E = [q/4\pi\varepsilon_o R^3]r$

$E = [3.2\text{x}10^{-12}/(4\pi\varepsilon_o(0.23)^3](0.14) = 0.331\ \text{N/C}$

3. (easy) An insulating plane of charge (Q = 0.5 C) measures 20 m by 30 m. While this is a large plane it is finite in size.

a) If one were to measure the E-field at a distance of 0.01 m from the plane, how would the magnitude compare to $E = \sigma/2\varepsilon_o$?

b) If one were to measure the E-field at a distance of 100 m from the plane, how would the magnitude compare to $E = \sigma/2\varepsilon_o$?

c) If one were to measure the E-field at a distance of $1\text{x}10^{20}$ m from the plane, how would the magnitude compare to $E = \sigma/2\varepsilon_o$?

a) Even though the plane is of finite size, at points very near the plane the E-field magnitude will be approximately equal to $\sigma/2\varepsilon_o$.

b) At point not near the plane (such as at 100 m) the E-field will be less than $\sigma/2\varepsilon_o$.

c) The E-field will decrease with distance from the plane. At $1\text{x}10^{20}$ m the field will be close to zero.

4. (easy) Two extremely large insulating planes each hold 1.8 C of excess charge. One plane is charged negatively and the other is charged positively. The planes are separated by a very small distance so that a uniform E-field is set up between them. Each plane is 1000 m wide and 1000 m long. Detremine the magnitude of the E-field in between the planes and outside the planes.

$E_{inside} = \sigma/\varepsilon_o = (Q/A)/\varepsilon_o$

$E_{inside} = (1.8/1000^2)/8.85\text{x}10^{-12}$

$E_{inside} = 2.03\text{x}10^5\ \text{N/C}$

$E_{outside} = 0$ (The E-fields from each plane cancel out)

5. (easy) An infinitely long line of charge carries 0.4 C along each meter of length. Find the E-field 0.3 m from the line of charge.

$E = \lambda/2\pi\varepsilon_o r$

$E = (Q/L)/2\pi\varepsilon_o r$

$E = (0.4/1)/(2\pi\varepsilon_o(0.3))$

$E = 2.4\text{x}10^{10}\ \text{N/C}$

6. (moderate) Two very long lines of charge are parallel to each other, separated by a distance x. They each have the same linear charge density. One is positive and the other is negative.

a) What is the magnitude of the E-field at a point half-way between the lines of charge?

b) How does the E-field at a point x/3 from the the the positive charge line (and 2x/3 from the negative charge line) compare to the E-field x/3 from the negative charge line (and 2x/3 from the positive charge line).

a) Each line would contribute to the E-field equally and in the same direction.

$E = 2(\lambda/2\pi\varepsilon_o r) = \lambda/\pi\varepsilon_o(x/2) = 2\lambda/\pi\varepsilon_o x$

b) Each point will have the same magnitude and direction for the E-field.

$E = E_+ + E_- = (\lambda/2\pi\varepsilon_o(x/3)) + (\lambda/2\pi\varepsilon_o(2x/3))$

$E = (3\lambda/2\pi\varepsilon_o x) + (3\lambda/4\pi\varepsilon_o x)$

$E = 9\lambda/4\pi\varepsilon_o x$ (magnitude)

7. (moderate) Repeat question #6 but with two positive lines of charge.

a) Each line would contribute to the E-field equally but in the opposite direction.

$E = (\lambda/2\pi\varepsilon_o(x/2) - \lambda/2\pi\varepsilon_o(x/2) = 0$

b) Each point will have the same magnitude but opposite direction for the E-field.

$E = E_1 - E_2 = (\lambda/2\pi\varepsilon_o(x/3)) - (\lambda/2\pi\varepsilon_o(2x/3))$

$E = (3\lambda/2\pi\varepsilon_o x) - (3\lambda/4\pi\varepsilon_o x)$

$E = 3\lambda/4\pi\varepsilon_o x$ (magnitude)

8. (moderate) A soccer goal, found is a city park, is made of tubing that supports an odd-shaped hanging net behind the goal, but has a rectangular opening in front. The height of the opening is 2.5 m and the width is 3.2 m. If a uniform E-field, with a mangnitude of 0.1 N/C, passes through the goal from the front to the back, entering at 90º to the plane of the goal opening, what is the flux through the net? Also, find the flux through the net if the E-field enters the goal at a 60° angle to the plane of the front of the goal. In both cases, assume that there is no charge found inside the goal itself.

No charge inside implies no total flux.

$\Phi_{total} = 0 = \Phi_{net} + \Phi_{front}$

$0 = \Phi_{net} + EAcos180$

$\Phi_{net} = - 0.1(2.5)(3.2)cos180 = 0.8\ Nm^2/C$

For part 2, the angle between the E-Field and the Area vector would be 30°.

$\Phi_{net} = -EAcos150 = -0.1(2.5)(3.2)cos150 = 0.7\ Nm^2/C$

9. (moderate) A cubic space (1.5 m on each side) contains positively charged particles. Imagine that the space is surrounded by a Gaussian surface of the exact same dimension as the cube and that the E-Field caused by the charges is normal to the faces of the Gaussian cube. If the E-field at each surface has a magnitude of 760 N/C, determine the number of charges per unit volume in the space described (ie., find the charge density,ρ).

$\Phi_{net} = EAcos0 = q/\varepsilon_o$

$760(6)(1.5)^2 = q/8.85x10^{-12}$

$q = 9.1x10^{-8}C$

Now find the volume of the cube:

$V = (1.5)^3 = 3.375\ m^3$

Finally, determine the charge density:

$\rho = q/V = 9.1x10^{-8}/3.375 = 2.7x10^{-8}C/m^3$