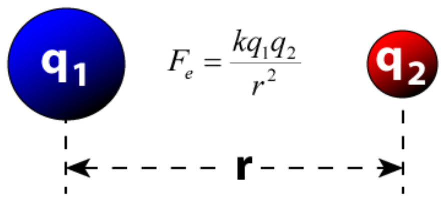
COULOMB'S LAW

(INVERSE SQUARE LAW)

The electrical force, like gravitational force, decrease inversely as the square of the distance between charged bodies. This relationship was discovered by Charles-Augustin de Coulomb, whose experiments in 1785 led him to it and called Coulomb's law. It states that, for two charged objects that are much smaller than the distance between them (Point charges), the magnitude of electrostatic force between the point charges is directly proportional to the product of charges and inversely proportional to the square of the distance between them.



i.e.

$$F \propto q_1 q_2 \qquad \text{(i)}$$

$$F \propto \frac{1}{r^2} \qquad \text{(ii)}$$

By combining (i) and (ii) we get

$$F \propto \frac{q_1 q_2}{r^2}$$

$$F = K\frac{q_1 q_2}{r^2} \qquad \text{(iii)}$$

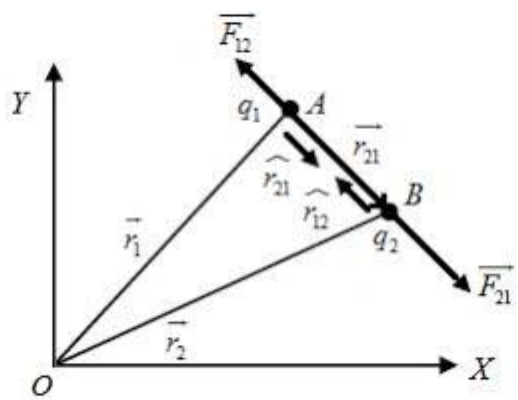
Important Points Regarding Coulomb's Law

1. It is applicable only for point charges.
2. The constant of proportionality K in SI units in vacuum is expressed as$\frac{1}{4\pi\varepsilon_0}$. If charges are dipped in a medium, then electrostatics force on one charge is$F = \frac{1}{4\pi\varepsilon_0\varepsilon_r}\frac{q_1q_2}{r^2}$.
3. ε_0 *and* ε are called permittivity of free space (vacuum) and absolute permittivity of the medium, respectively. The ratio $\frac{\varepsilon}{\varepsilon_0} = \varepsilon_r$ is called relative permittivity of the medium, which is dimensionless quantity. The value of ε_0 is $8.85x10^{-12}\ C^2/N\ m^2$
4. The value of relative permittivity ε_r varies between 1 and ∞. For vacuum, by definition it is equal to 1. For air, it is nearly equal to 1 and may be taken to be equal to 1 for calculations. For metals the value of ε_r is∞.
5. The value of $\frac{1}{4\pi\varepsilon_0} = \frac{1}{4(3.14)(8.85x10^{-12})} = 0.00899636547x10^{12} = 9x10^9 N\ m^2/C^2$

6. The force acting on one point charge due to the other point charge is always along the line joining these two charges. It is equal in magnitude and opposite in direction on two charges, irrespective of the medium, in which they lie.
7. The force is conservative in nature, i.e. work done by electrostatic force in moving a point charge along a close loop of any shape is zero.
8. Since the force is a central force, in the absence of any other external force, angular momentum of one particle with respect to the other particle is conserved.
9. The electrostatic force is a two body interaction, i.e., electrostatic force between two point charges is independent of presence or absence of other charges and so the principle of superposition is valid.

The Vector Form of Coulomb's Law

Force, being a vector, has both magnitude and direction. In the case of Coulomb's law, the direction of the force is determined by the relative sign of the two electric charges. The concept of position vector can be used to make calculations of coulomb's force more explicit.



As illustrated in above fig, suppose we have two pint charges $\boldsymbol{q_{1\,and}\,q_2}$ separated by a distance $\vec{\boldsymbol{r}}_{\boldsymbol{21}}$. For the measurement, we assume the two charges to have the same sign, so that they repel one another. Let us consider the force on charge $\boldsymbol{q_1}$ exerted by charge$\boldsymbol{q_2}$, which we write in our usual form as$\vec{\boldsymbol{F}}_{\boldsymbol{12}}$. $\hat{\boldsymbol{r}}_{\boldsymbol{21}}$ is the unit vector which represents the direction from $\boldsymbol{q_2}$ to $\boldsymbol{q_1}$. So, we can represent the force as

$$\vec{\boldsymbol{F}}_{\boldsymbol{12}} = \boldsymbol{K}\frac{\boldsymbol{q_1 q_2}}{\boldsymbol{r_{21}^2}}\,\hat{\boldsymbol{r}}_{\boldsymbol{21}} \qquad \text{(iv)}$$

From fig

$$\vec{\boldsymbol{r}}_{\boldsymbol{1}} + \vec{\boldsymbol{r}}_{\boldsymbol{21}} = \vec{\boldsymbol{r}}_{\boldsymbol{2}}$$

$$\vec{\boldsymbol{r}}_{\boldsymbol{21}} = \vec{\boldsymbol{r}}_{\boldsymbol{2}} - \vec{\boldsymbol{r}}_{\boldsymbol{1}}$$

So eq 6 become

$$\vec{\boldsymbol{F}}_{\boldsymbol{12}} = \boldsymbol{K}\frac{\boldsymbol{q_1 q_2}}{\boldsymbol{r_{21}^2}}\,\frac{\vec{\boldsymbol{r}}_{\boldsymbol{21}}}{\boldsymbol{r_{21}}} \qquad \therefore\, \hat{\boldsymbol{r}}_{\boldsymbol{21}} = \frac{\vec{\boldsymbol{r}}_{\boldsymbol{21}}}{\boldsymbol{r_{21}}}$$

$$\vec{F}_{12} = K\frac{q_1 q_2}{r_{21}^3}\,\vec{r}_2 - \vec{r}_1$$

$$\vec{F}_{12} = K\frac{q_1 q_2}{|\vec{r}_2 - \vec{r}_1|^3}\,\vec{r}_2 - \vec{r}_1 \qquad \text{(v)}$$

Similarly if the force on charge q_2 exerted by chargeq_1, which we write in our usual form as$\vec{F}_{21}$. $\hat{r}_{12}$ is the unit vector which represents the direction from q_2 to q_1. So, we can represent the force a

$$\vec{F}_{21} = K\frac{q_1 q_2}{r_{12}^2}\,\hat{r}_{12}$$

or

$$\vec{F}_{21} = K\frac{q_1 q_2}{|\vec{r}_1 - \vec{r}_2|^3}\,(\vec{r}_1 - \vec{r}_2) \qquad \text{(vi)}$$

Since

$$\vec{r}_{12} = -\vec{r}_{21}$$

$$\vec{r}_1 - \vec{r}_2 = -(\vec{r}_2 - \vec{r}_1)$$

So above equation become

$$\vec{F}_{21} = K\frac{q_1 q_2}{|\vec{r}_1 - \vec{r}_2|^3}\,(-(\vec{r}_2 - \vec{r}_1))$$

$$\vec{F}_{21} = K\frac{q_1 q_2}{|\vec{r}_1 - \vec{r}_2|^3}\,(\vec{r}_2 - \vec{r}_1)$$

From eq (vi)

$$\vec{F}_{21} = -\vec{F}_{12}$$

So Coulomb’s Law is accordance to Newton’s third law