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New Potential Functions for Mobile Robot Path Planning

S. S. Ge and Y. J. Cui

Abstract—This paper first describes the problem of goals nonreachable with obstacles nearby when using potential field methods for mobile robot path planning. Then, new repulsive potential functions are presented by taking the relative distance between the robot and the goal into consideration, which ensures that the goal position is the global minimum of the total potential.

I. INTRODUCTION

The potential field method has been studied extensively for autonomous mobile robot path planning in the past decade [1]–[16]. The basic concept of the potential field method is to fill the robot's workspace with an artificial potential field in which the robot is attracted to its goal position and is repulsed away from the obstacles [1]. This method is particularly attractive because of its mathematical elegance and simplicity. However, it has some inherent limitations. A systematic criticism of the inherent problems based on mathematical analysis was presented in [3], which includes the following: 1) trap situations due to local minima; 2) no passage between closely spaced obstacles; 3) oscillations in the presence of obstacles; and 4) oscillations in narrow passages. Besides the four problems mentioned above, there exists an additional problem, goals nonreachable with

obstacles nearby (GNRON). In most of the previous studies, the goal position is set relatively far away from obstacles. In these cases, when the robot is near its goal position, the repulsive force due to obstacles is negligible, and the robot will be attracted to the goal position by the attractive force. However, in many real-life implementations, the goal position needs to be quite close to an obstacle. In such cases, when the robot approaches its goal, it also approaches the obstacle nearby. If the attractive and repulsive potentials are defined as commonly used [2]–[4], the repulsive force will be much larger than the attractive force, and the goal position is not the global minimum of the total potential. Therefore, the robot cannot reach its goal due to the obstacle nearby.

To overcome this problem, the repulsive potential functions for path planning are modified by taking into account the relative distance between the robot and the goal. The new repulsive potential function ensures that the total potential has a global minimum at the goal position. Therefore, the robot will reach the goal finally. Note that we are not trying to tackle the common local minima problems due to obstacles between the robot and the goal. We shall restrict our attention to the formulation and solution of the GNRON problem only.

This paper is organized as follows. In Section II, the cause of the GNRON problem is analyzed after the introduction of the potential field methods. Section III presents the new repulsive potential function and its properties. In Section IV, the relationship between scaling parameters of the potential functions is presented. In Section V, safety issues of the new potential functions are discussed, and a control system directly making use of the new potentials is also suggested. Simulation results are presented in Section VI to show the problems of the conventional potential field method and the effectiveness of the new method.

II. POTENTIAL FIELD METHOD AND GNRON PROBLEM

For simplicity, we assume that the robot is of point mass and moves in a two-dimensional (2-D) workspace. Its position in the workspace is denoted by $\mathbf{q} = \begin{bmatrix} x & y \end{bmatrix}^T$.

Different potential functions have been proposed in the literature. The most commonly used attractive potential takes the form [1]–[3]

$$U_{\mathrm{att}}(\mathbf{q}) = \frac{1}{2}\,\xi\rho^m(\mathbf{q}, \mathbf{q}_{\mathrm{goal}}) \tag{1}$$

where ξ is a positive scaling factor, $\rho(\mathbf{q}, \mathbf{q}_{\mathrm{goal}}) = \|\mathbf{q}_{\mathrm{goal}} - \mathbf{q}\|$ is the distance between the robot $\mathbf{q}$ and the goal $\mathbf{q}_{\mathrm{goal}}$, and $m = 1$ or 2. For $m = 1$, the attractive potential is conic in shape and the resulting attractive force has constant amplitude except at the goal, where U_{att} is singular. For $m = 2$, the attractive potential is parabolic in shape. The corresponding attractive force is then given by the negative gradient of the attractive potential

$$\mathbf{F}_{\mathrm{att}}(\mathbf{q}) = -\nabla U_{\mathrm{att}}(\mathbf{q}) = \xi(\mathbf{q}_{\mathrm{goal}} - \mathbf{q}) \tag{2}$$

which converges linearly toward zero as the robot approaches the goal. One commonly used repulsive potential function takes the following form [1]:

$$U_{\mathrm{rep}}(\mathbf{q}) = \begin{cases} \dfrac{1}{2}\,\eta\left(\dfrac{1}{\rho(\mathbf{q}, \mathbf{q}_{\mathrm{obs}})} - \dfrac{1}{\rho_0}\right)^2, & \text{if } \rho(\mathbf{q}, \mathbf{q}_{\mathrm{obs}}) \leq \rho_0 \\ 0, & \text{if } \rho(\mathbf{q}, \mathbf{q}_{\mathrm{obs}}) > \rho_0 \end{cases} \tag{3}$$

where η is a positive scaling factor, $\rho(\mathbf{q}, \mathbf{q}_{\mathrm{obs}})$ denotes the minimal distance from the robot $\mathbf{q}$ to the obstacle, $\mathbf{q}_{\mathrm{obs}}$ denotes the point on the

obstacle such that the distance between this point and the robot is minimal between the obstacle and the robot, and ρ_0 is a positive constant denoting the distance of influence of the obstacle. The corresponding repulsive force is given by

$$\mathbf{F}_{\mathrm{rep}}(\mathbf{q}) = -\nabla U_{\mathrm{rep}}(\mathbf{q})$$

$$= \begin{cases} \eta \left(\dfrac{1}{\rho(\mathbf{q},\, \mathbf{q}_{\mathrm{obs}})} - \dfrac{1}{\rho_0} \right) \\ \quad \dfrac{1}{\rho^2(\mathbf{q},\, \mathbf{q}_{\mathrm{obs}})} \nabla \rho(\mathbf{q},\, \mathbf{q}_{\mathrm{obs}}), & \text{if } \rho(\mathbf{q},\, \mathbf{q}_{\mathrm{obs}}) \le \rho_0 \\ 0, & \text{if } \rho(\mathbf{q},\, \mathbf{q}_{\mathrm{obs}}) > \rho_0. \end{cases}$$

$$(4)$$

The total force applied to the robot is the sum of the attractive force and the repulsive force

$$\mathbf{F}_{\mathrm{total}} = \mathbf{F}_{\mathrm{att}} + \mathbf{F}_{\mathrm{rep}} \qquad (5)$$

which determines the motion of the robot.

When the above induced force is used, there are four commonly referred problems in the literature [3] as follows: 1) trap situations due to local minima; 2) no passage between closely spaced obstacles; 3) oscillations in the presence of obstacles; and 4) oscillations in narrow passages. However, the above list is not complete. In fact, there is an additional problem, goals nonreachable with obstacles nearby, encountered when the goal is very close to an obstacle. When the robot approaches its goal, it approaches the obstacle as well. As a consequence, the attractive force decreases, while the repulsive force increases. Thus, the robot will be repulsed away rather than reaching the goal. In the previous literature, this situation is seldom considered. But the problem does exist in reality and is worth investigating.

The essential cause of the GNRON problem is that the goal position is not a minimum of the total potential function. For example, consider a one-dimensional (1-D) case as shown in Fig. 1, where the robot, $\mathbf{q} = \begin{bmatrix} x & 0 \end{bmatrix}^T$, is moving along x-axis toward the goal $\mathbf{q}_{\mathrm{goal}} = \begin{bmatrix} 0 & 0 \end{bmatrix}^T$, while the obstacle $\mathbf{q}_{\mathrm{obs}} = \begin{bmatrix} x_{\mathrm{obs}} & 0 \end{bmatrix}^T$ is on the right-hand side of the goal. Both the robot and the goal are within the distance of influence of the obstacle. The attractive potential (for $m = 2$) and the repulsive potential are given by

$$U_{\mathrm{att}}(\mathbf{q}) = \frac{1}{2} \xi x^2 \qquad (6)$$

$$U_{\mathrm{rep}}(\mathbf{q}) = \frac{1}{2} \eta \left(\frac{1}{x_{\mathrm{obs}} - x} - \frac{1}{\rho_0} \right)^2. \qquad (7)$$

Assuming that $x_{\mathrm{obs}} = 0.5$, $\rho_0 = 2$, and $\xi = \eta = 1$, Fig. 2 shows the total potential $U_{\mathrm{total}}(\mathbf{q}) = U_{\mathrm{att}}(\mathbf{q}) + U_{\mathrm{rep}}(\mathbf{q})$ with respect to x. It is clear that $x = x_{\mathrm{goal}} = 0$ is not the minimum of the total potential function. In fact, the robot will be trapped at the minimum at $x = -0.5$ where the total force becomes zero and the forces at positions on both sides of the minimum are pointing to the minimum position. The robot cannot reach the goal, though there is no obstacle in its way.

To overcome this problem, a new family of repulsive potential functions are proposed which take the relative distance between the robot and the goal into account. The new repulsive potential function ensures that the total potential has a global minimum at the goal position.

III. New Repulsive Potential Functions

The GNRON problem arises because the global minimum of the total potential field is not at the goal position when the goal is within the influence distance of the obstacle. This problem is due to the fact that as the robot approaches the goal, the repulsive potential increases as well. It is found that if the repulsive potential approaches zero as the robot
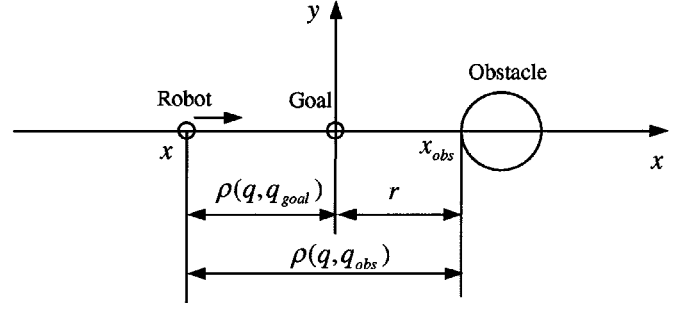

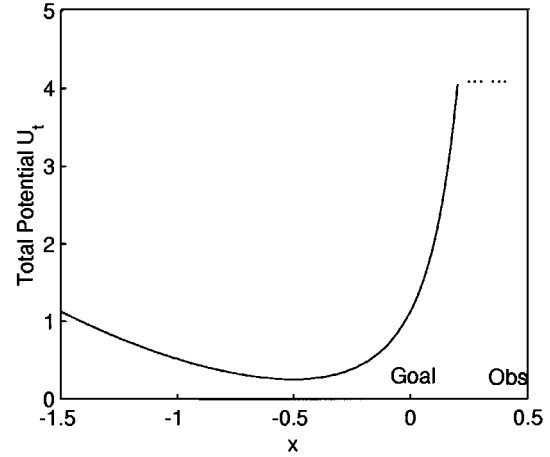Fig. 1. Locations of the robot, goal, and obstacle in a 1-D case.

Fig. 2. Total potential function in a 1-D case.

approaches the goal, the total potential will take the global minimum at the goal. This motivates us to construct a new repulsive potential function which takes the relative distance between the robot and the goal into consideration as

$$U_{\mathrm{rep}}(\mathbf{q})$$

$$= \begin{cases} \dfrac{1}{2} \eta \left(\dfrac{1}{\rho(\mathbf{q},\, \mathbf{q}_{\mathrm{obs}})} - \dfrac{1}{\rho_0} \right)^2 \rho^n(\mathbf{q},\, \mathbf{q}_{\mathrm{goal}}), & \text{if } \rho(\mathbf{q},\, \mathbf{q}_{\mathrm{obs}}) \le \rho_0 \\ 0, & \text{if } \rho(\mathbf{q},\, \mathbf{q}_{\mathrm{obs}}) > \rho_0 \end{cases}$$

$$(8)$$

where $\rho(\mathbf{q},\, \mathbf{q}_{\mathrm{obs}})$ is the minimal distance between the robot $\mathbf{q}$ and the obstacle, $\rho(\mathbf{q},\, \mathbf{q}_{\mathrm{goal}})$ is the distance between the robot and the goal $\mathbf{q}_{\mathrm{goal}}$, ρ_0 is the distance of influence of the obstacle, and n is a positive constant.

In comparison with (3), the introduction of $\rho(\mathbf{q},\, \mathbf{q}_{\mathrm{goal}})$ ensures that the total potential $U_{\mathrm{total}}(\mathbf{q}) = U_{\mathrm{att}}(\mathbf{q}) + U_{\mathrm{rep}}(\mathbf{q})$ arrives at its global minimum, 0, if and only if $\mathbf{q} = \mathbf{q}_{\mathrm{goal}}$. Fig. 3 shows the total potential distribution in a 2-D environment for $m = n = 2$ and $\xi = \eta = 1$, where the goal is at the origin of the coordinate system, the obstacle is a circle of radius 0.5 with its center at position $(1, 0)$. It is obvious that at the goal, the origin, the total potential reaches its global minimum zero. For other choices of n, we have similar potential distribution as shown in Fig. 3 by properly tuning the scaling parameters ξ and η. If ξ and η are not chosen properly, local minima do exist though the goal is the global minimum of the total potential. For $m = n = 2$, $\xi = 1$, and $\eta = 25$, a local minimum exists at point $(-1.08, 0)$ as shown in Fig. 4.

The potential function $U_{\mathrm{total}}(\mathbf{q})$ should have the property that the total force, the sum of the attractive force and the repulsive force, pushes the robot away from the obstacles and pulls toward the goal.
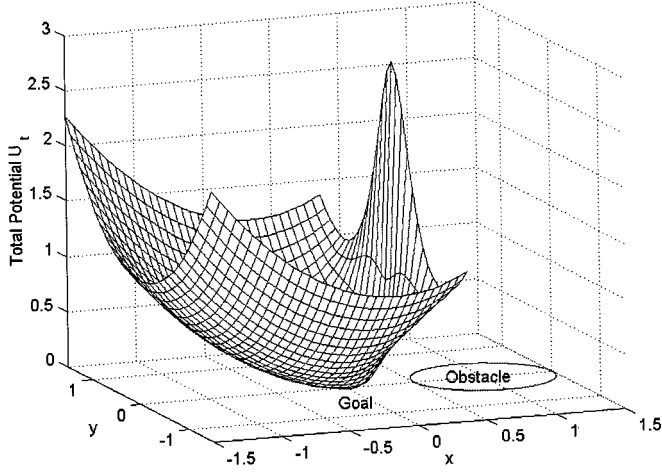
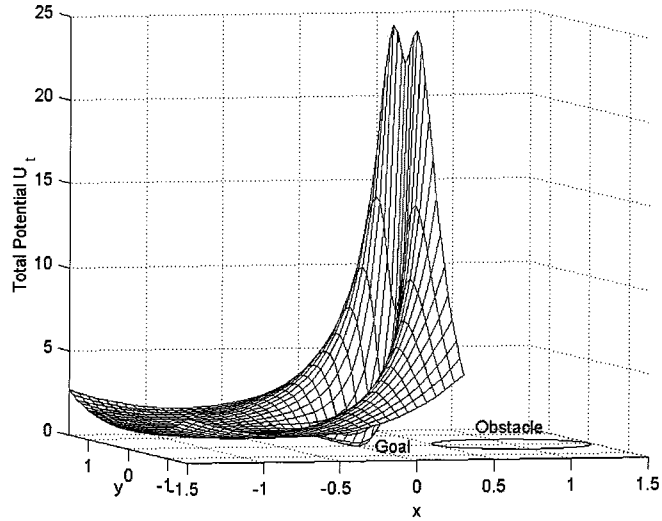Fig. 3.   Total potential in a 2-D environment without local minima.

Fig. 4.   Total potential in a 2-D environment with a local minimum.

According to (8), when the robot is not at the goal, i.e., $\mathbf{q} \neq \mathbf{q}_{\mathrm{goal}}$, the repulsive force is given by

$$\begin{aligned}\mathbf{F}_{\mathrm{rep}}(\mathbf{q}) &= -\nabla U_{\mathrm{rep}}(\mathbf{q}) \\ &= \begin{cases} F_{\mathrm{rep}1}\mathbf{n}_{OR} + F_{\mathrm{rep}2}\mathbf{n}_{RG}, & \text{if } \rho(\mathbf{q}, \mathbf{q}_{\mathrm{obs}}) \leq \rho_0 \\ 0, & \text{if } \rho(\mathbf{q}, \mathbf{q}_{\mathrm{obs}}) > \rho_0 \end{cases}\end{aligned}$$

$$(9)$$

where

$$F_{\mathrm{rep}1} = \eta \left(\frac{1}{\rho(\mathbf{q}, \mathbf{q}_{\mathrm{obs}})} - \frac{1}{\rho_0} \right) \frac{\rho^n(\mathbf{q}, \mathbf{q}_{\mathrm{goal}})}{\rho^2(\mathbf{q}, \mathbf{q}_{\mathrm{obs}})} \tag{10}$$

$$F_{\mathrm{rep}2} = \frac{n}{2} \eta \left(\frac{1}{\rho(\mathbf{q}, \mathbf{q}_{\mathrm{obs}})} - \frac{1}{\rho_0} \right)^2 \rho^{n-1}(\mathbf{q}, \mathbf{q}_{\mathrm{goal}}) \tag{11}$$

$\mathbf{n}_{OR} = \nabla \rho(\mathbf{q}, \mathbf{q}_{\mathrm{obs}})$ and $\mathbf{n}_{RG} = -\nabla \rho(\mathbf{q}, \mathbf{q}_{\mathrm{goal}})$ are two unit vectors pointing from the obstacle to the robot and from the robot to the goal, respectively. The relationship between the repulsive force and its two components is shown in Fig. 5. It is clear that while the component $F_{\mathrm{rep}1}\mathbf{n}_{OR}$ repulses the robot away from the obstacle, the other component $F_{\mathrm{rep}2}\mathbf{n}_{RG}$ attracts the robot toward the goal.

From (8), we can see that the new repulsive potential function degenerates to the conventional form (3) if $n = 0$. To take into account
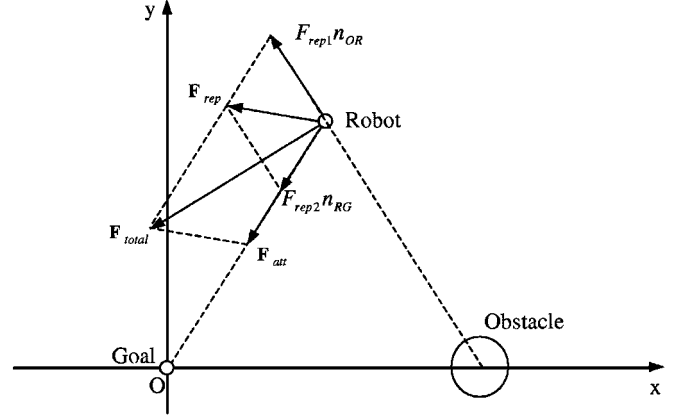

Fig. 5.   Repulsive force derived by the new potential function.

of the goal information in constructing the new repulsive potential, we need to choose $n > 0$.

Now, let us investigate the mathematical properties for different choices of n, i.e., $0 < n < 1$, $n = 1$, and $n > 1$. For $0 < n < 1$, we can see that (8) is not differentiable at the goal, $\mathbf{q} = \mathbf{q}_{\mathrm{goal}}$. According to (10) and (11), when $\rho(\mathbf{q}, \mathbf{q}_{\mathrm{obs}}) < \rho_0$ and $\rho(\mathbf{q}, \mathbf{q}_{\mathrm{goal}}) \neq 0$, i.e., when the robot is within the distance of influence of the obstacle and not at the goal, we have

$$F_{\mathrm{rep}1} = \eta \left(\frac{1}{\rho(\mathbf{q}, \mathbf{q}_{\mathrm{obs}})} - \frac{1}{\rho_0} \right) \frac{\rho^n(\mathbf{q}, \mathbf{q}_{\mathrm{goal}})}{\rho^2(\mathbf{q}, \mathbf{q}_{\mathrm{obs}})} \tag{12}$$

$$F_{\mathrm{rep}2} = \frac{n}{2} \eta \left(\frac{1}{\rho(\mathbf{q}, \mathbf{q}_{\mathrm{obs}})} - \frac{1}{\rho_0} \right)^2 \frac{1}{\rho^{1-n}(\mathbf{q}, \mathbf{q}_{\mathrm{goal}})}. \tag{13}$$

As the robot approaches the goal, $\rho(\mathbf{q}, \mathbf{q}_{\mathrm{goal}})$ approaches zero. Thus, the first component of the repulsive force $F_{\mathrm{rep}1}\mathbf{n}_{OR}$ approaches zero, while the second component $F_{\mathrm{rep}2}\mathbf{n}_{RG}$, which drives the robot to the goal, approaches infinity, as can be seen from (12) and (13), respectively.

For $n = 1$, $\rho(\mathbf{q}, \mathbf{q}_{\mathrm{obs}}) \leq \rho_0$, and $\rho(\mathbf{q}, \mathbf{q}_{\mathrm{goal}}) \neq 0$, (10) and (11) become

$$F_{\mathrm{rep}1} = \eta \left(\frac{1}{\rho(\mathbf{q}, \mathbf{q}_{\mathrm{obs}})} - \frac{1}{\rho_0} \right) \frac{\rho(\mathbf{q}, \mathbf{q}_{\mathrm{goal}})}{\rho(\mathbf{q}, \mathbf{q}_{\mathrm{obs}})} \tag{14}$$

$$F_{\mathrm{rep}2} = \frac{1}{2} \eta \left(\frac{1}{\rho(\mathbf{q}, \mathbf{q}_{\mathrm{obs}})} - \frac{1}{\rho_0} \right)^2. \tag{15}$$

As the robot approaches the goal, the first component of the repulsive force $F_{\mathrm{rep}1}\mathbf{n}_{OR}$ approaches zero, while the magnitude of the second component $F_{\mathrm{rep}2}\mathbf{n}_{RG}$, which drives the robot to the goal, approaches a constant

$$\frac{1}{2} \eta \left(\frac{1}{\rho(\mathbf{q}_{\mathrm{goal}}, \mathbf{q}_{\mathrm{obs}})} - \frac{1}{\rho_0} \right)^2.$$

For $n > 1$, the repulsive potential function (8) is differentiable at the goal and as the robot approaches the goal, the total force continuously approaches zero.

Fig. 6 shows the total potential functions for the case in Fig. 1, where $\mathbf{q}_{\mathrm{goal}} = \begin{bmatrix} 0 & 0 \end{bmatrix}^T$, $\mathbf{q}_{\mathrm{obs}} = \begin{bmatrix} 0.5 & 0 \end{bmatrix}^T$, $\xi = \eta = 1$, $m = 2$, and $n = 0.5, 1, 2, 3$, respectively. It is clear that if $n = 0.5$ and 1, the new potentials are not differentiable at the goal, while the new potentials with $n = 2$ and 3 are differentiable at the goal. Note that there is a local minimum at $x = -0.355$ for the case $n = 0.5$ for the particular set of parameters in Fig. 4. Of course, the local minima can be eliminated by properly choosing ξ and η as will be shown in Section IV.
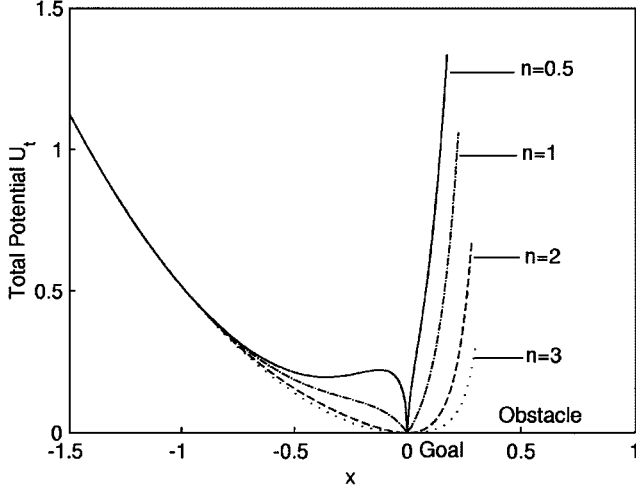
Fig. 6.   Potential functions in a 1-D case.

IV. RELATIONSHIP BETWEEN ξ AND η

Though the new potential functions guarantee the global minimum be at the goal, local minima still exist. To eliminate the local minima problem (the GNRON problem), the scaling parameters of the attractive and repulsive potential functions, ξ and η, have to be chosen properly. These two scaling parameters determine the relative intensity of the attractive force and the repulsive force. From Fig. 6, we can see that the total potential function for $n = 0.5$ has a local minimum at $x = -0.355$. This means that if the robot moves from the left of that local minimum toward the goal, it will be trapped at the local minimum position and cannot reach the goal though there are no obstacles in between the robot and the goal. Actually, for any $n > 0$, if ξ and η are not selected properly, the free path local minima problem (the GNRON problem) still exists, though the new repulsive potential function has a global minimum at the goal. In the remainder of this section, we shall discuss the ways to choose ξ and η to avoid this problem.

Obviously, if the goal is not within the influence distance of obstacles, ξ and η can be chosen to be any positive numbers. In the following analysis, we shall discuss the ways to choose ξ and η to avoid the local minima when the goal is within the influence distance of an obstacle.

Note that if the robot lies between the goal and the obstacle, the resultant virtual force will direct the robot toward the goal definitely, i.e., there is no free path minima between the goal and the nearby obstacle. From Fig. 5, we can see that as long as the robot does not lie on the line of the goal and the nearest boundary point on the obstacle, there will be a nonzero net total force which will keep the robot moving toward this line. This means that there exists no free path local minimum outside the line passing through the goal and the nearest boundary point on the obstacle, and the free path local minimum only occurs on this line. Accordingly, we shall consider the case where the robot, the goal, and the nearest point on the boundary of the obstacle are collinear, with the robot and the obstacle lying on different sides of the goal. The following analysis is based on this simplified situation for a better understanding of the problem.

Equation (2) can be rewritten as

$$\mathbf{F}_{\mathrm{att}}(\mathbf{q}) = F_{\mathrm{att}}\mathbf{n}_{RG} \tag{16}$$

where

$$F_{\mathrm{att}} = \xi\rho(\mathbf{q}, \mathbf{q}_{\mathrm{goal}}). \tag{17}$$

From (9), the total force can be written as

$$\begin{aligned} \mathbf{F}_{\mathrm{total}}(\mathbf{q}) &= \mathbf{F}_{\mathrm{att}}(\mathbf{q}) + \mathbf{F}_{\mathrm{rep}}(\mathbf{q}) \\ &= F_{\mathrm{att}}\mathbf{n}_{RG} + F_{\mathrm{rep}\,1}\mathbf{n}_{OR} + F_{\mathrm{rep}\,2}\mathbf{n}_{RG}. \end{aligned} \tag{18}$$

When the goal, robot, and obstacle are collinear, with the robot and the obstacle lying on different sides of the goal, we have

$$\mathbf{n}_{OR} = -\mathbf{n}_{RG}. \tag{19}$$

Substituting (19) into (18), we obtain

$$\mathbf{F}_{\mathrm{total}}(\mathbf{q}) = (F_{\mathrm{att}} - F_{\mathrm{rep}1} + F_{\mathrm{rep}2})\mathbf{n}_{RG}. \tag{20}$$

To eliminate the free path local minima, $\mathbf{F}_{\mathrm{total}}(\mathbf{q})$ should be pointing to the goal, i.e., the following condition should be satisfied:

$$F_{\mathrm{att}} - F_{\mathrm{rep}\,1} + F_{\mathrm{rep}\,2} > 0. \tag{21}$$

Substituting (17), (10), and (11) into (21) leads to

$$\begin{aligned} &\xi\rho(\mathbf{q}, \mathbf{q}_{\mathrm{goal}}) - \eta\left(\frac{1}{\rho(\mathbf{q}, \mathbf{q}_{\mathrm{obs}})} - \frac{1}{\rho_0}\right)\frac{\rho^n(\mathbf{q}, \mathbf{q}_{\mathrm{goal}})}{\rho^2(\mathbf{q}, \mathbf{q}_{\mathrm{obs}})} \\ &\quad + \frac{n}{2}\eta\left(\frac{1}{\rho(\mathbf{q}, \mathbf{q}_{\mathrm{obs}})} - \frac{1}{\rho_0}\right)^2\rho^{n-1}(\mathbf{q}, \mathbf{q}_{\mathrm{goal}}) > 0. \end{aligned} \tag{22}$$

Since $\xi, \eta > 0$, it can be written as

$$\begin{aligned} \frac{\xi}{\eta} > &\left(\frac{1}{\rho(\mathbf{q}, \mathbf{q}_{\mathrm{obs}})} - \frac{1}{\rho_0}\right)\frac{\rho^{n-1}(\mathbf{q}, \mathbf{q}_{\mathrm{goal}})}{\rho^2(\mathbf{q}, \mathbf{q}_{\mathrm{obs}})} \\ &- \frac{n}{2}\left(\frac{1}{\rho(\mathbf{q}, \mathbf{q}_{\mathrm{obs}})} - \frac{1}{\rho_0}\right)^2\rho^{n-2}(\mathbf{q}, \mathbf{q}_{\mathrm{goal}}). \end{aligned} \tag{23}$$

Let r be the constant distance between the goal and the obstacle nearby. We have $\rho(\mathbf{q}, \mathbf{q}_{\mathrm{goal}}) = \rho(\mathbf{q}, \mathbf{q}_{\mathrm{obs}}) - r$. Thus

$$\begin{aligned} \frac{\xi}{\eta} > &\left(\frac{1}{\rho(\mathbf{q}, \mathbf{q}_{\mathrm{obs}})} - \frac{1}{\rho_0}\right)\frac{(\rho(\mathbf{q}, \mathbf{q}_{\mathrm{obs}}) - r)^{n-1}}{\rho^2(\mathbf{q}, \mathbf{q}_{\mathrm{obs}})} \\ &- \frac{n}{2}\left(\frac{1}{\rho(\mathbf{q}, \mathbf{q}_{\mathrm{obs}})} - \frac{1}{\rho_0}\right)^2(\rho(\mathbf{q}, \mathbf{q}_{\mathrm{obs}}) - r)^{n-2}. \end{aligned} \tag{24}$$

Since we are considering the case where the robot and the obstacle lie on different sides of the goal and both the robot and the goal are within the influence distance of the obstacle, we have $r < \rho(\mathbf{q}, \mathbf{q}_{\mathrm{obs}}) < \rho_0$. The instant $\rho(\mathbf{q}, \mathbf{q}_{\mathrm{obs}}) = r$ is considered separately as the robot reaches the goal already.

Let k_n denote the supremum of the right-hand side of (24) and abbreviate $\rho(\mathbf{q}, \mathbf{q}_{\mathrm{obs}})$ as ρ, we have

$$\begin{aligned} k_n &= \sup_{r < \rho < \rho_0}\left\{\left(\frac{1}{\rho} - \frac{1}{\rho_0}\right)\frac{(\rho - r)^{n-1}}{\rho^2} - \frac{n}{2}\left(\frac{1}{\rho} - \frac{1}{\rho_0}\right)^2(\rho - r)^{n-2}\right\} \\ &= \sup_{r < \rho < \rho_0}\left\{\left(\frac{1}{\rho} - \frac{1}{\rho_0}\right)(\rho - r)^{n-2}\left(\frac{1}{\rho} - \frac{r}{\rho^2} - \frac{n}{2\rho} + \frac{n}{2\rho_0}\right)\right\}. \end{aligned} \tag{25}$$

Since $r < \rho < \rho_0$, we have $(1/\rho - 1/\rho_0) > 0$ and $(\rho - r)^{n-2} > 0$. Through some simple algebraic manipulations, we know that

$$\alpha(\rho) =: \left(\frac{1}{\rho} - \frac{r}{\rho^2} - \frac{n}{2\rho} + \frac{n}{2\rho_0}\right)\begin{cases} \leq 0, & \text{if } r < \rho \leq \rho_m \\ > 0, & \text{if } \rho_m < \rho < \rho_0 \end{cases} \tag{26}$$

where

$$\rho_m = \frac{2r}{1 - \frac{n}{2} + \sqrt{\left(1 - \frac{n}{2}\right)^2 + \frac{2nr}{\rho_0}}} > r. \tag{27}$$

Therefore, (25) can be written as

$$k_n = \sup_{\rho_m < \rho < \rho_0} \left\{ \left(\frac{1}{\rho} - \frac{1}{\rho_0} \right) (\rho - r)^{n-2} \alpha(\rho) \right\} \qquad (28)$$

which gives an accurate lower bound of the ratio ξ/η.

For $n = 2$, from (28), we have

$$k_2 = \sup_{\rho_m < \rho < \rho_0} \left\{ \left(\frac{1}{\rho} - \frac{1}{\rho_0} \right) \left(\frac{1}{\rho_0} - \frac{r}{\rho^2} \right) \right\}$$
$$= \left(\frac{2}{9\rho_0^2} + \frac{2r}{27\rho_0^3} \right) \sqrt{1 + \frac{3\rho_0}{r}} - \frac{2}{3\rho_0^2} + \frac{2r}{27\rho_0^3} \qquad (29)$$

which is the accurate lower bound of the ratio ξ/η. In actual implementations, by choosing the ratio ξ/η greater than k_2 in (29) for $n = 2$, there will be no free path local minima occurring.

For other choices of n, it is relatively complicated to compute k_n from (28). However, it is feasible to give another more conservative lower bound of the ratio ξ/η by finding a simpler expression $k_n' \geq k_n$. Thus, as long as $\xi/\eta > k_n'$, there will be no free path local minima occurring.

When $n \neq 2$, (28) can be written as

$$k_n = \sup_{\rho_m < \rho < \rho_0} \left\{ \left(\frac{1}{\rho} - \frac{1}{\rho_0} \right) (\rho - r)^{n-2} \alpha(\rho) \right\}$$
$$\leq \sup_{\rho_m < \rho < \rho_0} \left\{ \frac{1}{\rho} - \frac{1}{\rho_0} \right\} \sup_{\rho_m < \rho < \rho_0} \left\{ (\rho - r)^{n-2} \right\} \sup_{\rho_m < \rho < \rho_0} \left\{ \alpha(\rho) \right\}. \qquad (30)$$

Apparently by defining

$$k_n' =: \left(\frac{1}{\rho_m} - \frac{1}{\rho_0} \right) \sup_{\rho_m < \rho < \rho_0} \left\{ (\rho - r)^{n-2} \right\} \sup_{\rho_m < \rho < \rho_0} \left\{ \alpha(\rho) \right\} \qquad (31)$$

we have $k_n' \geq k_n$ because $\sup_{\rho_m < \rho < \rho_0} \{ 1/\rho - 1/\rho_0 \} = (1/\rho_m - 1/\rho_0)$.

For the case $n < 2$, we have

$$\sup_{\rho_m < \rho < \rho_0} \left\{ (\rho - r)^{n-2} \right\} = (\rho_m - r)^{n-2} \qquad (32)$$

$$\sup_{\rho_m < \rho < \rho_0} \left\{ \alpha(\rho) \right\}$$
$$= \begin{cases} \dfrac{n}{2\rho_0} + \dfrac{(1 - n/2)^2}{4r}, & \text{if } \dfrac{r}{\rho_0} \leq \left(\dfrac{1}{2} - \dfrac{n}{4} \right) \\ \alpha(\rho_0) = \dfrac{1}{\rho_0} - \dfrac{r}{\rho_0^2}, & \text{if } \dfrac{r}{\rho_0} > \left(\dfrac{1}{2} - \dfrac{n}{4} \right). \end{cases} \qquad (33)$$

Accordingly, k_n' in (31) can be expressed as

$$k_n' = \begin{cases} \left(\dfrac{1}{\rho_m} - \dfrac{1}{\rho_0} \right) (\rho_m - r)^{(n-2)} \\ \quad \left(\dfrac{n}{2\rho_0} + \dfrac{(1 - n/2)^2}{4r} \right), & \text{if } \dfrac{r}{\rho_0} \leq \left(\dfrac{1}{2} - \dfrac{n}{4} \right) \\ \left(\dfrac{1}{\rho_m} - \dfrac{1}{\rho_0} \right) (\rho_m - r)^{(n-2)} \\ \quad \left(\dfrac{1}{\rho_0} - \dfrac{r}{\rho_0^2} \right), & \text{if } \dfrac{r}{\rho_0} > \left(\dfrac{1}{2} - \dfrac{n}{4} \right). \end{cases} \qquad (34)$$

For the case $n > 2$, we have

$$\sup_{\rho_m < \rho < \rho_0} \left\{ (\rho - r)^{n-2} \right\} = (\rho_0 - r)^{n-2} \qquad (35)$$

$$\sup_{\rho_m < \rho < \rho_0} \left\{ \alpha(\rho) \right\} = \alpha(\rho_0) = \frac{1}{\rho_0} - \frac{r}{\rho_0^2} = \frac{(\rho_0 - r)}{\rho_0^2}. \qquad (36)$$

Similarly, (31) becomes

$$k_n' = \left(\frac{1}{\rho_m} - \frac{1}{\rho_0} \right) \frac{(\rho_0 - r)^{n-1}}{\rho_0^2}. \qquad (37)$$

Therefore, for $n \neq 2$, as long as we choose the ratio $\xi/\eta > k_n'$, there will be no free path local minima occurring.

V. SAFETY ISSUES

The main objectives of this paper are as follows: 1) to present the GNRON problem encountered when using potential fields for path planning and 2) provide new repulsive potential functions to overcome this problem. This task can be accomplished by assuming that the robot moves at constant speed, and the total virtual force applied to the robot only determines the direction of its motion. The path obtained in this way is actually the negative gradient flow in the workspace from the starting position to the goal. Since the potential approaches infinity near the obstacle, the path cannot approach the obstacle due to high repulsive forces. Therefore, the path obtained in such a manner is a safe path that avoids obstacles automatically.

If the proposed potential functions are used for robot control directly, i.e., the potential functions generate control signals for the robot system as in [5], [7], and [17], it is important to guarantee that the robot is protected against collision with obstacles, especially as it approaches the goal which is close to an obstacle.

The mobile robot can be described by the following dynamic model [5], [7], [15], [17]:

$$M(\mathbf{q})\ddot{\mathbf{q}} + B(\mathbf{q}, \dot{\mathbf{q}}) + G(\mathbf{q}) = \tau \qquad (38)$$

where $M(\mathbf{q})$ is the robot's inertia matrix, $B(\mathbf{q}, \dot{\mathbf{q}})$ represents fictitious forces, $G(\mathbf{q})$ represents the gravitational forces, and τ is the control input. To control the motion of the robot, the feedback law [5], [6], [17]

$$\tau = G(\mathbf{q}) - \nabla U_{\text{total}} + d \qquad (39)$$

is applied to (38), where d is a dissipative external input torque. In order to obtain smooth torque input, the parameters m and n for the attractive and repulsive potentials should be set $m, n \geq 2$. To avoid unbounded torques when the robot is close to the surface of an obstacle, the repulsive potential function is modified slightly as follows:

$$U_{\text{rep}}(\mathbf{q}) = \begin{cases} \dfrac{1}{2} \eta \left(\dfrac{1}{\rho(\mathbf{q}, \mathbf{q}_{\text{obs}}) - \epsilon} - \dfrac{1}{\rho_0} \right)^2 \\ \quad \rho^n(\mathbf{q}, \mathbf{q}_{\text{goal}}), & \text{if } \rho(\mathbf{q}, \mathbf{q}_{\text{obs}}) \leq \rho_0 \\ 0, & \text{if } \rho(\mathbf{q}, \mathbf{q}_{\text{obs}}) > \rho_0 \end{cases} \qquad (40)$$

where ϵ is a small positive parameter which guarantees that at the surface of the obstacles, the repulsive potential and repulsive force are sufficiently large but bounded. The potential function-based control theory of Koditschek guarantees the safety of motion when using the feedback law (39). Since our main concern is with the path planning problem, safety issues could not be discussed in detail here. For more information, interested readers can refer to [5], [6], and [17].

VI. SIMULATION RESULTS

In the following simulations, the paths are found by assuming that the robot moves at constant speed, and the resultant virtual force applied to it only determines the direction of its motion. The workspace is within a 10 m × 10 m room and there are some rectangular obstacles scattered in it. Define the origin of the global frame at the lower left corner of the room. Set the starting position of the robot at point
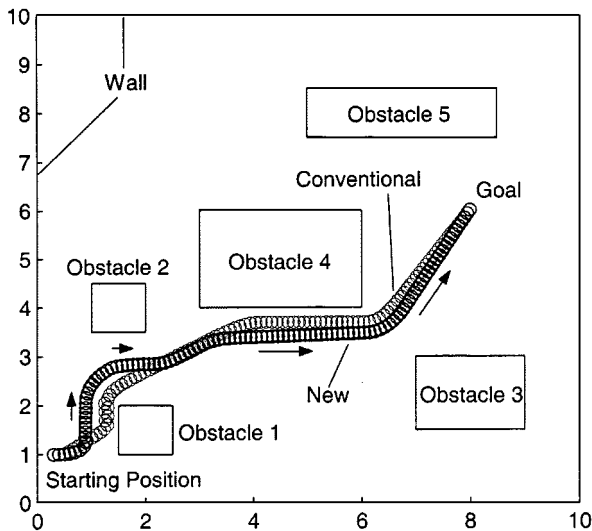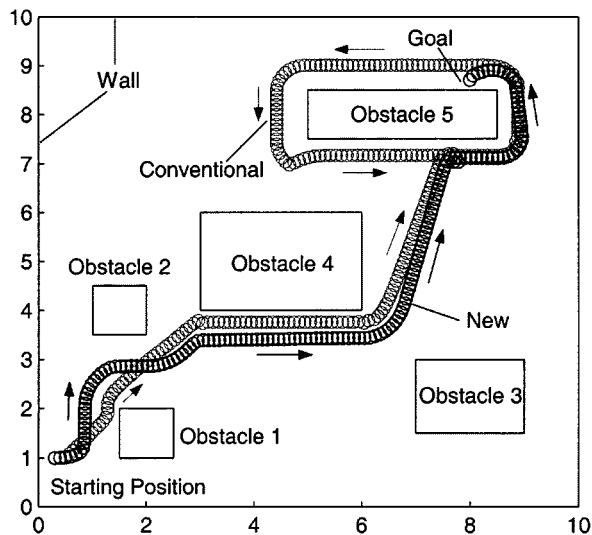
Fig. 7. Goal is far away from obstacles.

Fig. 8. Goal is near an obstacle.

(0.3, 1). For the potential functions, we choose $m = n = 2$, $\xi = 1$, and $\eta = 0.1$.

A. Goal Far Away from Obstacles

In the first case, the goal is out of the influence range of the obstacles, i.e., $r \geq \rho_0 = 0.8$ with the goal at point (8, 6). Fig. 7 shows the simulation results in this case, where the gray and black paths represent the results obtained using the conventional repulsive potential function (3) and the new repulsive potential function (8), respectively. Both functions can successfully drive the robot to the goal while avoiding the obstacles.

B. Goal Close to an Obstacle

In the second simulation study, the goal (8, 8.7) is chosen close to an obstacle and within its influence range, i.e., $r = 0.2 < \rho_0 = 0.8$. When the conventional potential function is used, the robot will be trapped at point (8.04, 9.00). Then, it switches to a wall-following mode in an effort to escape from the trap. As shown by the gray path in Fig. 8, the robot cannot arrive at the goal position, and is trapped in some closed loop. When the new repulsive potential function is used,

the robot can successfully arrive at the goal as shown by the black path in Fig. 8.

Although the new repulsive potential functions assure that the goal is the global minimum of the total potential and an analytic solution has been found for the GNRON problem, they cannot eliminate other local minima due to obstacles between the robot and the goal. Since the local minima problem due to obstacles between the robot and the goal is different from the GNRON problem, other path planning approaches such as wall following may be used to solve the problem.

VII. CONCLUSION

In this paper, an additional problem associated with potential filed methods, GNRON problem, has been presented for mobile robot path planning. To solve the problem, new repulsive potential functions have been provided by taking the relative distance between the robot and the goal into consideration, which ensures that the goal position is the global minimum of the total potential. By choosing the parameters of the potential functions carefully, we can eliminate all the free path local minima, and at the same time, ensure that the robot can reach the goal while avoiding collision with obstacles. Simulation results have verified that the new repulsive potential function can solve the GNRON problem effectively.

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