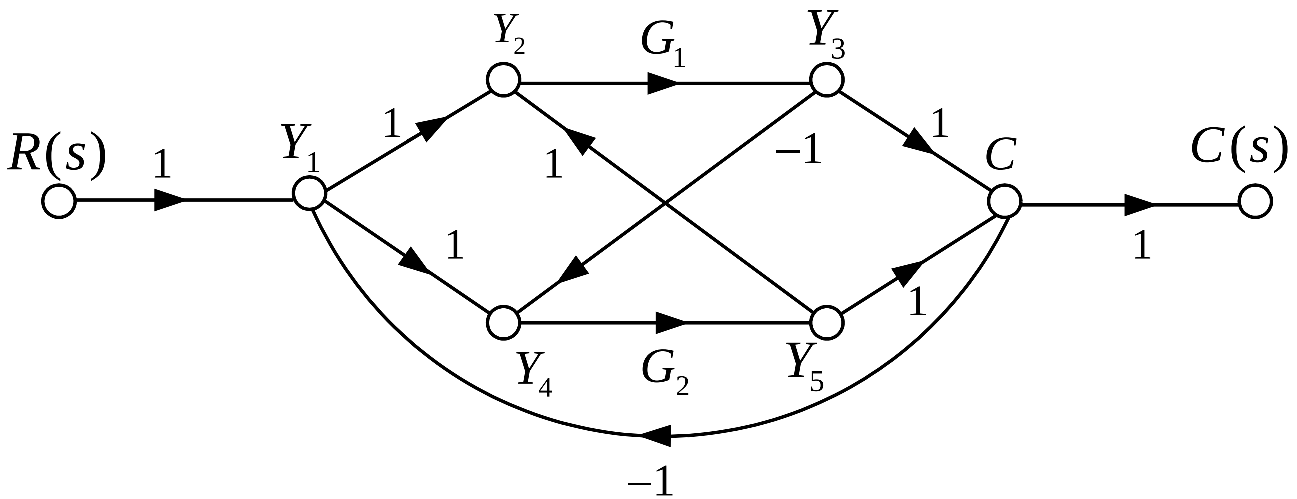
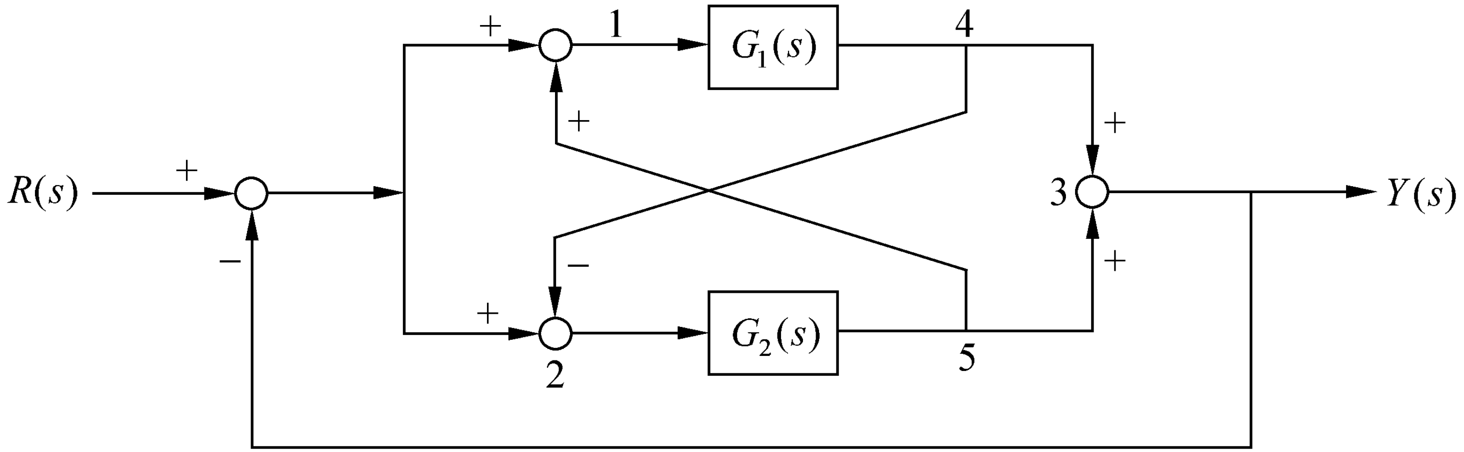
2.4 控制系统的信号流图

2.4.1 信号流图

2.4.2 梅逊公式


R(s)
1
Y1
1
1
Y2
G1
Y3
1
−1
1
C
C(s)
1
1
Y4
G2
Y5
−1

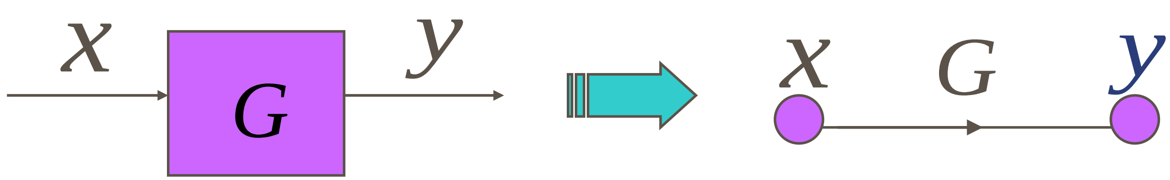

R(s)
+
−
+
1
G1(s)
4
+
+
3
Y(s)
−
+
+
G2(s)
2
5

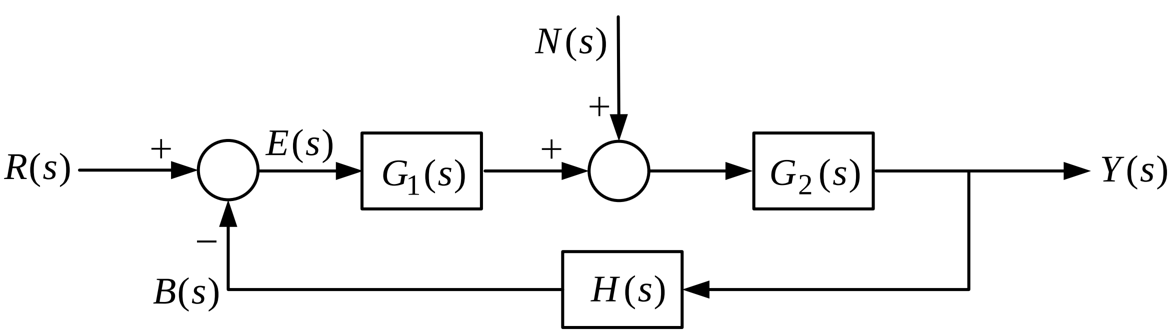
2.4.1 信号流图

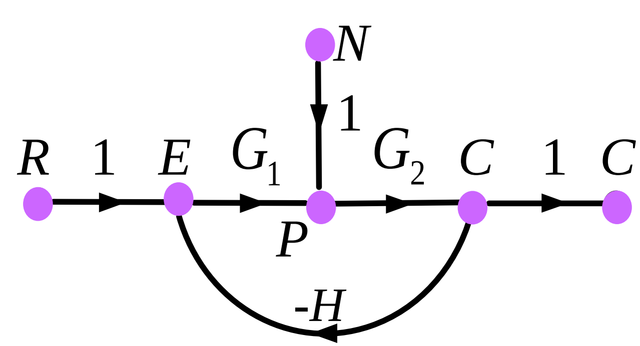
信号流图可以表示系统的结构和变量传送过程中的数学关系。它也是控制系统的一种数学模型。在求复杂系统的传递函数时较为方便。

1. 信号流图

组成： 信号流图由节点和支路组成的信号传递网络。

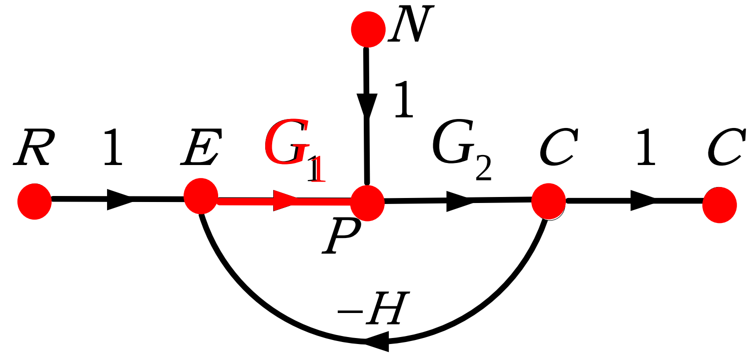




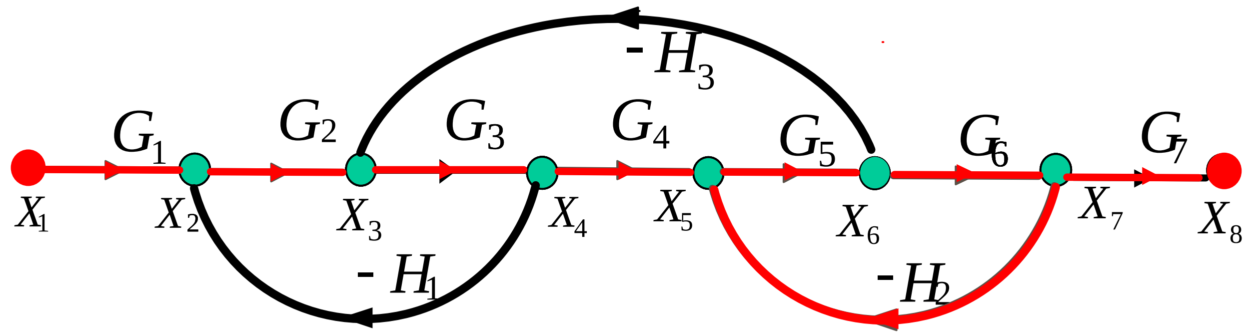


2. 信号流图中的术语

A signal-flow graph is a diagram consisting of nodes that are connected by several directed branches and is a graphical representation of a set of linear relations.

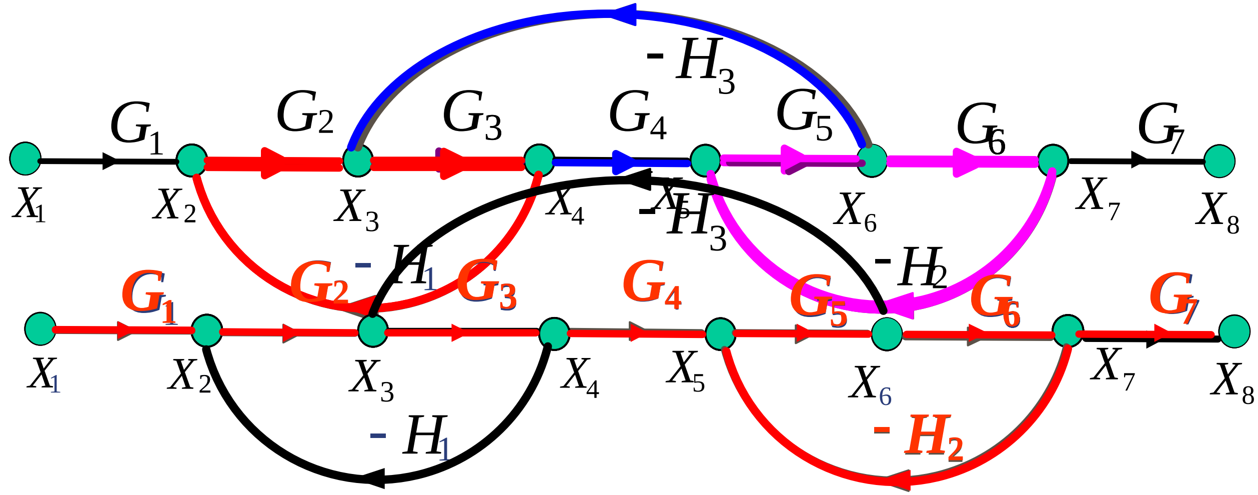


- Node: 节点 — The input and output points or junctions are all called nodes.
- Branch: 支路 — is an unidirectional path segment with an arrow. A signal can only transmit through a branch in the direction of the arrow.
- Branch Gain: 支路增益 — The relation between two nodes is called a branch gain and usually a branch gain is written next to the directional arrow.



[Terms]:

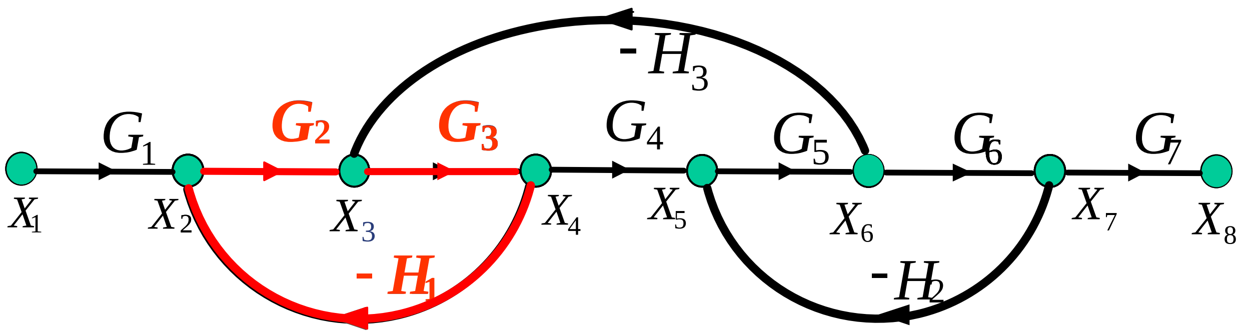
• Input Node: 输入节点	is a node with the leaving branches, without any entering branches .
• Output Node: 输出节点	is a node with the entering branches, without any leaving branches.
• Path: 通路	is a branch or a continuous sequence of branches that can be traversed from one node to the other node.
• Forward Path: 前向通路	is a path that starts at an input node and ends at an output node, and along which no node is traversed more than once.



- Loop: 回路 — is a path that the starting node and ending node are the same node, and along this loop no other node is encountered more than once.
- Non Touching Loops: 互不接触回路 — if two loops do not have any common node, we call them Non touching loops.
- Path Gain: 通路增益 — is the products of the branch gains encountered in one path.
- Forward Path Gain: 前向通路增益 — is the products of all of the branch gains encountered in one forward path.

- Loop Gain: is the products of all of the branch gains encountered in one loop.

  回路增益

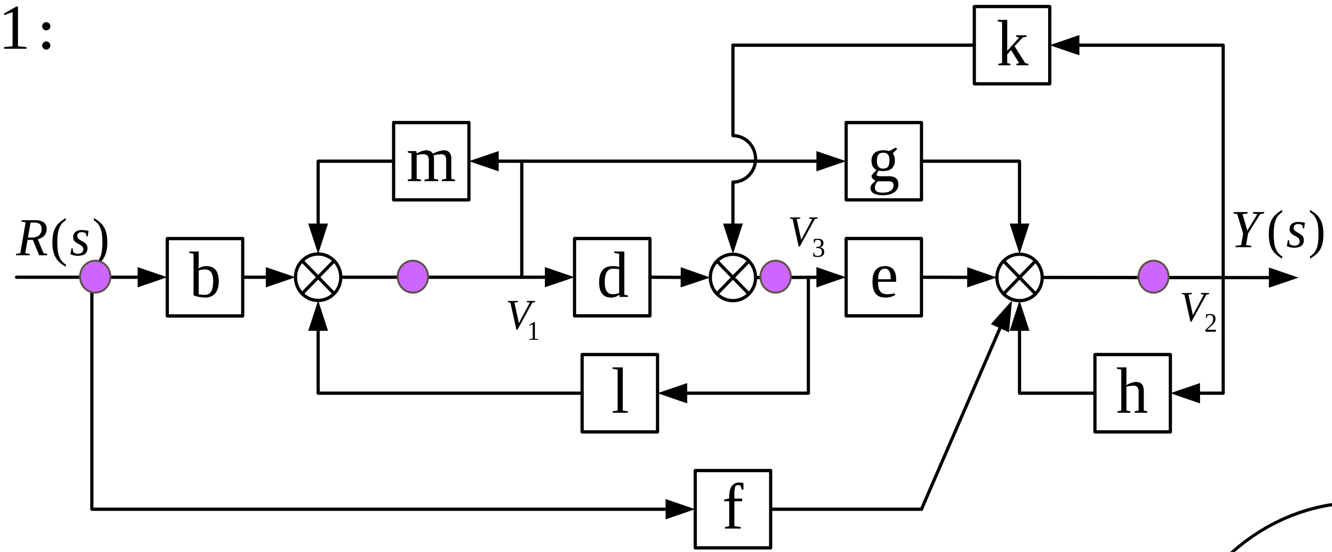


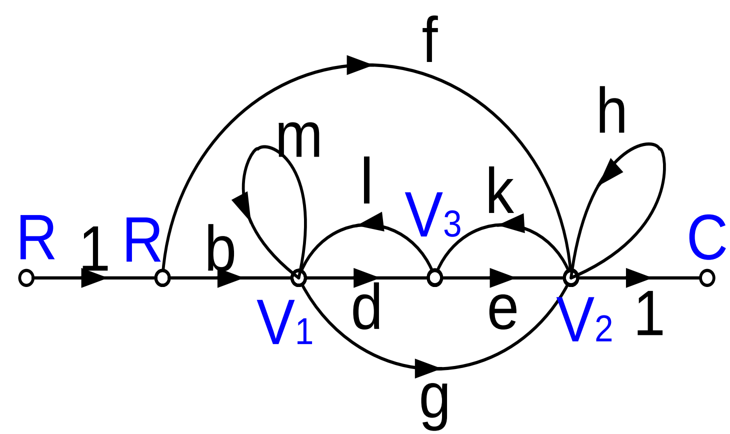
3. 信号流图的绘制

1）根据方块图绘制

2）按微分方程拉氏变换后的代数方程所表示的变量间数学关系绘制

例1：

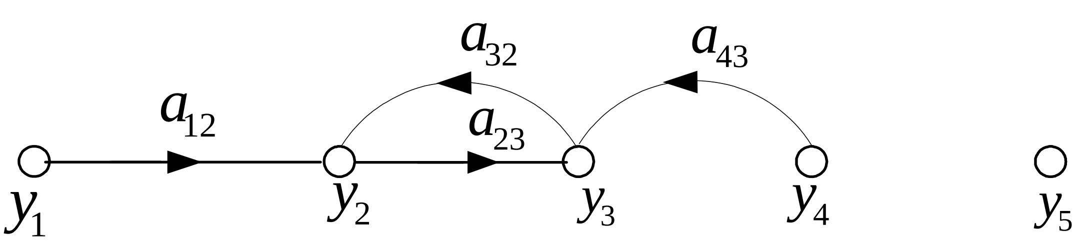




例2：Get a signal-flow graph from the algebraic equations

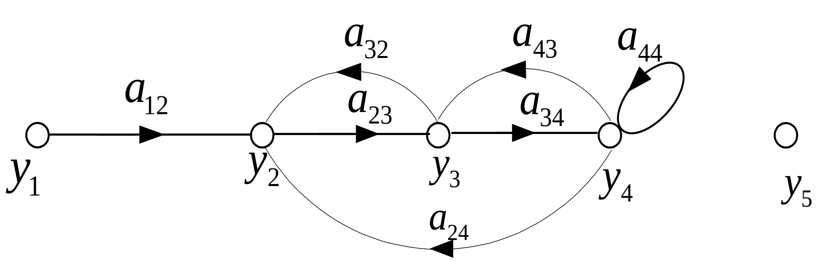
$$y_2 = a_{12}y_1 + a_{32}y_3 \qquad y_3 = a_{23}y_2 + a_{43}y_4$$

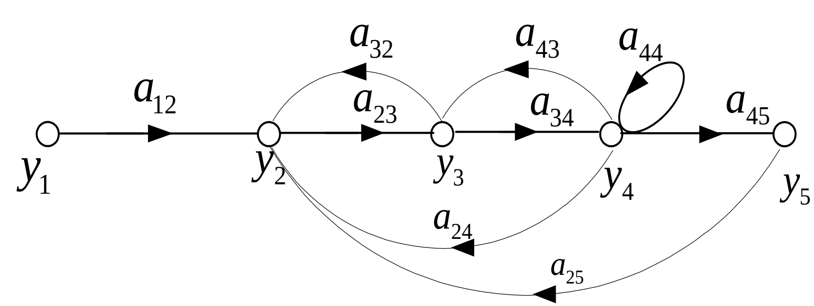
$$y_4 = a_{24}y_2 + a_{34}y_3 + a_{44}y_4 \qquad y_5 = a_{25}y_2 + a_{45}y_4$$



- use the nodes to express the variables

- use the branches to express the relationships between the variables





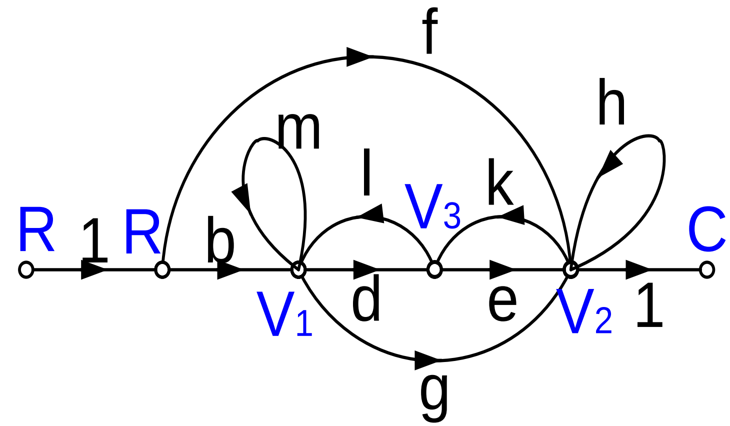
2.4.2 梅逊增益公式

1. 梅逊公式的推导

$$V_1 = mV_1 + lV_3 + bR$$

$$C = V_2 = gV_1 + hV_2 + eV_3 + fR$$

$$V_3 = dV_1 + kV_2$$



以R为输入，V_2为输出则方程可整理为

$$(1-m)V_1 - lV_3 = bR$$

$$-gV_1 + (1-h)V_2 - eV_3 = fR$$

$$-dV_1 - kV_2 + V_3 = 0$$

写成矩阵形式为

$$\begin{bmatrix} 1-m & 0 & -l \\ -g & 1-h & -e \\ -d & -k & 1 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix} = \begin{bmatrix} b \\ f \\ 0 \end{bmatrix} R$$

根据克莱姆法则 C ? $= V_2 = \dfrac{\Delta_2}{\Delta}$

$$\Delta = \begin{vmatrix} 1-m & 0 & -l \\ -g & 1-h & -e \\ -d & -k & 1 \end{vmatrix} = (1-m)(1-h) - gkl - dl(1-h) - (1-m)ke$$

$$= 1 - m - h + mh - gkl - dl + dlh - ke + mke$$

$$= 1 - (m + dl + ke + h + gkl) + mh + dlh + mke$$

和

$$\Delta_2 = \begin{vmatrix} 1-m & bR & -l \\ -g & fR & -e \\ -d & 0 & 1 \end{vmatrix} = (1-m)\, fR + debR - dlfR + gbR$$

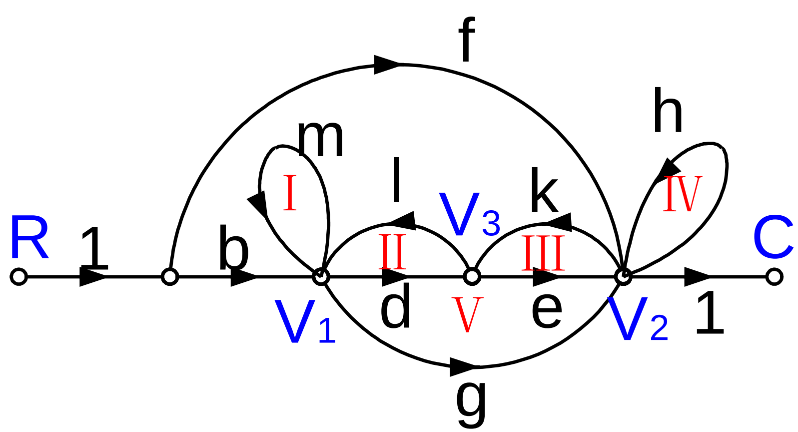
$$= [bde + f(1 - m - dl) + bg]R$$

由克莱姆法则得

$$C = V_2 = \frac{\Delta_2}{\Delta} = \frac{[bde + f(1-m-dl) + bg]R}{1-(m+dl+ke+h+gkl)+mh+dlh+mke}$$

于是传递函数为

$$\Phi(s) = \frac{C(s)}{R(s)} = \frac{\Delta_2}{\Delta \times R} = \frac{bde + f(1-m-dl) + bg}{1-(m+dl+ke+h+gkl)+mh+dlh+mke}$$



2. 梅逊公式

用梅逊公式可不必简化信号流图而直接求得从输入节点到输出节点之间的总传输。（即总传递函数）

其表达式为：
$$P=\frac{1}{\Delta}\sum_{k=1}^{n}P_k\Delta_k$$

式中： P: 总传输（即总传递函数）；

n: 从输入节点到输出节点的前向通道总数；

P_k: 第k个前向通道的总传输；

Δ: 流图特征式；

$$\Delta=1-\sum L_a+\sum L_bL_c-\sum L_dL_eL_f+\ldots$$（正负号间隔）

$$\Delta = 1 - \sum L_a + \sum L_b L_c - \sum L_d L_e L_f + \ldots$$（正负号间隔）

$\sum L_a$：流图中所有不同回路的回路传输之和；

$\sum L_b L_c$：所有互不接触回路中，每次取其中两个回路传输乘积之和；

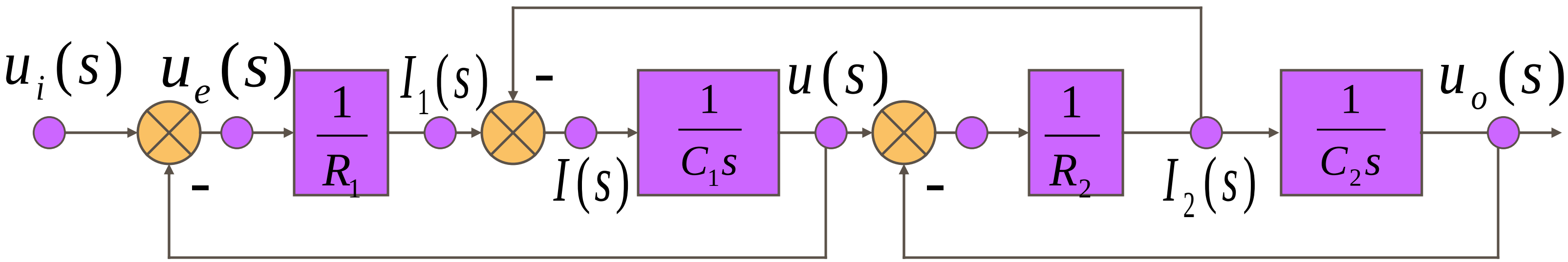
$\sum L_d L_e L_f$：所有互不接触回路中，每次取其中三个回路传输乘积之和；

Δ_k：第k个前向通道的特征式的余子式；其值为Δ中除去与第k个前向通道接触的回路后的剩余部分；

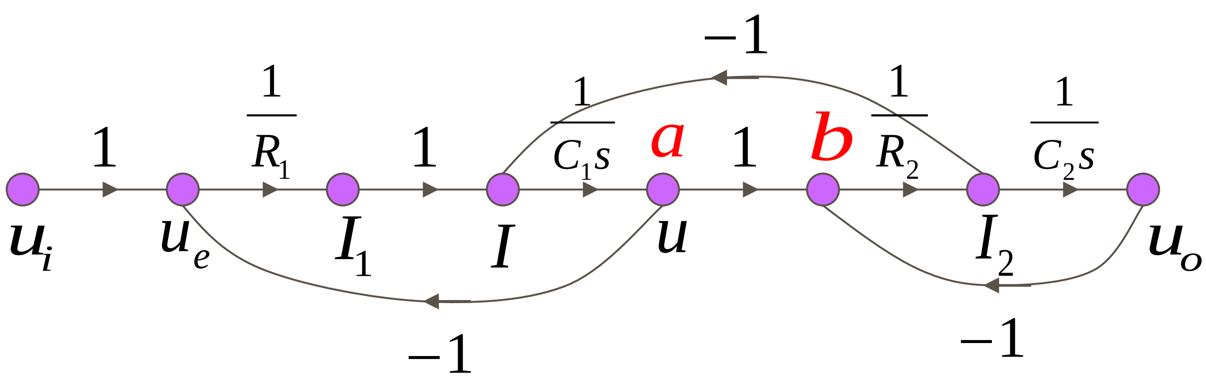
总之，传递函数的分母△取决于信号流图中回路的拓扑结构。

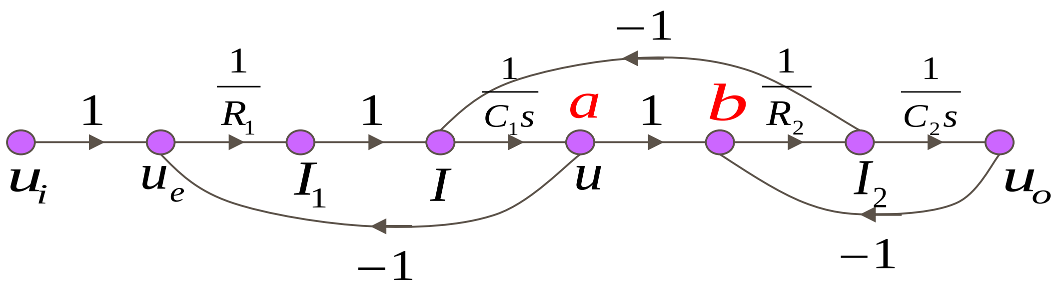
分子取决于前向通路及与该前向通路无关的回路的拓扑结构。

例：用Mason公式计算总传递函数。



[解]：先在结构图上标出节点，再根据逻辑关系画出信号流图。





有一个前向通道；　$P_1=\dfrac{1}{R_1C_1R_2C_2s^2}$

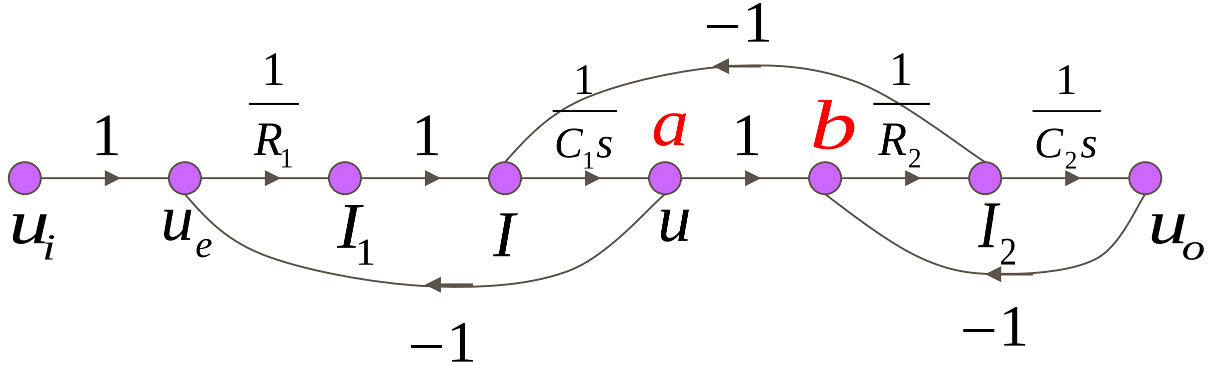
有三个回路；　$\sum L_a=\dfrac{-1}{R_1C_1s}+\dfrac{-1}{R_2C_2s}+\dfrac{-1}{R_2C_1s}$

有两个互不接触回路；$\sum L_bL_c=\dfrac{-1}{R_1C_1s}\times\dfrac{-1}{R_2C_2s}=\dfrac{1}{R_1R_2C_1C_2s^2}$

$$\therefore \Delta=1+\frac{1}{R_1C_1s}+\frac{1}{R_2C_2s}+\frac{1}{R_2C_1s}+\frac{1}{R_1R_2C_1C_2s^2}$$

$$\Delta_i=1$$

总传输为：$P=$? $\dfrac{1}{\Delta}\sum_{k=1}^{1}P_k\Delta_k=\dfrac{1}{R_1R_2C_1C_2s^2+(R_1C_1+R_2C_2+R_1C_2)s+1}$

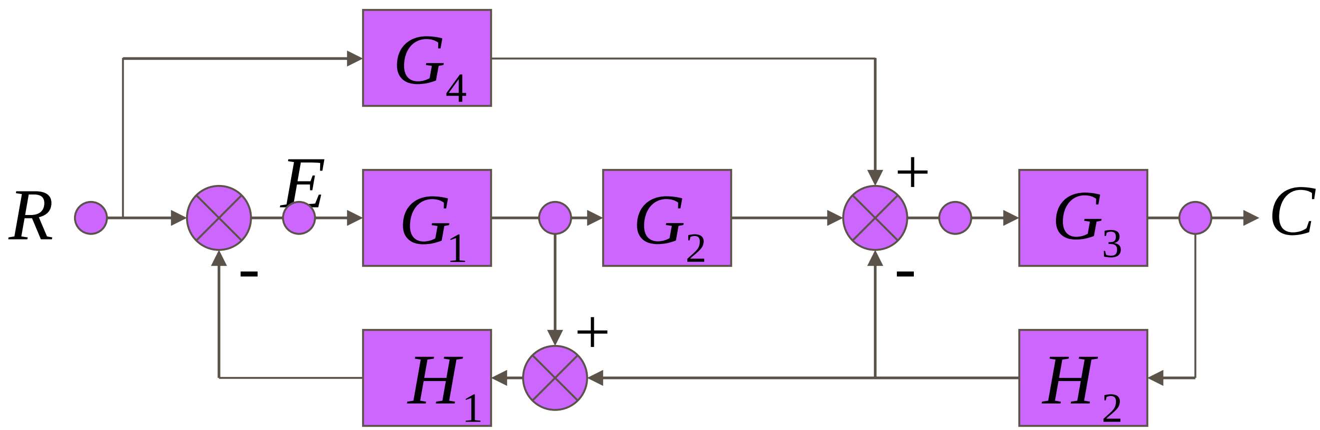


讨论：信号流图中，a点和b点之间的传输为1，是否可以将该两点合并。使得将两个不接触回路变为接触回路？如果可以的话，总传输将不一样。

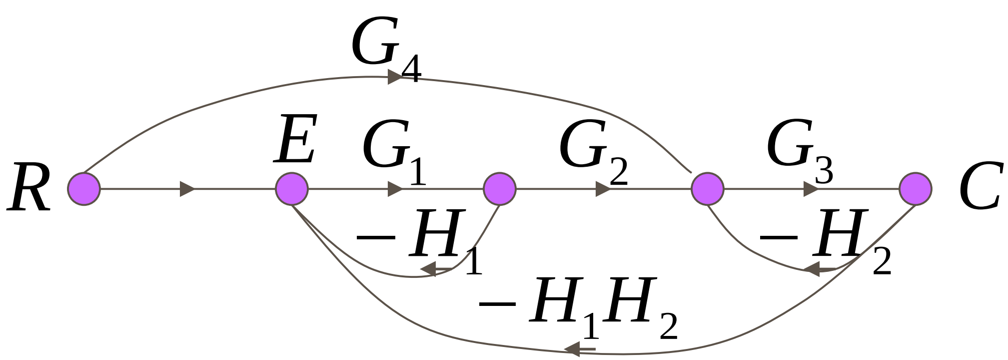
不能合并。因为a、b两点的信号值不一样。

上图中，u_i和u_e，I_1和I，a和b可以合并。为什么？

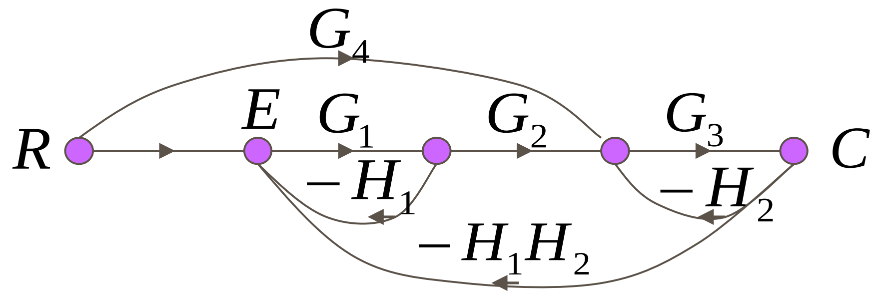
例：使用Mason公式计算下列方块图的传递函数 $\dfrac{C(s)}{R(s)},\dfrac{E(s)}{R(s)}$



[解]：在结构图上标出节点，画出信号流图。



求 $\dfrac{C(s)}{R(s)}$：



有二条前向通道 $P_1 = G_1G_2G_3,\quad P_2 = G_3G_4$

有三个回路 $-G_1H_1, -G_3H_2, -G_1G_2G_3H_1H_2$

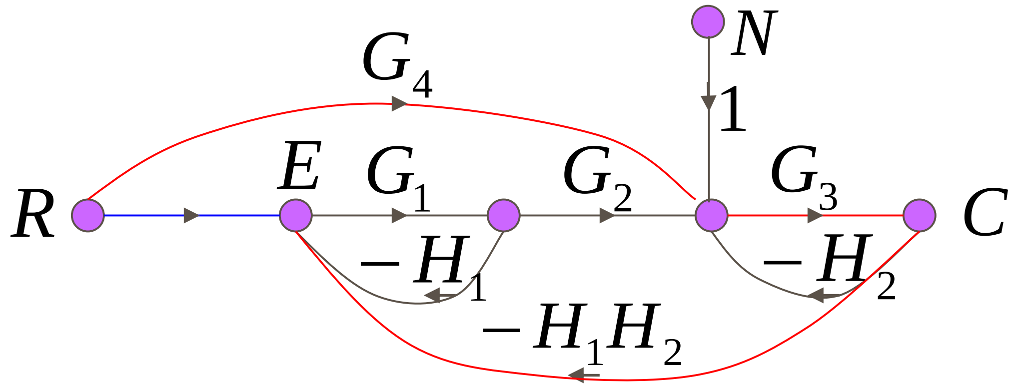
有一个互不接触回路

$$\Delta = 1 - \sum L_a + \sum L_bL_c = 1 + G_1H_1 + G_3H_2 + G_1G_2G_3H_1H_2 + G_1G_3H_1H_2$$

$$\Delta_1 = 1, \Delta_2 = 1 + G_1H_1$$

$$\therefore P = \frac{1}{\Delta}\sum_{k=1}^{2} P_k\Delta_k = \frac{G_1G_2G_3 + G_3G_4 + G_1G_3G_4H_1}{1 + G_1H_1 + G_3H_2 + G_1G_2G_3H_1H_2 + G_1G_3H_1H_2}$$

求 $\dfrac{E(s)}{R(s)}$:



Δ 不变。

$P_1 = 1, \quad \Delta_1 = 1 + G_3 H_2$ （兰线表示）

$P_2 = -G_3 G_4 H_1 H_2, \quad \Delta_2 = 1$ （红线表示）

$$\therefore P = \frac{1 + G_3 H_2 - G_3 G_4 H_1 H_2}{\Delta}$$

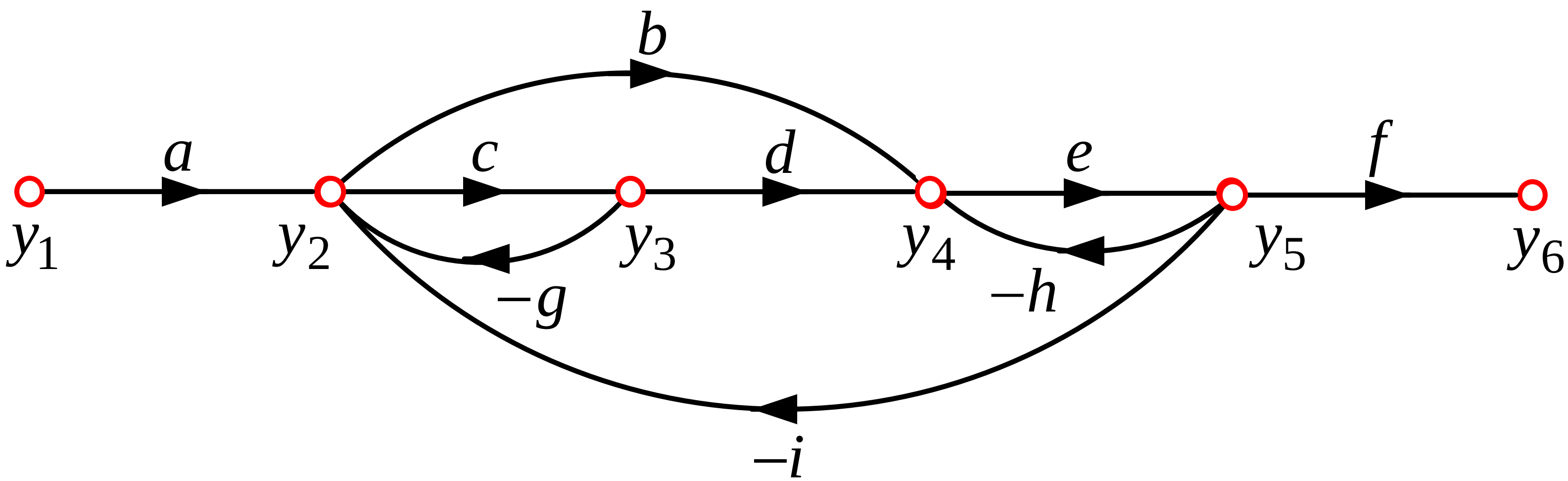
注意：Δ 不变，为什么?

Δ是流图特征式，也就是传递函数的特征方程式。对于一个给定的系统，特征方程式总是不变的。

注意：梅逊公式只能求系统的总增益，即输出对输入的增益。而输出对混合节点（中间变量）的增益就不能直接应用梅逊公式。也就是说对混合节点，不能简单地通过引出一条增益为一的支路，而把非输入节点变成输入节点。

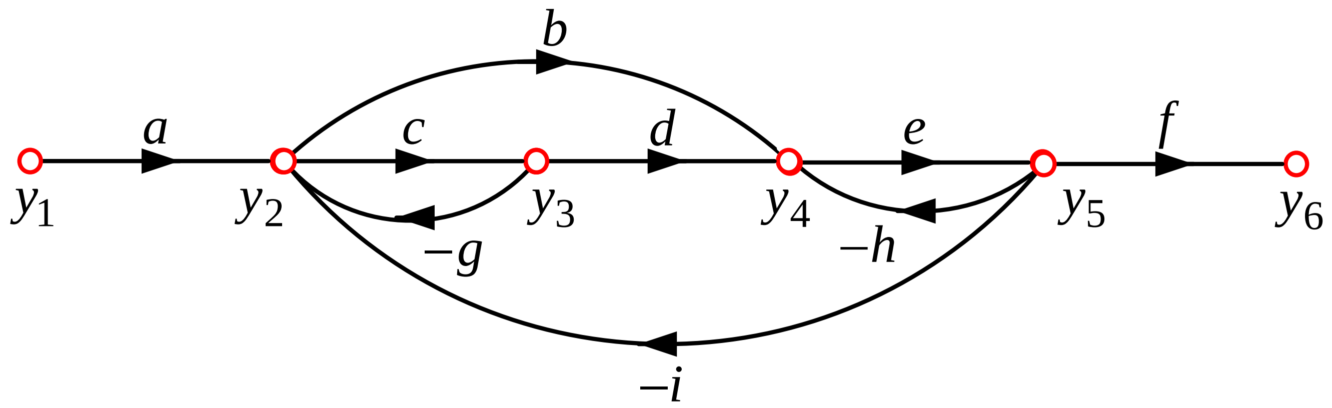
例：一系统的信号流图如图所示，

试求增益 y_6/y_1 ，y_3/y_1 ，y_5/y_2 。



$$\Delta = 1 + cg + cdei + bei + eh + cgeh$$

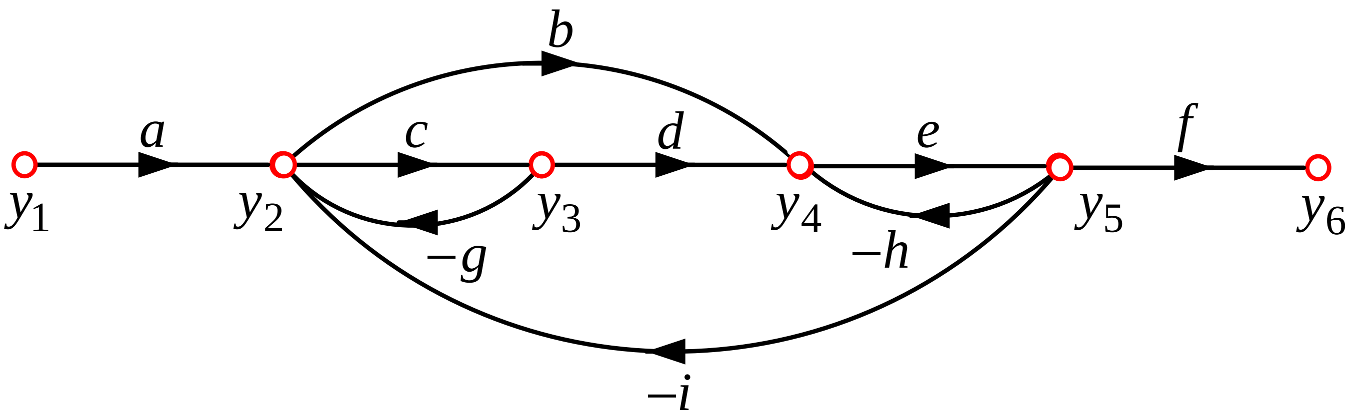
1. 计算增益 y_6/y_1



$P_1 = acdef$　　$P_2 = abef$　　$y_6/y_1 = \frac{1}{\Delta}\left(P_1\Delta_1 + P_2\Delta_2\right)$

$\Delta_1 = 1$　　$\Delta_2 = 1$　　$= \frac{acdef + abef}{\Delta}$

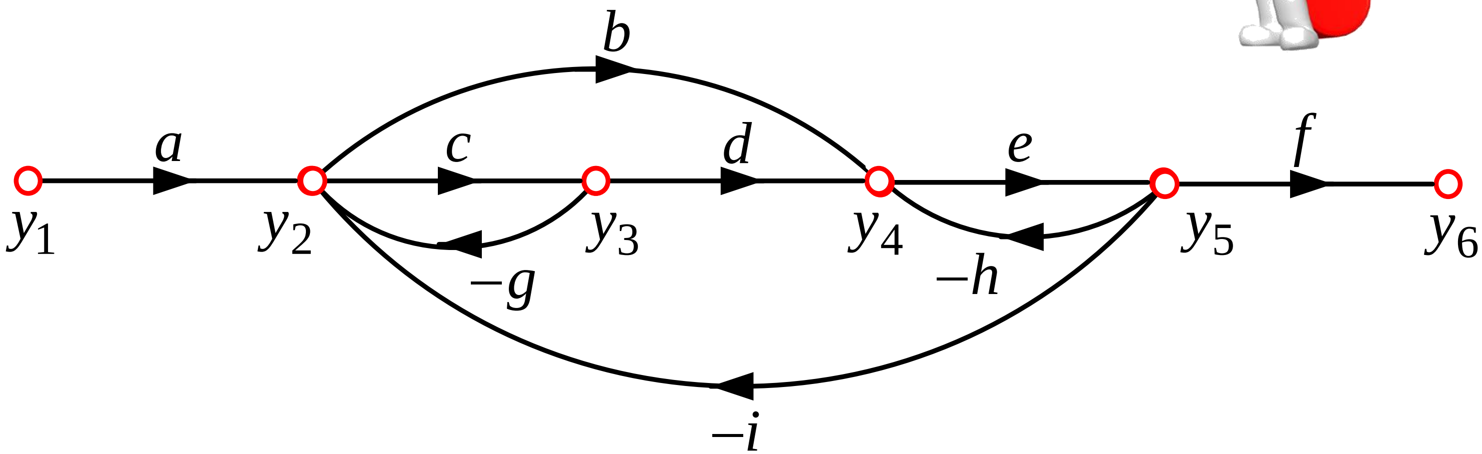
2． 计算增益 y_3/y_1



$$P_1 = ac \qquad \Delta_1 = 1 + eh \qquad y_3/y_1 = \frac{P_1\Delta_1}{\Delta} = \frac{ac(1+eh)}{\Delta}$$

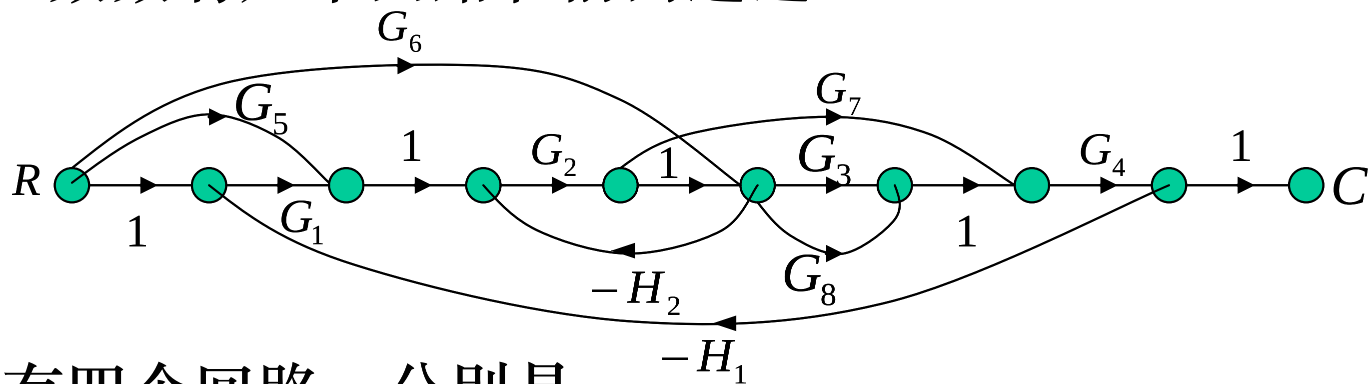
3． 计算增益 y_5/y_2

在图中先应用梅森增益公式分别求出增益 y_5/y_1 和 y_2/y_1 ，然后再计算



$$y_5/y_2 = \frac{y_5/y_1}{y_2/y_1} \qquad y_5/y_2 = \frac{cde+be}{1+eh}$$

例： 数数有几个回路和前向通道。



- 有四个回路，分别是：

$$-G_2H_2, -G_1G_2G_3G_4H_1, -G_1G_2G_7G_4H_1, -G_1G_2G_8G_4H_1$$

它们都是互相接触的。

$$\therefore \Delta = 1 + G_2H_2 + G_1G_2G_3G_4H_1 + G_1G_2G_7G_4H_1 + G_1G_2G_8G_4H_1$$

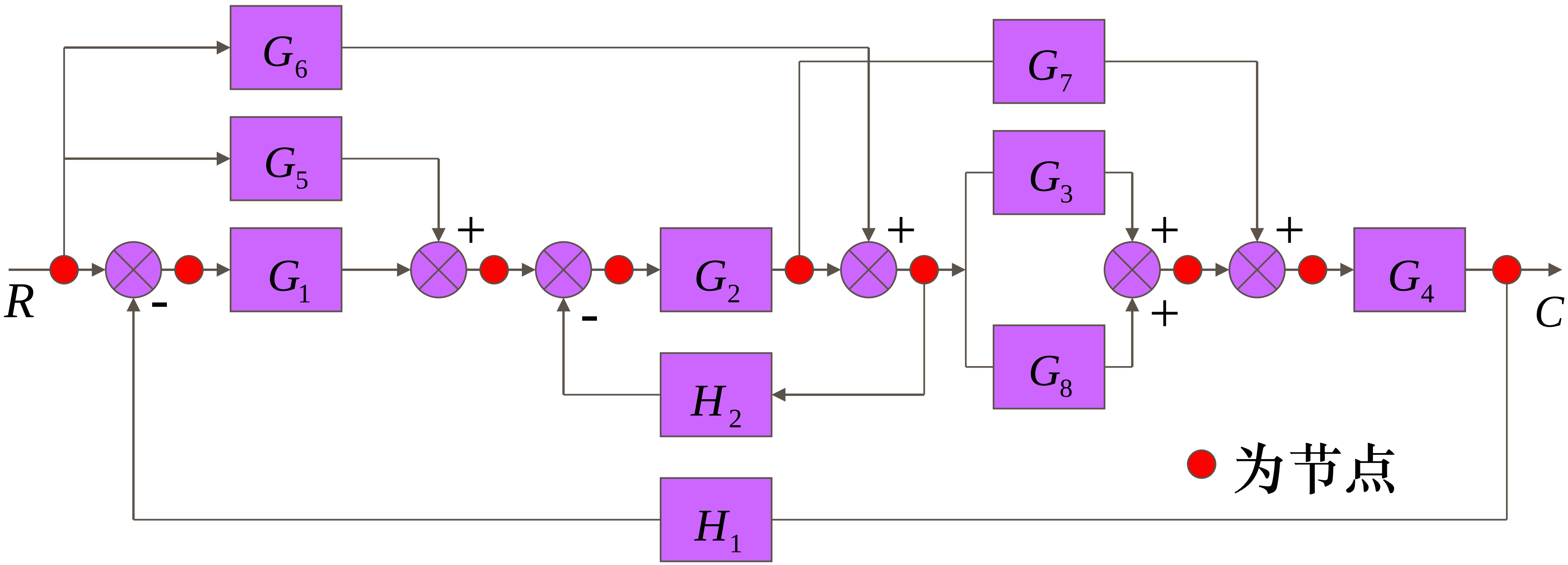
- 有九条前向通道，分别是：

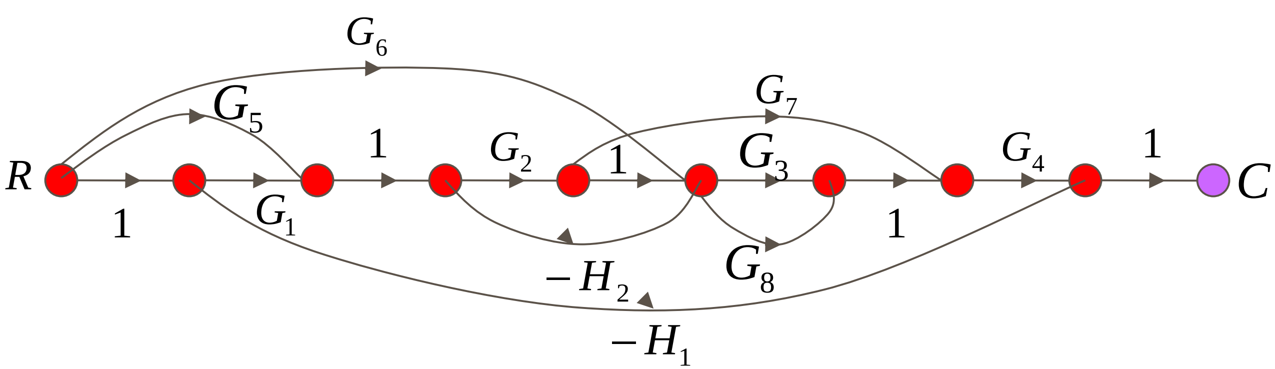
$$P_1 = G_1G_2G_3G_4 \qquad P_4 = G_5G_2G_3G_4 \qquad P_7 = G_6G_3G_4$$

$$P_2 = G_1G_2G_7G_4 \qquad P_5 = G_5G_2G_7G_4 \qquad P_8 = G_6G_8G_4$$

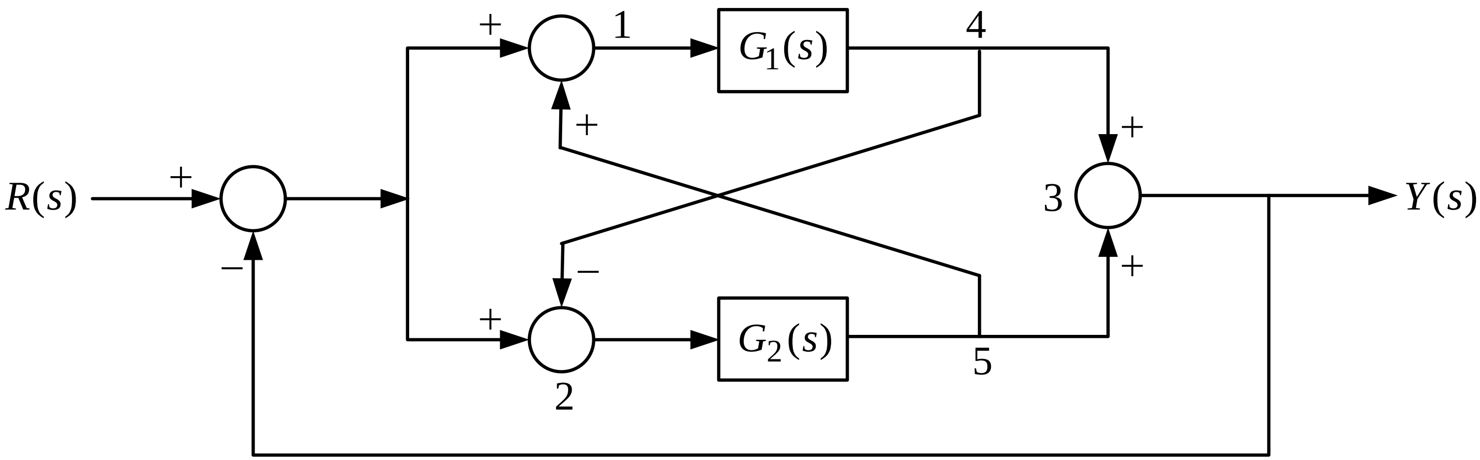
$$P_3 = G_1G_2G_8G_4 \qquad P_6 = G_5G_2G_8G_4 \qquad P_9 = -G_6H_2G_2G_7G_4$$

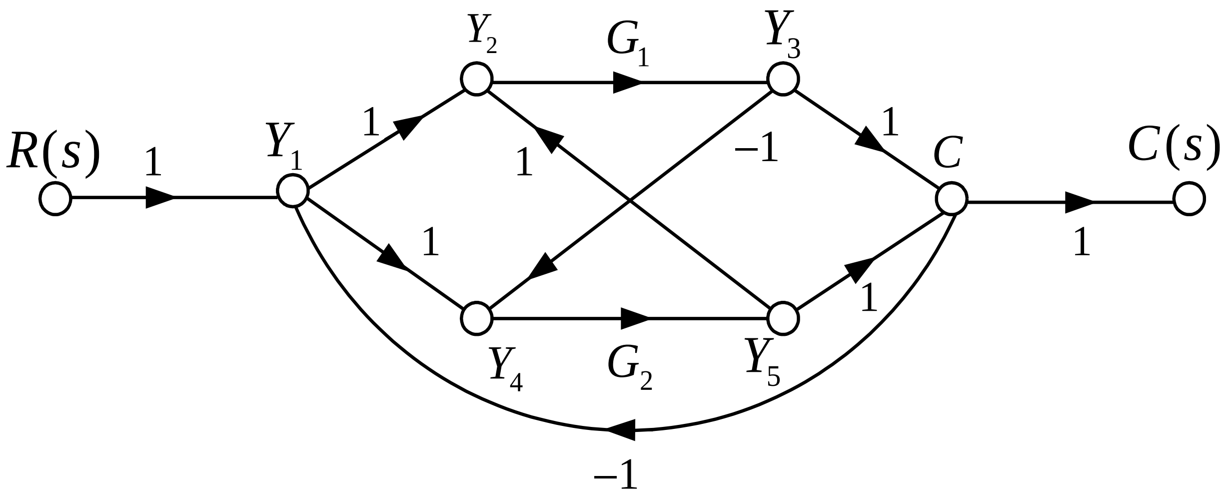
- 对应的结构图为：





注意：①信号流图与结构图的对应关系；②仔细确定前向通道和回路的个数。





$$\Phi = C/R = \frac{G_1(s)+G_2(s)}{1+G_1(s)+G_2(s)+G_1(s)G_2(s)}$$