

Exercise 11 – Differential equations

Exercise 9.1

Consider the differential equation: $y' = -x^2 y^2$

This equation can be analytically solved and the general solution is: $y = \frac{3}{A + x^3}$

If we apply the initial condition $y(0)=1$ the solution becomes: $y = \frac{3}{3 + x^3}$

Write a program which implements the three methods (Euler, Runge-Kutta II and Runge-Kutta IV) to solve the equation.

The program should ask as input:

- which method has to be used
- the range
- the stepsize

and write as output a file where evaluations of the function y at each step are performed.

Compare results obtained in the range $[0,1]$ and stepsize $Dx=0.1, 0.01$ with the exact solution.

Exercise 9.2

Modify your program in order to solve the differential equation:

$$y' = y \sin x + \sin 2x \quad \text{with} \quad y(0) = -2$$

Compare your results with the exact solution:

$$y = 2 - 2 \cos x - 2e^{1 - \cos x}$$