Exercise no. 1

1.1)

Given the parabola $y = 2.5x^2 - 0.9x + 0.4$

and the circle $(x - 0.5)^2 + (y - 0.5)^2 = r^2$

assume a value of 0.1 for the radius of the circle and use the acceptance/rejection Monte Carlo method to calculate the area obtained from the intersection of the area above the parabola and the area inside the circle, as shown in the picture (red area).

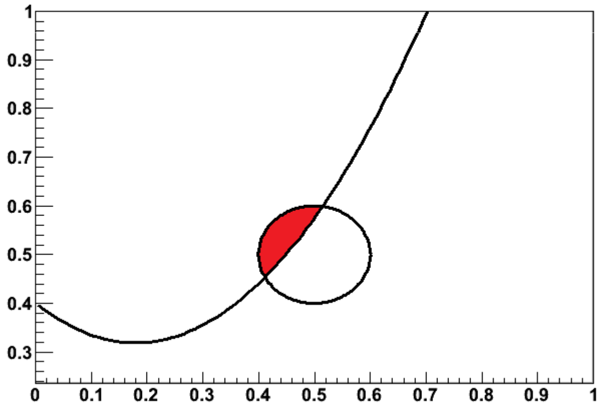
You are requested to generate N=10^6 couples (x,y) with x,y from a uniform distribution (Use the standard *rand* function) and to count the *hits* which occur when your x,y simultaneously satisfy

$$y > 2.5x^2 - 0.9x + 0.4$$

$$(x - 0.5)^2 + (y - 0.5)^2 < r^2$$

Your program should print the estimation of the intersection area I and its error to the screen :

$$I = \frac{N_{hit}}{N} \pm \frac{\sqrt{N_{hit}}}{N}$$



1.2)

Let the radius r vary and implement an algorithm which searches for a value of r so that the value of the integral found satisfies the criterion

$$|I - 0.1| < 0.001$$

Your program should print the chosen value for r, and the corresponding estimation of the intersection area to the screen.

Hint: You could use the bisection algorithm with initual values

$$r_{low} = 0.1 \qquad r_{high} = 0.4$$

Exercise no. 2:

Given the system of first order differential equations:

$$y'(x) = x^2 \sin(y) + e^x \cos(z)$$

$$z'(x) = 2xy + e^z$$

and initial conditions :

$$y(0) = 1 \qquad z(0) = -1$$

2.1)

Find approximate solution for y(x); z(x) over interval [0, 1] with stepsize h =0.1 using the 2nd order Runge Kutta method

$$k_1 = hf\,(x_0, y_0, z_0)$$

$$l_1 = hg\,(x_0, y_0, z_0)$$

$$k_2 = hf\,(x_0 + \tfrac{1}{2}h, y_0 + \tfrac{1}{2}k_1, z_0 + \tfrac{1}{2}l_1)$$

$$l_2 = hg\,(x_0 + \tfrac{1}{2}h, y_0 + \tfrac{1}{2}k_1, z_0 + \tfrac{1}{2}l_1)$$

$$y_1 = y_0 + k_2$$

$$z_1 = z_0 + l_2$$

Your program should print to the screen the final table in the form
y(0.1) = ... z(0.1) = ...
y(0.2) = ... z(0.2) = ...
... ...
y(1.0) = ... z(1.0) = ...

2.2)

Implement an iterative process which divides the stepsize in half at each iteration (h $\rightarrow$ h/2 $\rightarrow$ h/4 ...) until the precision 10^{-4} is reached. For a given iteration N you need to compare values of the solution $y_N(x)$ and $z_N(x)$ with corresponding values from the previous iteration $y_{N-1}(x)$ and $z_{N-1}(x)$ and continue until:

2.2a)

Both the following conditions are verified:

$$\left|y_N(1.0) - y_{N-1}(1.0)\right| < 10^{-4} \qquad \left|z_N(1.0) - z_{N-1}(1.0)\right| < 10^{-4}$$

2.2b)

All the following conditions are verified:

$$\left|y_N(0.1) - y_{N-1}(0.1)\right| < 10^{-4} \qquad \left|z_N(0.1) - z_{N-1}(0.1)\right| < 10^{-4}$$

$$\left|y_N(0.2) - y_{N-1}(0.2)\right| < 10^{-4} \qquad \left|z_N(0.2) - z_{N-1}(0.2)\right| < 10^{-4}$$

$$\ldots \qquad \ldots$$

$$\left|y_N(1.0) - y_{N-1}(1.0)\right| < 10^{-4} \qquad \left|z_N(1.0) - z_{N-1}(1.0)\right| < 10^{-4}$$

For both 2.2a) and 2.2b) your program is requested to print the final table

y(0.1) = ... z(0.1) = ...
y(0.2) = ... z(0.2) = ...
... ...
y(1.0) = ... Z(1.0)= ...

and the value of the stepsize h used to produce it.

Hint: In order to solve exercise 2.2b we suggest you to use global arrays (see the last slide of lesson 6)