

## Exercise 1

Given the function  $f(x) = \cos(x) - p \sqrt{x} e^{-x}$   
with  $p$  in  $[1,5]$

1.1) Write a program which asks the user to insert a value for the parameter  $p$  and calculates the root of the function in the interval  $[0,2]$  with the bisection method and a precision of  $10^{-6}$

1.2) Write a program which searches that value of  $p$  in  $[1,5]$  so that

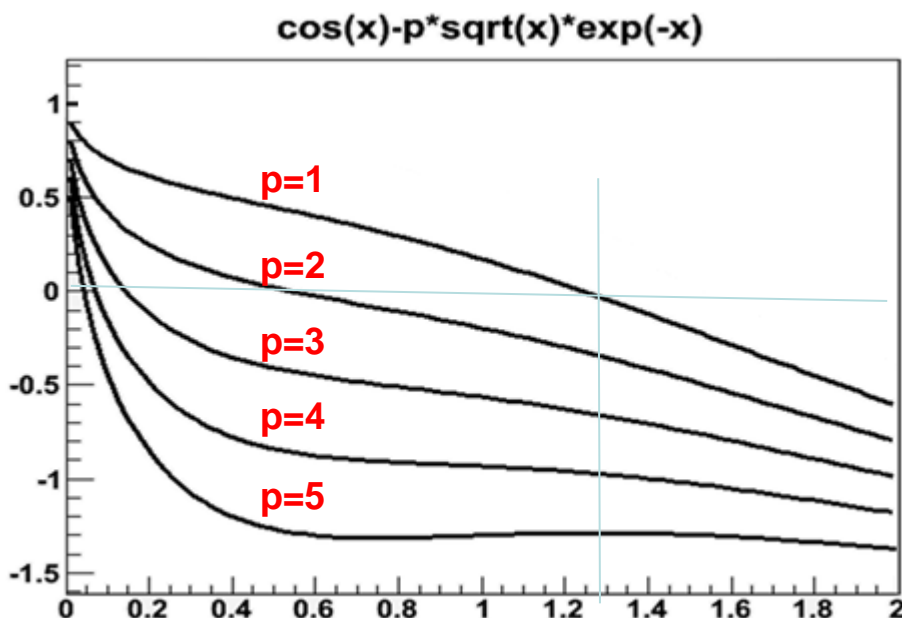
$$\int_0^2 f(x) dx = 0$$

using the Simpson rule with a precision of  $10^{-7}$ . The condition to be fulfilled is:

$$|I(\text{iteration } N) - I(\text{iteration } N-1)| < \text{precision}$$

and the value of the parameter such that

$$|I| < 10^{-5}$$



## Exercise 2

2.1) Write a program which simulates the Brownian Motion.

The simulation should model the motion in the following way. At time  $t=0$  a particle is at the point  $x_i=0, y_i=0$ . Then the particle starts to move with a given speed  $v$  and angle  $\theta$ . Extract two random numbers, one uniformly distributed in the interval  $[0, 2\pi]$  to be used as angle  $\theta$  and another from a Gaussian distribution with  $\mu=0$  and  $\sigma=1$  using the acceptance/rejection polar method to be used as particle speed. Assume that this constant speed is kept for one second and evaluate the new position of the particle:

$$x_n = x_{n-1} + |v| \cos(\theta)$$

$$y_n = y_{n-1} + |v| \sin(\theta)$$

Repeat the operation with new angle  $\theta$  and speed  $v$  generated in the same way, until the particle leaves a circle with radius 10 ( $x=0, y=0$  is the center of this circle).

Repeat the experiment for  $N=10^4$  particles. Evaluate the maximum, minimum and mean time employed by the particles to leave the circle. Print them to the screen.

2.2) Verify the uniformity of the final angle  $\theta$  distribution of the particles once they have left the circle in the following way.

If  $N$  is the total number of particle and  $B$  is the number of bins in which the  $\theta$  angle is subdivided, then  $N_m = N/B$  will be the average number of particle expected in each bin.

In order to prove that the distribution is uniform, you need to verify that the number of particle in each bin  $N_i$  verifies the rule  $|N_i - N_m| < 3 * \sqrt{N_i}$