第二次书面作业

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1

1.1

列出真值表

Tab. 1: (1)

P	Q	$\neg P \vee \neg Q$	$P \leftrightarrow \neg Q$	$(\neg P \vee \neg Q) \rightarrow (P \leftrightarrow \neg Q)$
T	T	F	F	T
T	F	T	T	T
F	T	T	T	T
F	F	T	F	F

因此得到析取范式为$P \vee Q$ 合取范式为$P \vee Q$

1.2

这个式子本身是析取范式，于是他的析取范式就是它本身

$$(P \wedge \neg Q \wedge S) \vee (\neg P \wedge Q \wedge R)$$

对这个式子取反，得到

$(\neg P \wedge \neg Q) \vee (\neg P \wedge Q \wedge \neg R) \vee (P \wedge Q \wedge \neg S) \vee (P \wedge \neg Q \wedge S) \vee (P \wedge Q \wedge S \wedge R)$
再次取反得到合取范式

$(P \vee Q) \wedge (P \vee \neg Q \vee R) \wedge (\neg P \vee \neg Q \vee S) \wedge (\neg P \vee Q \vee \neg S) \wedge (\neg P \vee \neg Q \vee \neg S \vee \neg R)$

2

1. 对于所有的x存在y使得$x + y = 0$ 正确
2. 存在一个x对于所有y使得$x + y = 0$ 错误

3

3.1

写出真值表

Tab. 2: (1)

P	Q	R	$P \vee \neg Q$	$(P \vee \neg Q) \rightarrow R$	$P \wedge R$	$((P \vee \neg Q) \rightarrow R) \rightarrow (P \wedge R)$
T	T	T	T	T	T	T
T	T	F	T	F	F	T
T	F	T	T	T	T	T
T	F	F	T	F	F	T
F	T	T	F	T	F	F
F	T	F	F	T	F	F
F	F	T	T	T	F	F
F	F	F	T	F	F	T

因此可以得到合取范式 $(P \vee \neg Q) \wedge (P \vee Q \vee \neg R)$

因此得到子句集为$\{(P \vee \neg Q), (P \vee Q \vee \neg R)\}$

3.2

$$\begin{aligned} G =&(\forall x)\{(\neg P(x) \vee (\forall y[\neg P(y) \vee P(f(x,y))])) \wedge (\neg(\forall y)[\neg Q(x,y) \vee P(y)])\} \\ =&(\forall x)\{(\neg P(x) \vee (\forall y[\neg P(y) \vee P(f(x,y))])) \wedge (\neg(\forall z)[\neg Q(x,z) \vee P(z)])\} \\ =&(\forall x)\{(\neg P(x) \vee (\forall y[\neg P(y) \vee P(f(x,y))])) \wedge ((\exists z)[Q(x,z) \wedge \neg P(z)])\} \\ =&(\forall x \forall y \exists z)\{(\neg P(x) \vee \neg P(y) \vee P(f(x,y))) \wedge Q(x,z) \wedge \neg P(z)\} \end{aligned} \tag{1}$$

得到子句集$\{(\neg P(x) \vee \neg P(y) \vee P(f(x,y))), Q(x,z), \neg P(g(x,y))\}$

4

1. 存在，$\sigma = \{a/x, b/y, b/z\}$
2. 存在，$\sigma = \{g(f(v), g(u))/x\}$
3. 如果存在$f(x) = x$的解$x = x_0$则存在，$\sigma = \{x_0/x, x_0/y\}$，否则不存在
4. 存在，$\sigma = \{b/x, b/y, b/z\}$

5

5.1

$$
\begin{aligned}
G =&\neg(\exists x P(x) \vee \exists y Q(y)) \vee (\exists z(P(z) \vee Q(z))) \\
=&(\forall x \neg P(x) \wedge \forall y \neg P(y)) \vee (\exists z(P(z) \vee Q(z))) \\
=&(\forall x \forall y \exists z)(\neg P(x) \wedge \neg P(y)) \vee P(z) \vee Q(z) \\
=&(\forall x \forall y \exists z)((\neg P(x) \vee P(z) \vee Q(z)) \wedge (\neg P(y) \vee P(z) \vee (Q(z))) \\
=&(\neg P(x) \vee P(g(x,y)) \vee Q(z)) \wedge (\neg P(y) \vee P(g(x,y)) \vee (Q(g(x,y)))
\end{aligned} \tag{2}
$$

5.2

$$
\begin{aligned}
G =&(\forall x)(\neg P(x) \vee ((\forall y)(\neg(\forall z) Q(z,y) \vee \neg(\forall z)(R(y,z))) \\
=&(\forall x)(\neg P(x) \vee ((\forall y)((\exists z)(\neg Q(z,y)) \vee (\exists t)(\neg R(y,t))) \\
=&(\forall x \forall y \exists z \exists t)(\neg P(x) \vee \neg Q(z,y) \vee \neg R(y,t)) \\
=&\neg P(x) \vee \neg Q(g(x,y),y) \vee \neg R(y, g(x,y))
\end{aligned} \tag{3}
$$

5.3

$$
\begin{aligned}
G =&\neg(\forall x P(x)) \vee ((\exists x \forall s \forall t)(Q(x,t) \vee R(x,s,t))) \\
=&(\exists r \neg P(r)) \vee ((\exists x \forall s \forall t)(Q(x,t) \vee R(x,s,t))) \\
=&\exists r \exists x \forall s \forall t(\neg P(r) \vee Q(x,t) \vee R(x,s,t)) \\
=&\neg P(f(s,t)) \vee Q(g(s,t),t) \vee R(g(s,t),s,t)
\end{aligned} \tag{4}
$$

6

推理过程比较简单因此直接采用简单的因果推理

$$\because Read(Liming), \forall x Read(x) \rightarrow Smart(x)$$

$$\therefore Smart(Liming)$$

$$\because Smart(Liming) \wedge \neg Poor(Liming), \forall x \neg Poor(x) \wedge Smart(x) \rightarrow Happy(x)$$

$$\therefore Happy(Liming)$$

$$\because \forall x Happy(x) \rightarrow Exciting(x)$$

$$\therefore Exciting(Liming)$$

得证李明过着激动人心的生活