

线控作业1

张蔚桐 2015011493 自55

2017 年 3 月 3 日

1

2

从图1中可以预计，系统的传递函数可以表示为 $G(s) = \frac{K}{s(T_1s+1)(T_2s+1)}$, $T_1 > 0, T_2 > 0$ 可以根据相频图估计转折频率 $\omega_1 = 18.14\text{rad/s}$, $\omega_2 = 1103\text{rad/s}$ 可得 $T_1 = \frac{1}{\omega_1} = 0.0551\text{s}$, $T_2 = \frac{1}{\omega_2} = 0.9066 \times 10^{-3}\text{s}$ ，同时，考虑辐频在 $\omega = 1\text{rad/s}$ 附近的增益可得 $20\lg(K) = -9.2$; $K = 0.346$ 因此系统传递函数可以表示为

$$G(s) = \frac{0.346}{s(0.551s+1)(0.9066 \times 10^{-3}s+1)}$$

，对应的Bode图如图2所示，可以看出和实际bode图1还是基本符合的

3

如图3显然这是一个惯性环节的图像，可以直接看出

$$G(s) = \frac{0.102}{(0.001s+1)}$$

MATLAB仿真之后的图像如图4所示，和原图3基本一致

4

可以看出系统的传递函数为 $G(s) = \frac{K}{s(Ts+1)}$, $T > 0$ ，并由图5可以得到 $20\lg(K) = 7.196$, $K = 2.29$, $T = \frac{1}{20} = 0.05$ 因此

$$G(s) = \frac{2.29}{s(0.05s+1)}$$

MATLAB作图如图6所示，和实际情况相差不多

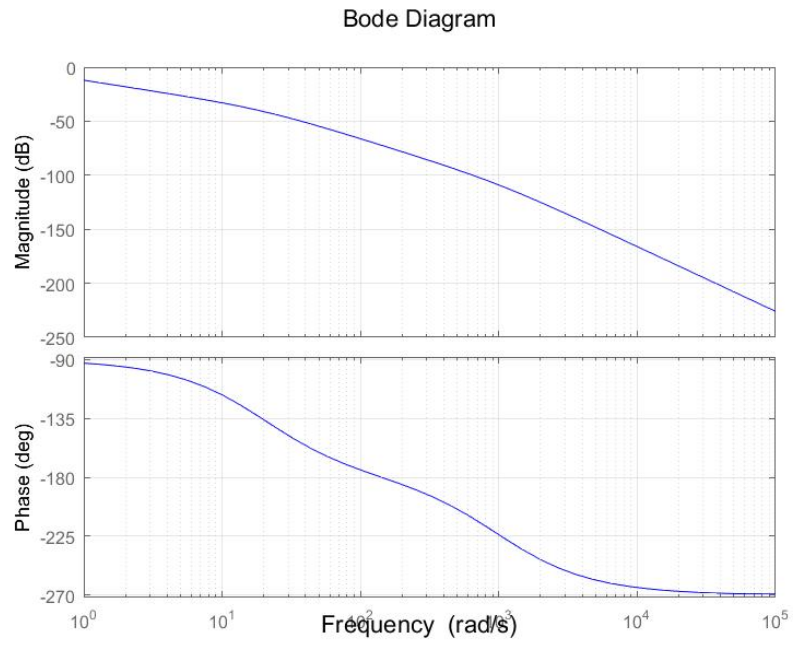


图 1: 原图像

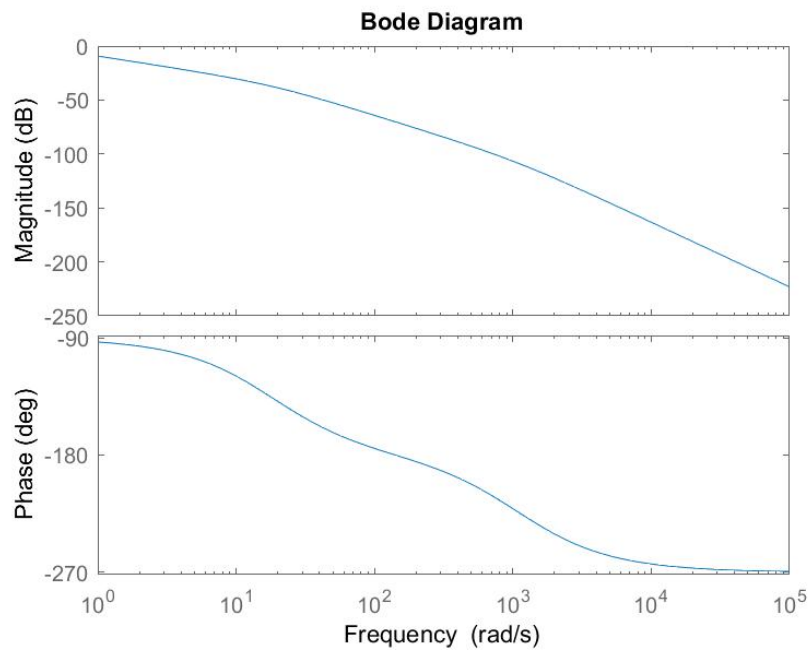


图 2: 理论图像

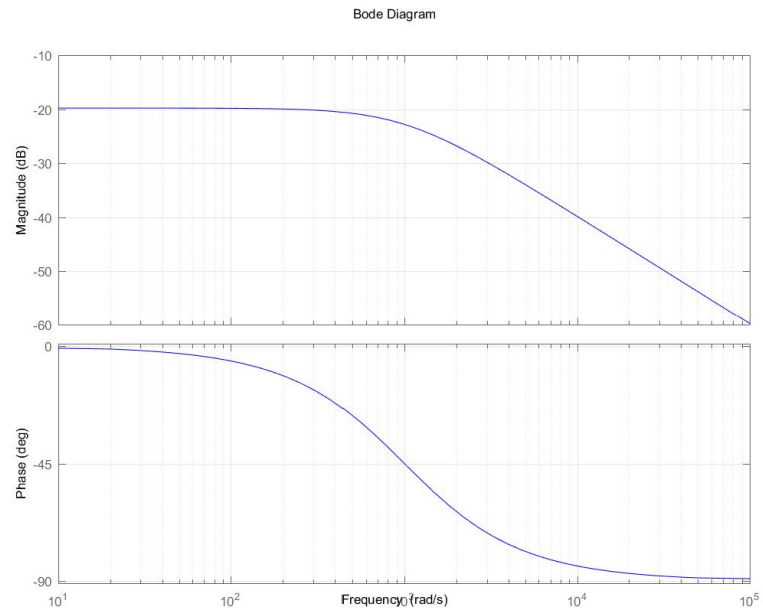


图 3: 原图像

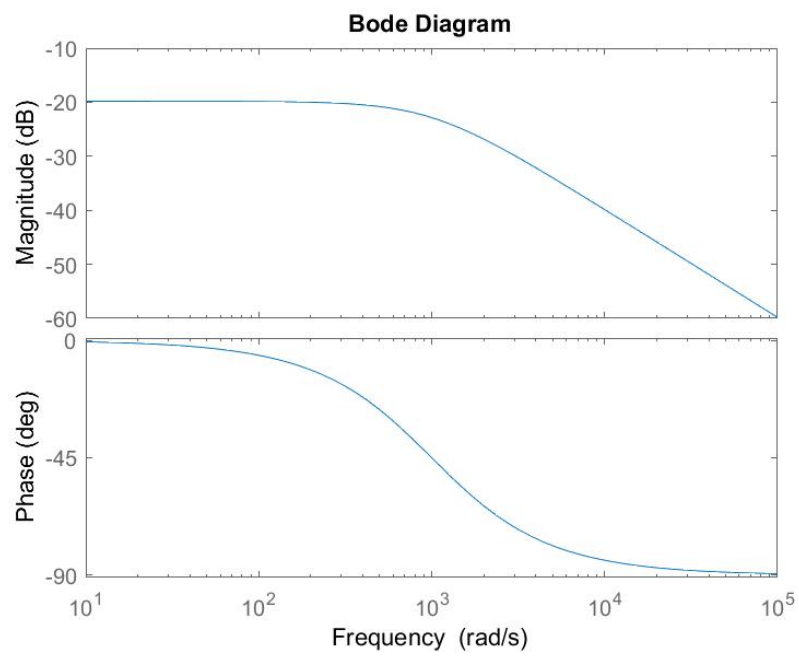


图 4: 理论图像

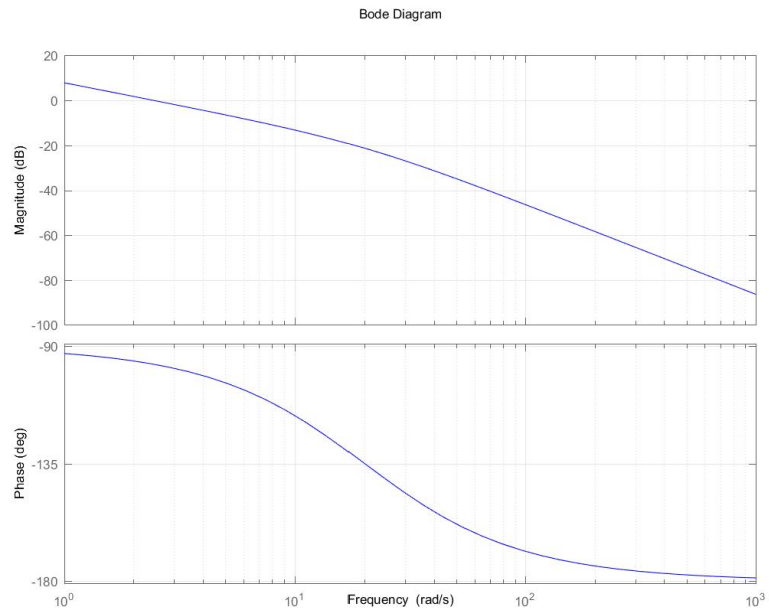


图 5: 原图像

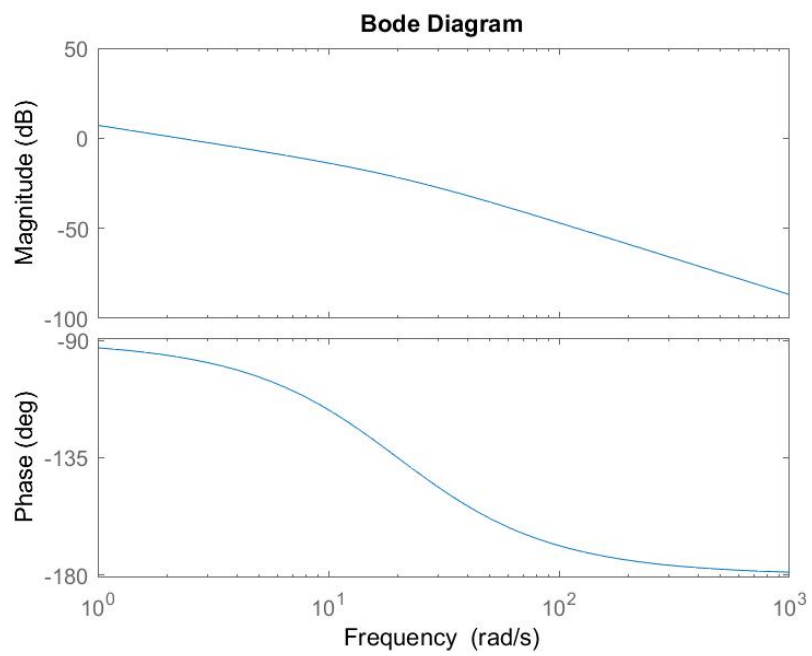


图 6: 理论图像

5

根据KCL, KVL, 可以得到

$$\begin{cases} \dot{x}_3 = \frac{1}{C_3}x_4 \\ \dot{x}_4 = -\frac{1}{L_1}x_1 - \frac{1}{L_1}x_3 + \frac{1}{L_1}u \\ \dot{x}_2 = -\frac{1}{C_2R}x_1 - \frac{1}{C_2R}x_2 + \frac{1}{C_2R}u \\ \dot{x}_1 = \frac{1}{C_1}x_4 - \frac{C_2}{C_1}\dot{x}_2 = \frac{1}{C_1R}x_1 + \frac{1}{C_1R}x_2 + \frac{1}{C_1}x_4 - \frac{1}{C_1R}u \end{cases}$$

并进一步得到

$$y = RC_2\dot{x}_2 = -x_1 - x_2 + u$$

因此得到

$$\begin{cases} \mathbf{A} = \begin{pmatrix} \frac{1}{RC_1} & \frac{1}{RC_1} & 0 & \frac{1}{C_1} \\ -\frac{1}{RC_2} & -\frac{1}{RC_2} & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{C_3} \\ -\frac{1}{L_1} & 0 & -\frac{1}{L_1} & 0 \end{pmatrix}; \mathbf{b} = \begin{pmatrix} -\frac{1}{RC_1} & \frac{1}{RC_2} & 0 & \frac{1}{L_1} \end{pmatrix}^T \\ \mathbf{c}^T = \begin{pmatrix} -1 & -1 & 0 & 0 \end{pmatrix}; \mathbf{d} = \begin{pmatrix} 1 \end{pmatrix} \end{cases}$$

6

由泊肃叶定律, 可以得到状态方程为

$$\begin{cases} c_1\dot{x}_1 = u_1 + \frac{\rho g}{R_2}x_2 - \frac{\rho g}{R_2}x_1 - \frac{\rho g}{R_1}x_1 \\ c_2\dot{x}_2 = u_2 - \frac{\rho g}{R_2}x_2 + \frac{\rho g}{R_2}x_1 \end{cases}$$

输出方程为

$$y = \frac{\rho g}{R_1}x_1$$

其中 ρ 是液体的密度, g 是重力加速度

7

记滑块1的位置为 F_1 滑块2的位置为 F_2 , 由牛顿第二定律得

$$\begin{cases} m_2\ddot{F}_2 = u(t) + k(F_1 - F_2 - l) - \xi_2\dot{F}_2 \\ m_1\ddot{F}_1 = -k(F_1 - F_2 - l) - \xi_1\dot{F}_1 \end{cases}$$

于是可以处理得到

$$\begin{cases} m_2 F_2 s^2 = U(s) + kF_1 - kF_2 - \xi F_2 s \\ m_1 F_1 s^2 = -kF_1 + kF_2 - \xi_1 F_1 s \end{cases}$$

于是

$$(m_1 s^2 + \xi_1 s + k)F_1 = k \frac{U(s) + kF_1}{m_2 s^2 + \xi_2 s + k}$$

整理得到

$$G(s) = \frac{k}{(m_1 s^2 + \xi_1 s + k)(m_2 s^2 + \xi_2 s + k) - k^2}$$

状态方程采用更简单的方式求取，设弹簧压缩的长度为 x_3 ，两个物体的速度为 x_1, x_2 第1个物体的位移是 x_4 ，由牛顿力学可得

$$\begin{cases} m_1 \dot{x}_1 = -\xi_1 x_1 + kx_3 \\ m_2 \dot{x}_2 = -\xi_2 x_2 - kx_3 + u \\ \dot{x}_3 = x_2 - x_1 \\ \dot{x}_4 = x_1 \end{cases}$$

于是得到

$$\begin{cases} \mathbf{A} = \begin{pmatrix} -\frac{\xi_1}{m_1} & 0 & \frac{k}{m_1} & 0 \\ 0 & \frac{\xi_2}{m_2} & -\frac{k}{m_2} & 0 \\ -1 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{pmatrix}; \mathbf{b} = \begin{pmatrix} 0 & 1 & 0 & 0 \end{pmatrix}^T \\ \mathbf{c}^T = \begin{pmatrix} 0 & 0 & 0 & 1 \end{pmatrix}; \mathbf{d} = 0 \end{cases}$$

8

$$\begin{cases} \mathbf{A} = \begin{pmatrix} 0 & 1 \\ \frac{g}{l} & 0 \end{pmatrix}; \mathbf{b} = \begin{pmatrix} 0 & 1 \end{pmatrix}^T \end{cases}$$

9

设 $\ddot{x} + 7\dot{x} + 14x = u$ 同时

$$\begin{cases} x_1 = x \\ x_2 = \dot{x} \\ x_3 = \ddot{x} \end{cases}$$

于是得到

$$\begin{cases} \mathbf{A} = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -8 & -14 & -7 \end{pmatrix}; \mathbf{b} = \begin{pmatrix} 0 & 0 & 1 \end{pmatrix}^T \\ \mathbf{c}^T = \begin{pmatrix} 3 & 0 & 0 \end{pmatrix}; \mathbf{d} = 0 \end{cases}$$

考察实现，传递函数可以化为

$$\frac{Y(s)}{U(s)} = \frac{3s^{-3}}{1 + 7s^{-1} + 14s^{-2} + 8s^{-3}}$$

记

$$e(s) = \frac{u(s)}{(1 + 7s^{-1} + 14s^{-2} + 8s^{-3})}$$

于是得到

$$\begin{cases} Y(s) = 3s^{-3}e(s) \\ e(s) = u(s) - 7s^{-1}e(s) - 14s^{-2}e(s) - 8s^{-3}e(s) \end{cases}$$

可以得到模拟电路图如图7为其模拟结构图

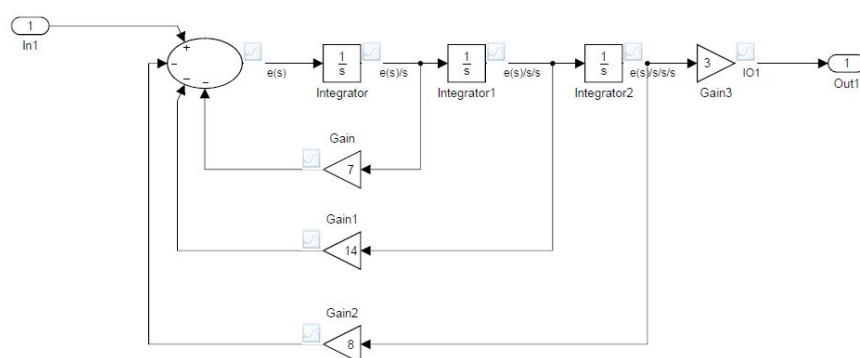
10

显然

$$G(s) = \frac{4}{s+1} + \frac{-2}{s+2}$$

于是可以得到其系统结构图8并进一步可以得到解耦标准型:

$$\begin{cases} \mathbf{A} = \begin{pmatrix} -1 & 0 \\ 0 & -2 \end{pmatrix}, \mathbf{b} = \begin{pmatrix} 4 & -2 \end{pmatrix}^T \\ \mathbf{c}^T = \begin{pmatrix} 1 & 1 \end{pmatrix}, \mathbf{d} = 0 \end{cases}$$



The diagram shows a control system with two parallel feedback loops. The input signal $In1$ is split into two paths. The top path consists of a gain block of 4, a summing junction (+), an integrator block ($1/s$), and a feedback gain block of -1. The bottom path consists of a gain block of -2, a summing junction (+), an integrator block ($1/s$), and a feedback gain block of -2. The outputs of both feedback loops are summed at a third summing junction (+) to produce the output $Out1$.

图 8: 11.模拟结构图

11

11.1

$$\left\{ \begin{array}{l} \mathbf{A} = \begin{pmatrix} 0 & 1 & 0 & 0 \\ -2 & -3 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & -4 & -4 \end{pmatrix}; \mathbf{b} = \begin{pmatrix} 0 & 1 & 0 & 1 \end{pmatrix}^{\mathbf{T}} \\ \mathbf{c}^{\mathbf{T}} = \begin{pmatrix} 1 & 0 & 1 & 2 \end{pmatrix}; \mathbf{d} = 0 \end{array} \right.$$

11.2

$$\left\{ \begin{array}{l} \mathbf{A} = \begin{pmatrix} 0 & 1 & 0 & 0 \\ -2 & -3 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & -4 & -4 \end{pmatrix}, \mathbf{b} = \begin{pmatrix} 0 & 1 & 0 & 0 \end{pmatrix}^{\mathbf{T}} \\ \mathbf{c}^{\mathbf{T}} = \begin{pmatrix} 0 & 0 & 1 & 2 \end{pmatrix}, \mathbf{d} = 0 \end{array} \right.$$

11.3

$$\left\{ \begin{array}{l} \mathbf{A} = \begin{pmatrix} 0 & 1 & 0 & 0 \\ -2 & -3 & -1 & -2 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & -4 & -4 \end{pmatrix}, \mathbf{b} = \begin{pmatrix} 0 & 1 & 0 & 0 \end{pmatrix}^{\mathbf{T}} \\ \mathbf{c}^{\mathbf{T}} = \begin{pmatrix} 1 & 0 & 0 & 0 \end{pmatrix}, \mathbf{d} = 0 \end{array} \right.$$

12

$$G(s) = \begin{pmatrix} 1 & 1 & 1 \\ -2 & -3 & -4 \end{pmatrix} \begin{pmatrix} s+2 & 0 & 0 \\ 0 & s+3 & 0 \\ 0 & 0 & s+4 \end{pmatrix}^{-1} \begin{pmatrix} 1 & -1 \\ -5 & 4 \\ 5 & -3 \end{pmatrix}$$

$$G(s) = \begin{pmatrix} \frac{1}{s+2} & \frac{1}{s+3} & \frac{1}{s+4} \\ \frac{-2}{s+2} & \frac{-3}{s+3} & \frac{-4}{s+4} \end{pmatrix} \begin{pmatrix} 1 & -1 \\ -5 & 4 \\ 5 & -3 \end{pmatrix}$$

$$G(s) = \begin{pmatrix} \frac{1}{s+2} - \frac{5}{s+3} + \frac{5}{s+4} & -\frac{1}{s+2} + \frac{4}{s+3} - \frac{3}{s+4} \\ -\frac{2}{s+2} + \frac{10}{s+3} - \frac{20}{s+4} & \frac{2}{s+2} - \frac{12}{s+3} + \frac{12}{s+4} \end{pmatrix}$$

13

$$\begin{cases} \dot{x}_1 = -2x_1 - 50x_2 - 50x_3 + 10u \\ \dot{x}_2 = x_1 + x_4 + x_2 + x_3 \\ \dot{x}_3 = x_1 + x_4 + x_2 \\ \dot{x}_4 = -x_2 - x_3 + 3x_4 \end{cases}$$

于是得到

$$\begin{cases} \mathbf{A} = \begin{pmatrix} -2 & -50 & -50 & 0 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 0 & 1 \\ 0 & -1 & -1 & 3 \end{pmatrix}, \mathbf{b} = \begin{pmatrix} 0 & 0 & 0 & 10 \end{pmatrix}^T \\ \mathbf{c}^T = \begin{pmatrix} 0 & 1 & 1 & 0 \end{pmatrix}, \mathbf{d} = 0 \end{cases}$$

14

显然有

$$\mathbf{x} = \begin{pmatrix} e^{2t} & te^{2t} & \frac{t^2}{2}e^{2t} \\ 0 & e^{2t} & te^{2t} \\ 0 & 0 & e^{2t} \end{pmatrix} \mathbf{x}_0, \text{ where } \mathbf{x}_0 = (x_1(0), x_2(0), x_3(0))^T$$

15

$$e^{At} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos(\theta t) & -\sin(\theta t) \\ 0 & \sin(\theta t) & \cos(\theta t) \end{pmatrix}$$

16

将 $\Phi(t)$ 对角化得到

$$\Phi(t) = \begin{pmatrix} -0.5 & 0.5 \\ 1 & 1 \end{pmatrix} \begin{pmatrix} e^{-t} & 0 \\ 0 & e^{3t} \end{pmatrix} \begin{pmatrix} -0.5 & 0.5 \\ 1 & 1 \end{pmatrix}^{-1}$$

于是

$$A = \begin{pmatrix} -0.5 & 0.5 \\ 1 & 1 \end{pmatrix} \begin{pmatrix} -1 & 0 \\ 0 & 3 \end{pmatrix} \begin{pmatrix} -0.5 & 0.5 \\ 1 & 1 \end{pmatrix}^{-1} = \begin{pmatrix} 1 & 1 \\ 4 & 1 \end{pmatrix}$$

17

$$\mathbf{x}(t) = \begin{pmatrix} e^{-t} & 0 \\ 0 & e^{-2t} \end{pmatrix} \begin{pmatrix} 2 \\ 3 \end{pmatrix} + \int_0^t \begin{pmatrix} e^{-(t-\tau)} & 0 \\ 0 & e^{-2(t-\tau)} \end{pmatrix} \begin{pmatrix} 1 \\ 1 \end{pmatrix} d\tau$$

整理并换元得

$$\mathbf{x}(t) = \begin{pmatrix} 2e^{-t} \\ 3e^{-2t} \end{pmatrix} + \int_0^t \begin{pmatrix} e^{-(\tau)} \\ e^{-2(\tau)} \end{pmatrix} d\tau \Rightarrow \mathbf{x}(t) = \begin{pmatrix} 2e^{-t} \\ 3e^{-2t} \end{pmatrix} + \begin{pmatrix} 1 - e^{-t} \\ 1 - \frac{1}{2}e^{-2(\tau)} \end{pmatrix}$$

$$\mathbf{x}(t) = \begin{pmatrix} 1 + e^{-t} \\ 1 + \frac{5}{2}e^{-2t} \end{pmatrix}$$