Basic Definitions:

Generalized Coordinate: $q = [q_1, q_2, \cdots, q_m]^T$

Coordinate: $x = [x_1, x_2, \cdots, x_n]^T$

Constrain: $x = x(q) = \left[x_1(q_1, q_2, \cdots, q_m), x_2(q_1, q_2, \cdots, q_m), \cdots, x_n(q_1, q_2, \cdots, q_m) \right]^T$

Jacobin Matrix:

$$\dot{x} = \frac{\partial x}{\partial q}\dot{q} = J\dot{q} = \begin{bmatrix} \frac{\partial x_1}{\partial q_1} & \frac{\partial x_1}{\partial q_2} & \cdots & \frac{\partial x_1}{\partial q_m} \\ \frac{\partial x_2}{\partial q_1} & \frac{\partial x_2}{\partial q_2} & \cdots & \frac{\partial x_2}{\partial q_m} \\ \cdots & \cdots & \cdots & \cdots \\ \frac{\partial x_n}{\partial q_1} & \frac{\partial x_n}{\partial q_2} & \cdots & \frac{\partial x_n}{\partial q_m} \end{bmatrix} \dot{q} \qquad J = J(x, q)$$

Accleration: $\ddot{x} = J\ddot{q} + \dot{J}\dot{q} = J\ddot{q} + D \qquad D = D(q, \dot{q})$

EOM: equation of motion

by Virtual Power: $J^T \cdot F^c = 0$

EOM: $M\ddot{x} = F^c + F^a$

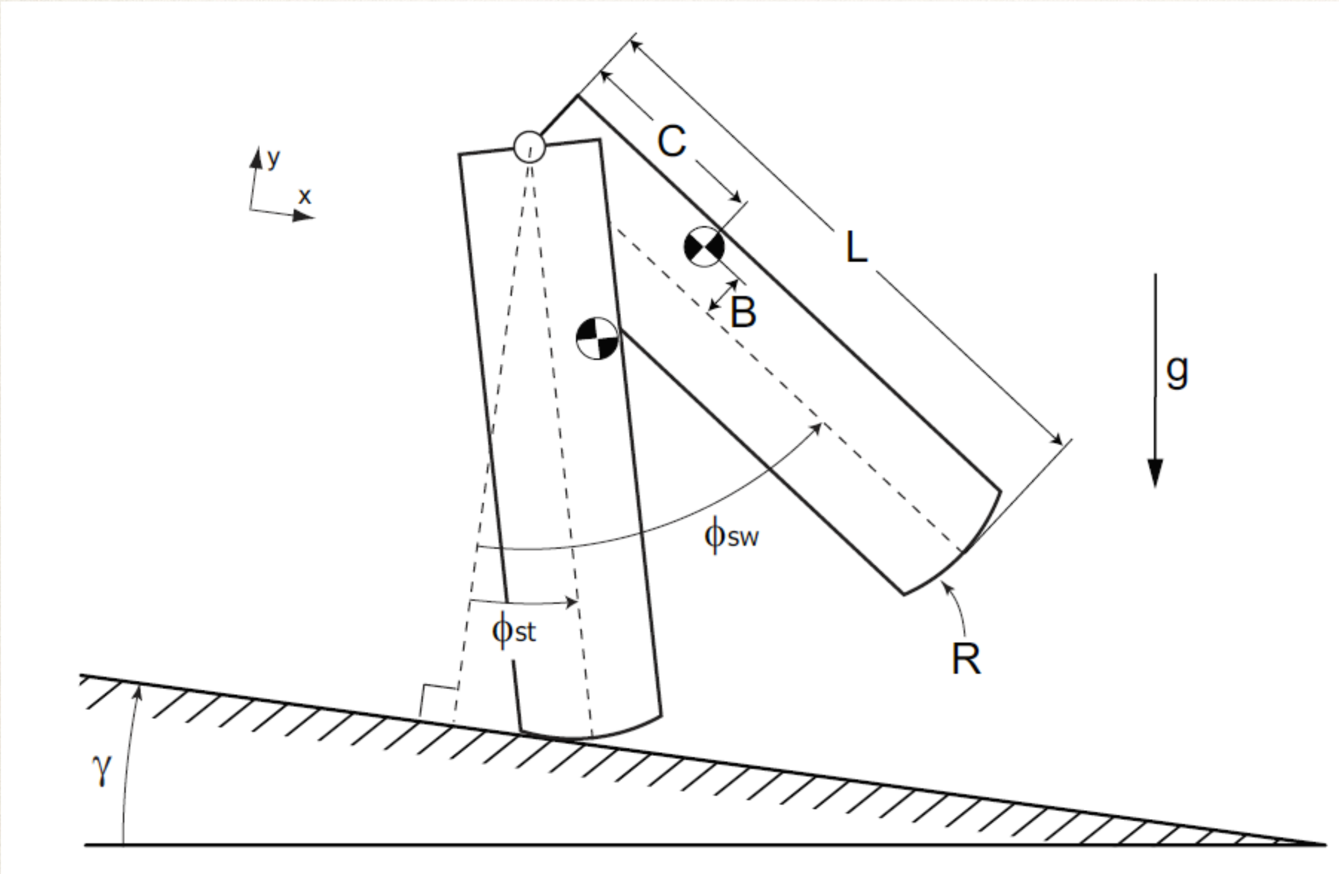
$$\begin{cases} MJ\ddot{q} + MD = F^c + F^a \\ J^T F^c = 0 \end{cases}$$

$J^T MJ\ddot{q} + J^T MD = J^T F^a$ when $(J^T MJ)$ is singular:

$$\boxed{\ddot{q} = (J^T MJ)^{-1}(J^T F^a - J^T MD)}$$

Constrain Force: $F^c = MJ\ddot{q} - MD - F^a$

Experiment model



参数:

L - 腿长

m - 腿部质量

B - 质心水平偏移

C - 质心竖直偏移

R - 脚半径

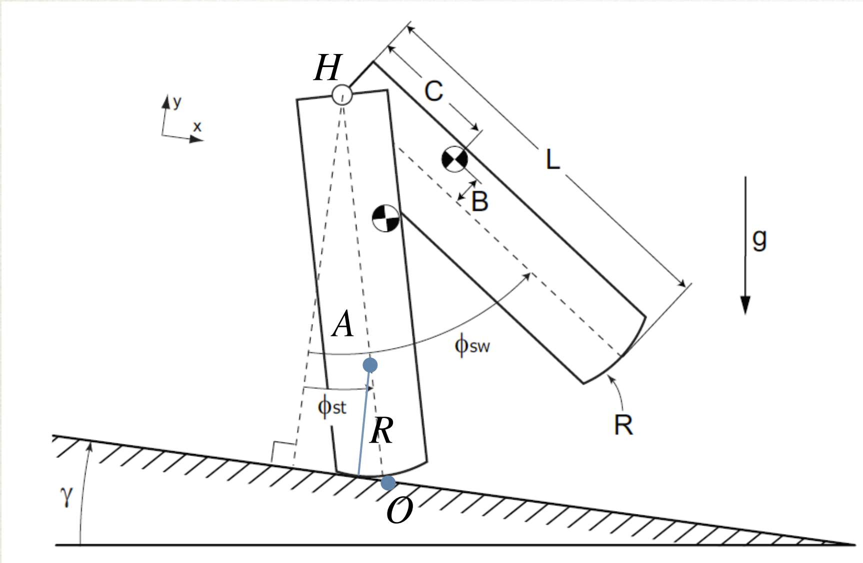
γ - 斜坡角度

变量:

ϕ_{st} - 支撑腿角度

ϕ_{sw} - 摆动腿角度

Experiment Model:EOM



$$\begin{cases} P_h = \begin{bmatrix} -R\phi_{st} - (L-R)\sin(\phi_{st}) \\ R + (L-R)\cos(\phi_{st}) \end{bmatrix} \\ R_{st} = \begin{bmatrix} \cos(\phi_{st}) & -\sin(\phi_{st}) \\ \sin(\phi_{st}) & \cos(\phi_{st}) \end{bmatrix} \\ R_{sw} = \begin{bmatrix} \cos(\phi_{sw}) & -\sin(\phi_{sw}) \\ \sin(\phi_{sw}) & \cos(\phi_{sw}) \end{bmatrix} \end{cases} \Rightarrow \begin{cases} P_{st} = P_h + R_{st}\begin{bmatrix} B \\ C \end{bmatrix} \\ P_{sw} = P_h + R_{sw}\begin{bmatrix} B \\ C \end{bmatrix} \end{cases}$$

$$\begin{cases} X = \left[P_{st}(1), P_{st}(2), \phi_{st}, P_{sw}(1), P_{sw}(2), \phi_{sw}\right]^T \\ q = [\phi_{st}, \phi_{sw}]^T \end{cases} \Rightarrow J = \frac{\partial X}{\partial q}$$

$$\begin{cases} M = diag[m, m, I, m, m, I] \\ F^a = mg\left[\sin(\gamma), -\cos(\gamma), 0, \sin(\gamma), -\cos(\gamma), 0\right]^T \end{cases}$$

$$\Rightarrow \boxed{\ddot{q} = (J^T M J)^{-1}(J^T F^a - J^T M D)}$$