INF240 Mandatory Exercise 3

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Computer problem 18.3.2

We want to decode the given vector that has been encoded with Hamming [15,11] Code and passed through a noisy channel:

$$y = 0 \quad 1 \quad 1 \quad 0 \quad 0 \quad 0 \quad 1 \quad 1 \quad 0 \quad 0 \quad 0 \quad 1 \quad 0 \quad 1 \quad 0$$

The resulting parity check matrix is:

$$H = \begin{matrix} 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 1 & 0 & 1 & 1 & 0 & 0 & 1 & 1 & 0 & 0 & 1 & 0 \\ 1 & 0 & 1 & 1 & 1 & 0 & 1 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 1 \end{matrix}$$

We use the algorithm described on p.416-417 in our book to decode.

We compute the syndrome s by taking the dot product of the received vector and the transposed parity check matrix, $s = yH^T$.

$$H^T = \begin{matrix} 0 & 1 & 0 & 1 \\ 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 \\ 1 & 0 & 0 & 1 \\ 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{matrix}$$

Now we compute the dot product of yH^T and get:

$$s = 1 \quad 1 \quad 0 \quad 0$$

Next we find out at what position in the parity check matrix the error is by comparing each column with the transposed syndrome.
We can see by looking at H that the error is in column 8.
We flip the 8th bit in y from 1 to 0 and get the decoded vector:

$$decodedVector = 0 \quad 1 \quad 1 \quad 0 \quad 0 \quad 0 \quad 1 \quad 0 \quad 0 \quad 0 \quad 0 \quad 1 \quad 0 \quad 1 \quad 0$$

Since this is a [15,11] Hamming Code we discard the last 4 bits of the decoded vector to obtain the original message:

$$originalMessage = 0 \quad 1 \quad 1 \quad 0 \quad 0 \quad 0 \quad 1 \quad 0 \quad 0 \quad 0 \quad 0$$

Program output:

```
Transposed matrix:
0 1 0 1
0 1 1 0
0 1 1 1
0 0 1 1
1 0 0 1
1 0 1 0
1 0 1 1
1 1 0 0
1 1 0 1
1 1 1 0
1 1 1 1
1 0 0 0
0 1 0 0
0 0 1 0
0 0 0 1
Syndrome vector: 1 1 0 0
Error at pos 8
Flipped bit from 1 to 0
Decoded vector is: 0 1 1 0 0 0 1 0 0 0 0 1 0 1 0
Original message was: 0 1 1 0 0 0 1 0 0 0 0
```