

Problem Sheet
Mathematik 1
Vorlesung vom 23/10/10

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1. Fill in the table and prove the derived rules from the slides.

2. Simplify the following expressions:

$$\begin{aligned}
 \text{(a)} \quad & \cos\left(\frac{\pi}{6} + x\right) - \cos\left(\frac{\pi}{6} - x\right) \\
 &= \cos\left(\frac{\pi}{6}\right) * \cos(x) - \sin\left(\frac{\pi}{6}\right) * \sin(x) - (\cos\left(\frac{\pi}{6}\right) * \cos(x) + \sin\left(\frac{\pi}{6}\right) * \sin(x)) \\
 &= \cos\left(\frac{\pi}{6}\right) * \cos(x) - \sin\left(\frac{\pi}{6}\right) * \sin(x) - \cos\left(\frac{\pi}{6}\right) * \cos(x) - \sin\left(\frac{\pi}{6}\right) * \sin(x) \\
 &= -\sin\left(\frac{\pi}{6}\right) * \sin(x) - \sin\left(\frac{\pi}{6}\right) * \sin(x) \\
 &= -2\left(\sin\left(\frac{\pi}{6}\right) * \sin(x)\right) \\
 &= -2 * (0.5 * \sin(x)) \\
 &= -\sin(x)
 \end{aligned}$$

$$\text{(b)} \quad \cos(x - 330^\circ) - \cos(120^\circ - x) + \sin(270^\circ - x)$$

$$\text{(c)} \quad \sin\left(\frac{2\pi}{3} - x\right) + \cos\left(\frac{5\pi}{6} - x\right)$$

$$\text{(d)} \quad \frac{1 - \cos^2(2x)}{2 \sin(x)}$$

3. Prove the following identities:

$$\text{(a)} \quad \frac{\sin x + \cos x \tan x}{\cos x - \sin x \tan x} = \tan(x + y)$$

$$\text{(b)} \quad \tan\left(\frac{\pi}{4} + x\right) = \frac{\cot x + 1}{\cot x - 1}$$

$$\text{(c)} \quad \frac{\tan x}{\tan(2x)} = \frac{1}{2} - \frac{1}{2} \tan^2 x$$

$$\text{(d)} \quad \tan^2\left(\frac{\pi}{4} + \frac{x}{2}\right) = \frac{1 + \sin x}{1 - \cos x}$$

$$\text{(e)} \quad \cos x * \cos y = \frac{1}{2} (\cos(x + y) + \cos(x - y))$$

$$\text{(f)} \quad \sin^2(45^\circ + 2x) = \frac{1 + \sin(2x)}{2}$$

$$\text{(g)} \quad \tan^2 x = \frac{1 - \cos(2x)}{1 + \cos(2x)}$$

$$\text{(h)} \quad \cot^2 x - \tan^2 x = \frac{4 * \cot(2x)}{\sin(2x)}$$

$$\text{(i)} \quad \frac{\sin(2x) + \sin x}{\cos(2x) + \cos x} = \tan\left(\frac{3}{2}x\right)$$

4. (Arens et al., 4.12 page 138) One of the following identities contains a typo. Correct it and prove both identities.

$$\text{(a)} \quad \sin(x + y) \sin^2\left(\frac{x - y}{2}\right) = \frac{1}{2} \sin(x + y) - \frac{1}{4} \sin(2x) - \frac{1}{4} \sin(2y)$$

$$\text{(b)} \quad \cos(3(x + y)) = 4 \cos^3(x + y) - 3 \cos x * \cos y - 3 \sin x * \sin y$$