Problem Sheet
Mathematik 1
Vorlesung vom 23/10/10

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1. Fill in the table and prove the derived rules from the slides.

α (degrees)	α (in radians)	$\sin\alpha$	$\cos\alpha$	$\tan\alpha$	$\cot\alpha$
$0°$	0	0	1	0	–
$30°$	$\frac{\pi}{6}$	–	–	–	–
$45°$	$\frac{\pi}{4}$	–	–	–	–
$60°$	$\frac{\pi}{3}$	–	–	–	–
$90°$	$\frac{\pi}{2}$	–	–	–	–

2. Simplify the following expressions:

   (a) $\cos(\frac{\pi}{6}+x)-\cos(\frac{\pi}{6}-x)$

$$
\begin{aligned}
&= \cos(\frac{\pi}{6}) * \cos(x) - \sin(\frac{\pi}{6}) * \sin(x) - (\cos(\frac{\pi}{6}) * \cos(x) + \sin(\frac{\pi}{6}) * \sin(x)) \\
&= \cos(\frac{\pi}{6}) * \cos(x) - \sin(\frac{\pi}{6}) * \sin(x) - \cos(\frac{\pi}{6}) * \cos(x) - \sin(\frac{\pi}{6}) * \sin(x) \\
&= -\sin(\frac{\pi}{6}) * \sin(x) - \sin(\frac{\pi}{6}) * \sin(x) \\
&= -2(\sin(\frac{\pi}{6}) * \sin(x)) \\
&= -2 * (0.5 * \sin(x)) \\
&= -\sin(x)
\end{aligned}
$$

   (b) $\cos(x-330°)-\cos(120°-x)+\sin(270°-x)$

   (c) $\sin(\frac{2\pi}{3}-x)+\cos(\frac{5\pi}{6}-x)$

   (d) $\frac{1-\cos^2(2x)}{2\sin(x)}$

3. Prove the following identities:

   (a) $\frac{\sin x+\cos x\tan x}{\cos x-\sin x\tan x}=\tan(x+y)$

   (b) $\tan(\frac{\pi}{4}+x)=\frac{cotx+1}{cotx-1}$

   (c) $\frac{\tan x}{\tan(2x)}=\frac{1}{2}-\frac{1}{2}\tan^2 x$

   (d) $\tan^2(\frac{\pi}{4}+\frac{x}{2})=\frac{1+\sin x}{1-\cos x}$

   (e) $\cos x*\cos y=\frac{1}{2}(\cos(x+y)+\cos(x-y))$

   (f) $\sin^2(45°+2x)=\frac{1+\sin(2x)}{2}$

   (g) $\tan^2 x=\frac{1-\cos(2x)}{1+\cos(2x)}$

   (h) $cot^2x-\tan^2 x=\frac{4*cot(2x)}{\sin(2x)}$

   (i) $\frac{\sin(2x)+\sin x}{\cos(2x)+\cos x}=\tan(\frac{3}{2}x)$

4. (Arens et al., 4.12 page 138) One of the following identities contains a typo. Correct it and prove both identities.

   (a) $\sin(x+y)\sin^2(\frac{x-y}{2})=\frac{1}{2}\sin(x+y)-\frac{1}{4}\sin(2x)-\frac{1}{4}\sin(2y)$

   (b) $cos(3(x+y))=4\cos^3(x+y)-3\cos x*cosy-3\sin x*\sin y$