Bachelorarbeit

Topological Entropy of Formal Languages

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1 Martingale Technique

Definition 1. A *martingale* is a family $(f_i, \mathcal{F}_i)_{i\in\{0,\dots,n\}}$ such that

- f_i is integrable for all $i \in \{0,\dots,n\}$,
- f_i is $\mathcal{F}_i$ measurable for all $i \in \{0,\dots,n\}$, and
- $f_i = \mathbb{E}[f_{i+1}|\mathcal{F}_i]$ for all $i \in \{0,\dots,n-1\}$.

Lemma 2.

$$\mu(\{x \in X \mid |f(x) - \mathbb{E}(f)| \geq c\}) \leq 2\exp\left(-\frac{c^2}{4\sum_{i=1}^n \|d_i\|_\infty^2}\right)$$

$$\mu(\{|f - \mathbb{E}(f)| \geq c\}) \leq 2\exp\left(-\frac{c^2}{4\sum_{i=1}^n \|d_i\|_\infty^2}\right)$$

Theorem 3. *If an mm-space (X, d, μ) has length l, then the concentration function of X satisfies*

$$\alpha_X(\varepsilon) \leq 2\exp\left(-\frac{\varepsilon^2}{16l^2}\right).$$

Theorem 4. *Let G be a compact group with a bi-invariant metric d, and let*

$$\{e\} = G_0 < G_1 < \cdots < G_n = G$$

be a chain of subgroups. Denote the diameter of G_i/G_{i-1} with respect to the factor metric by a_i. Then the concentration function of the mm-space (G, d, μ), where μ is the normalized Haar measure, satisfies

$$\alpha_X(\varepsilon) \leq 2\exp\left(-\frac{\varepsilon^2}{16\sum_{i=1}^n a_i^2}\right).$$

Theorem 5. *The normalized counting measure on the groups $\mathrm{SL}_{2^n}(q)$ concentrates with respect to the rank-metric, i.e. for all $r > 0$*

$$\lim_{n\to\infty} \alpha_{\mathrm{SL}_{2^n}}(r) = 0.$$

2 Fun

Consider an n-dimensional cube with 2^k nodes on each edge. Then its diameter $\nabla_{n,k}$ and length $L_{n,k}$ are

$$\nabla_{n,k} = \sqrt{(2^k - 1) \cdot n} \qquad\qquad L_{n,k} = \sqrt{\sum_{i=0}^{k-1} 2^{2i} \cdot n}.$$

Henceforth

$$\lim_{n\to\infty} \frac{L_{n,k}}{\nabla_{n,k}} = \frac{L_{1,k}}{\nabla_{1,k}} \qquad \text{and} \qquad \lim_{k\to\infty} \frac{L_{n,k}}{\nabla_{n,k}} = \frac{1}{\sqrt{3}}.$$

ERKLÄRUNG

Hiermit erkläre ich, dass ich die am heutigen Tag eingereichte Diplomarbeit zum Thema "Topological Entropy of Formal Languages" selbstständig erarbeitet, verfasst und Zitate kenntlich gemacht habe. Andere als die angegebenen Hilfsmittel wurden von mir nicht benutzt.

Datum Unterschrift