TECHNISCHE UNIVERSITÄT DRESDEN

DRESDEN concept

Institute of Theoretical Computer Science Chair of Automata Theory

UNIFICATION MODULO BOOLEAN RINGS

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Structure

Equational Unification

BR-Unification

Equational Unification

$$S := \left\{ f(a, x) \stackrel{?}{\approx} x \right\}$$

Equational Unification

$I := \{f(x, x) \approx x\}$

$$S := \left\{ f(a, x) \overset{?}{\approx} x \right\}$$

Equational Unification

$\mathrm{I} := \{\mathrm{f}(\mathrm{x}, \mathrm{x}) \approx \mathrm{x}\}$

$$\mathrm{S} := \left\{\mathrm{f}(\mathrm{a}, \mathrm{x}) \overset{?}{\approx} \mathrm{x}\right\} \qquad \Rightarrow \qquad \mathrm{S} := \left\{\mathrm{f}(\mathrm{a}, \mathrm{x}) \overset{?}{\approx}_{\mathrm{I}} \mathrm{x}\right\}$$

Unification Classes

Let S be an I-unification problem ...

$$\text{elementary:}\quad \mathrm{S} := \left\{ \mathrm{f}(\mathrm{y}, \mathrm{x}) \overset{?}{\approx}_{\mathrm{I}} \mathrm{x} \right\}$$

$$\text{with constants:}\quad \mathrm{S} := \left\{ \mathrm{f}(\mathrm{a}, \mathrm{x}) \overset{?}{\approx}_{\mathrm{I}} \mathrm{x} \right\}$$

$$\text{general:}\quad \mathrm{S} := \left\{ \mathrm{f}(\mathrm{g}(\mathrm{a}), \mathrm{x}) \overset{?}{\approx}_{\mathrm{I}} \mathrm{x} \right\}$$

MGU?

$$C := \{f(x, y) \approx f(y, x)\}$$

$$S := \left\{ f(x, y) \overset{?}{\approx}_C f(a, b) \right\}$$

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$$\sigma_1 := \{x \mapsto a, y \mapsto b\} \qquad \sigma_2 := \{x \mapsto b, y \mapsto a\}$$

More General

$$\sigma < \sigma'$$

iff

$$\exists \delta : \delta(\sigma) = \sigma'$$

More General

$$\begin{array}{ccc} \sigma < \sigma' & & \sigma \lesssim_{\mathrm{E}}^{\mathrm{X}} \sigma' \\ \text{iff} & \Rightarrow & \text{iff} \\ \exists \delta : \delta(\sigma) = \sigma' & & \exists \delta : \delta(\sigma(\mathrm{x})) \approx_{\mathrm{E}} \sigma'(\mathrm{x}) \text{ for all } \mathrm{x} \in \mathrm{X} \end{array}$$

Minimal Complete Sets

Let S be an E-unification problem. A minimal complete set of S is a set of substitutions $\mathcal{M}$ that satisfy the following properties.

- each $\sigma \in \mathcal{M}$ is an E-unifier of S
- for all E-unifiers θ of S there exists a $\sigma \in \mathcal{M}$ such that $\sigma \lesssim_{\mathrm{E}}^{\mathrm{X}} \theta$
- for all $\sigma, \sigma' \in \mathcal{M}$, $\sigma \lesssim_{\mathrm{E}}^{\mathrm{X}} \sigma'$ implies $\sigma = \sigma'$.

Unification Type of $\approx_{\mathrm{E}}$

unitary iff for all E-unification problems S there exists a minimal complete set of cardinality ≤ 1.

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infinitary	iff for all E-unification problems S there exists a minimal complete set, and there exists an E-unification problem for which this set is infinite.

Unification Type of $\approx_E$

unitary iff for all E-unification problems S there exists a minimal complete set of cardinality ≤ 1.

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infinitary iff for all E-unification problems S there exists a minimal complete set, and there exists an E-unification problem for which this set is infinite.

zero iff there exists an E-unification problem that does not have a minimal complete set.

BR-Unification