TECHNISCHE UNIVERSITÄT DRESDEN
Institute of Theoretical Computer Science Chair of Automata Theory

DRESDEN concept

UNIFICATION MODULO BOOLEAN RINGS

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Structure

Equational Unification

BR-Unification

Equational Unification

$$S := \left\{ f(a, x) \overset{?}{\approx} x \right\}$$

Equational Unification

$\mathrm{I} := \{\mathrm{f}(\mathrm{x}, \mathrm{x}) \approx \mathrm{x}\}$

$$\mathrm{S} := \left\{\mathrm{f}(\mathrm{a}, \mathrm{x}) \overset{?}{\approx} \mathrm{x}\right\}$$

Equational Unification

$\mathrm{I} := \{\mathrm{f}(\mathrm{x}, \mathrm{x}) \approx \mathrm{x}\}$

$$\mathrm{S} := \left\{\mathrm{f}(\mathrm{a}, \mathrm{x}) \overset{?}{\approx} \mathrm{x}\right\} \qquad \Rightarrow \qquad \mathrm{S} := \left\{\mathrm{f}(\mathrm{a}, \mathrm{x}) \overset{?}{\approx}_{\mathrm{I}} \mathrm{x}\right\}$$

Unification Classes

Let S be an I-unification problem ...

elementary: $S := \left\{ f(y, x) \overset{?}{\approx}_I x \right\}$

with constants: $S := \left\{ f(a, x) \overset{?}{\approx}_I x \right\}$

general: $S := \left\{ f(g(a), x) \overset{?}{\approx}_I x \right\}$

MGU?

$$\mathrm{C} := \{\mathrm{f}(\mathrm{x},\mathrm{y}) \approx \mathrm{f}(\mathrm{y},\mathrm{x})\}$$

$$\mathrm{S} := \left\{\mathrm{f}(\mathrm{x},\mathrm{y}) \overset{?}{\approx}_{\mathrm{C}} \mathrm{f}(\mathrm{a},\mathrm{b})\right\}$$

MGU?

$$C := \{f(x,y) \approx f(y,x)\}$$
$$S := \left\{ f(x,y) \overset{?}{\approx}_C f(a,b) \right\}$$

$$\sigma_1 := \{x \mapsto a, y \mapsto b\} \qquad \sigma_2 := \{x \mapsto b, y \mapsto a\}$$

More General

$$\sigma < \sigma'$$

iff

$$\exists \delta : \delta(\sigma) = \sigma'$$

More General

$$\begin{array}{ccc} \sigma < \sigma' & & \sigma \lesssim^{\mathrm{X}}_{\mathrm{E}} \sigma' \\ \text{iff} & \Rightarrow & \text{iff} \\ \exists \delta : \delta(\sigma) = \sigma' & & \exists \delta : \delta(\sigma(\mathrm{x})) \approx_{\mathrm{E}} \sigma'(\mathrm{x}) \text{ for all } \mathrm{x} \in \mathrm{X} \end{array}$$

Minimal Complete Sets

Let S be an E-unification problem. A minimal complete set of S is a set of substitutions $\mathcal{M}$ that satisfy the following properties.

- each $\sigma \in \mathcal{M}$ is an E-unifier of S
- for all E-unifiers θ of S there exists a $\sigma \in \mathcal{M}$ such that $\sigma \lesssim_{\mathrm{E}}^{\mathrm{X}} \theta$
- for all $\sigma, \sigma' \in \mathcal{M},\ \sigma \lesssim_{\mathrm{E}}^{\mathrm{X}} \sigma'$ implies $\sigma = \sigma'$.

BR-Unification