Make-up Exam

The exam should be done individually. You write your solutions on paper by yourself, scan (or photo capture through a mobile application such as CamScanner) and submit them as a single .pdf file. Your solutions have to be handwritten.

Solutions must be submitted electronically as .pdf file before **4:30 pm on June 26**. **No credit will be given to solutions obtained verbatim from the Internet or other sources.**

1. Given an array of size n that has the following specifications:
   - each element in the array contains either a policeman or a thief.
   - each policeman can catch only one thief.
   - a policeman cannot catch a thief who is more than K units away from the policeman

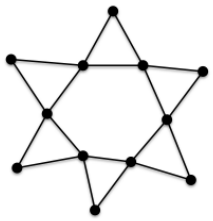
a) (10p) Devise a greedy algorithm that finds the maximum number of thieves that can be caught.

b) (20p) Prove that your algorithm satisfies the greedy-choice property.

For the array [P, T, T, P, T] and $k = 1$, your algorithm should output 2

For the array [T, T, P, P, T, P] and $k = 2$, your algorithm should output 3.

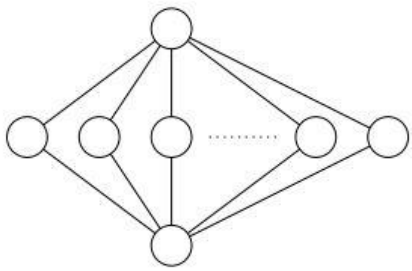
2. A star graph is an undirected graph of the following form, i.e. let n be the number of vertices, a star graph consists of a center cycle with n/2 vertices where each pair of connected vertices are also connected to a third vertex:



a) (10p) How long would it take Prim's or Kruskal's algorithms to find the minimum spanning tree of a given star graph (as a function of n)?

b) (25p) Describe and analyze a more efficient algorithm for finding the minimum spanning tree of a star graph. To get full credit, you need to argue the running time.

3. Consider a pole-graph which is an undirected graph with positive edge weights, consisting of two poles connected through a layer of nodes as follows:



a) (10p) How long would it take Dijkstra's algorithm to find the single-source-shortest-paths from one of the poles in a pole-graph to all other nodes (as a function of n where n is the number of vertices in a pole-graph)?

b) (25p) Describe and analyze a more efficient algorithm for solving the single-source-shortest-paths problem on a pole-graph. To get full credit, you need to argue the running time.