

ANKARA UNIVERSITY COMPUTER ENGINEERING
2019-2020 COM364 AUTOMATA THEORY
FINAL EXAMINATION

02.06.2020

Important Note: Please write your answers clearly and explain your reasoning. Points will be deducted if your final result is correct but how you obtained it is not clear.

1. [18 points] Give a regular expression for the following language

$$L = \{0^m 1^n \mid m + n \text{ is even}\}$$

2. [15 points] The explanation below is a proof for showing that the language of 0^*1^* is not a regular language. Is this proof correct or incorrect? If it is not correct, explain what the mistake is.

We can show that 0^*1^* is not regular by using proof by contradiction. Assume it is regular. According to the pumping lemma, there has to be a pumping length, so let p represent the pumping length for this language. We choose $s = 0^p 1^p$. Obviously, $s \in 0^*1^*$ and $|s| = 2p \geq p$. We should be able to split this s into three parts as $s = xyz$ such that the three conditions below are true:

- (i) $|y| > 0$,
- (ii) $|xy| \leq p$, and
- (iii) for each $i \geq 0$, $xy^i z \in 0^*1^*$

Because of the first two conditions, it has to be the case that y is not empty and it contains only 0s. So, when we consider xy^2z for this string, we know that there will be more 0s than 1s. Then, $xy^2z \notin 0^*1^*$ which means that we have a contradiction. Therefore, 0^*1^* cannot be regular. ■

3. Consider the context-free grammar (CFG) given below for the following questions.

$$\begin{aligned} S &\rightarrow VX|Y \\ V &\rightarrow 0V1|01 \\ X &\rightarrow X2|2 \\ Y &\rightarrow 0Y2|0Z2 \\ Z &\rightarrow Z1|1 \end{aligned}$$

- a. [14 points] Describe the language of this grammar mathematically or with at most one or two sentences. In other words, what kind of strings is generated with this grammar?
- b. [15 points] Is this grammar ambiguous? Why or why not?
- c. [20 points] Give the PDA which recognizes the language of this grammar.
- d. [18 points] Convert the given grammar into an equal one in Chomsky Normal Form (CNF). Show your steps clearly.