HW1

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Setting

```python
import math
import numpy as np
import pandas as pd
import matplotlib.pyplot as plt
from pandas.plotting import scatter_matrix
```

Exercise 1

(a) CEO Salary

- **Type**: Regression
- **Reason**: We are mainly interested in understanding which predictors (profit, employees, industry) affect salary, not just predicting new values.
- **Goal**: Inference (understand which factors affect salary)
- **n** = 500
- **p** = 3

(b) Product Success/Failure

- **Type**: Classification

- **Reason**: The purpose is to predict whether a new product will succeed, based on previous products' data.
- **Goal**: Prediction (predict whether new product will succeed)
- **n** = 20
- **p** = 13

(c) Exchange Rate Prediction

- **Type**: Regression
- **Reason**: The response variable is % change in USD/Euro, which is continuous.
- **Goal**: Prediction (predict % change in USD/Euro)
- **n** = 52
- **p** = 3

Exercise 2

(a)

```
data = {
    "Obs": [1, 2, 3, 4, 5, 6],
    "X1": [0, 2, 0, 0, -1, 1],
    "X2": [3, 0, 1, 1, 0, 1],
    "X3": [0, 0, 3, 2, 1, 1],
    "Y": ["Red", "Red", "Red", "Green", "Green", "Red"]
}
df = pd.DataFrame(data)
test_point = np.array([0, 0, 0])
df["Distance"] = np.sqrt((df["X1"] - test_point[0])**2 +
                         (df["X2"] - test_point[1])**2 +
                         (df["X3"] - test_point[2])**2)
df_sorted = df.sort_values("Distance").reset_index(drop=True)
df_sorted
```

	Obs	X1	X2	X3	Y	Distance
0	5	-1	0	1	Green	1.414214
1	6	1	1	1	Red	1.732051
2	2	2	0	0	Red	2.000000
3	4	0	1	2	Green	2.236068
4	1	0	3	0	Red	3.000000
5	3	0	1	3	Red	3.162278

(b)

K=1, KNN assigns the test point the same class as the single closest observation. Since Obs 5 (Green) is the nearest, the predicted class is **Green**.

(c)

K=3, the test point is classified according to the majority among its three closest neighbors. Since 2 out of 3 are Red, the prediction is **Red**.

(d)

If the Bayes decision boundary is highly nonlinear, the best K would be small. Because small K gives KNN higher flexibility, allowing it to trace the nonlinear decision boundary more accurately.

Exercise 3

(a)

```
college = pd.read_csv("College.csv")
print(college.head())
print(college.info())
```

```
                  Unnamed: 0 Private  Apps  Accept  Enroll  Top10perc  \
0  Abilene Christian University     Yes  1660    1232     721         23
1            Adelphi University     Yes  2186    1924     512         16
2                Adrian College     Yes  1428    1097     336         22
3           Agnes Scott College     Yes   417     349     137         60
```

```
4     Alaska Pacific University     Yes   193     146      55         16

   Top25perc  F.Undergrad  P.Undergrad  Outstate  Room.Board  Books  Personal  \
0         52         2885          537      7440        3300    450      2200
1         29         2683         1227     12280        6450    750      1500
2         50         1036           99     11250        3750    400      1165
3         89          510           63     12960        5450    450       875
4         44          249          869      7560        4120    800      1500

   PhD  Terminal  S.F.Ratio  perc.alumni  Expend  Grad.Rate
0   70        78       18.1           12    7041         60
1   29        30       12.2           16   10527         56
2   53        66       12.9           30    8735         54
3   92        97        7.7           37   19016         59
4   76        72       11.9            2   10922         15
<class 'pandas.core.frame.DataFrame'>
RangeIndex: 777 entries, 0 to 776
Data columns (total 19 columns):
 #   Column       Non-Null Count  Dtype
---  ------       --------------  -----
 0   Unnamed: 0   777 non-null    object
 1   Private      777 non-null    object
 2   Apps         777 non-null    int64
 3   Accept       777 non-null    int64
 4   Enroll       777 non-null    int64
 5   Top10perc    777 non-null    int64
 6   Top25perc    777 non-null    int64
 7   F.Undergrad  777 non-null    int64
 8   P.Undergrad  777 non-null    int64
 9   Outstate     777 non-null    int64
 10  Room.Board   777 non-null    int64
 11  Books        777 non-null    int64
 12  Personal     777 non-null    int64
 13  PhD          777 non-null    int64
 14  Terminal     777 non-null    int64
 15  S.F.Ratio    777 non-null    float64
 16  perc.alumni  777 non-null    int64
 17  Expend       777 non-null    int64
 18  Grad.Rate    777 non-null    int64
dtypes: float64(1), int64(16), object(2)
memory usage: 115.5+ KB
None
```

(b)

```
college2 = pd.read_csv("College.csv", index_col=0)
college = pd.read_csv("College.csv")
college3 = college.rename({"Unnamed: 0": "College"}, axis=1)
college3 = college3.set_index("College")
college = college3
college
```

	Private	Apps	Accept	Enroll	Top10perc	Top25perc	F.Undergrad	F
College								
Abilene Christian University	Yes	1660	1232	721	23	52	2885	5
Adelphi University	Yes	2186	1924	512	16	29	2683	1
Adrian College	Yes	1428	1097	336	22	50	1036	9
Agnes Scott College	Yes	417	349	137	60	89	510	6
Alaska Pacific University	Yes	193	146	55	16	44	249	8
...	...	...	...	...	...	...	...	.
Worcester State College	No	2197	1515	543	4	26	3089	2
Xavier University	Yes	1959	1805	695	24	47	2849	1
Xavier University of Louisiana	Yes	2097	1915	695	34	61	2793	1
Yale University	Yes	10705	2453	1317	95	99	5217	8
York College of Pennsylvania	Yes	2989	1855	691	28	63	2988	1

(c)

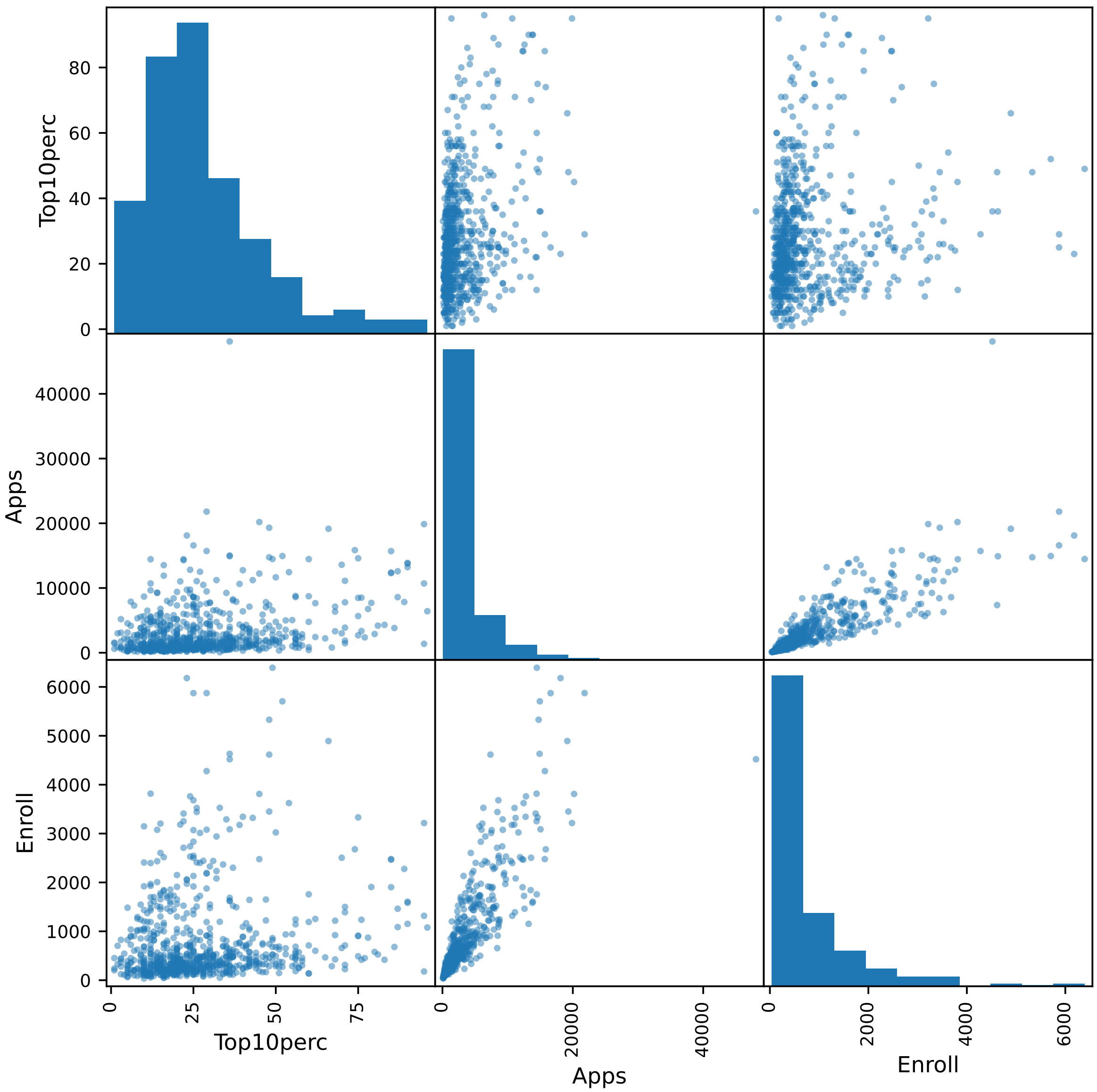
```
summary = college.describe()
summary
```

	Apps	Accept	Enroll	Top10perc	Top25perc	F.Undergrad	P.Undergrad
count	777.000000	777.000000	777.000000	777.000000	777.000000	777.000000	777.000000
mean	3001.638353	2018.804376	779.972973	27.558559	55.796654	3699.907336	855.298584
std	3870.201484	2451.113971	929.176190	17.640364	19.804778	4850.420531	1522.431887
min	81.000000	72.000000	35.000000	1.000000	9.000000	139.000000	1.000000
25%	776.000000	604.000000	242.000000	15.000000	41.000000	992.000000	95.000000
50%	1558.000000	1110.000000	434.000000	23.000000	54.000000	1707.000000	353.000000
75%	3624.000000	2424.000000	902.000000	35.000000	69.000000	4005.000000	967.000000
max	48094.000000	26330.000000	6392.000000	96.000000	100.000000	31643.000000	21836.000000

Apps	Accept	Enroll	Top10perc	Top25perc	F.Undergrad	P.Undergrad

(d)

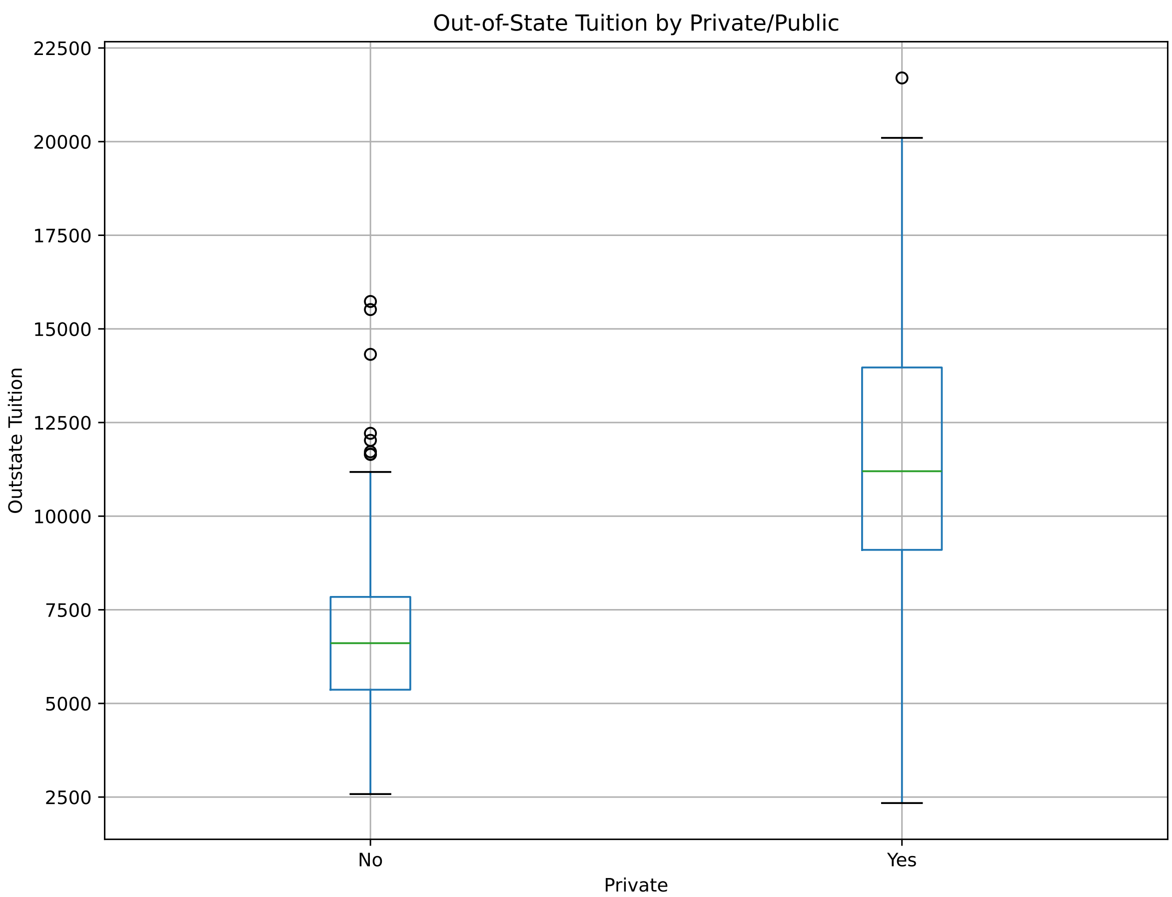
```
cols = ["Top10perc", "Apps", "Enroll"]
scatter_matrix(college[cols], figsize=(8, 8), diagonal='hist')
plt.show()
```



(e)

```
college.boxplot(column="Outstate", by="Private", figsize=(10,8))
plt.title("Out-of-State Tuition by Private/Public")
plt.suptitle("")
plt.xlabel("Private")
```

```
plt.ylabel("Outstate Tuition")
plt.show()
```

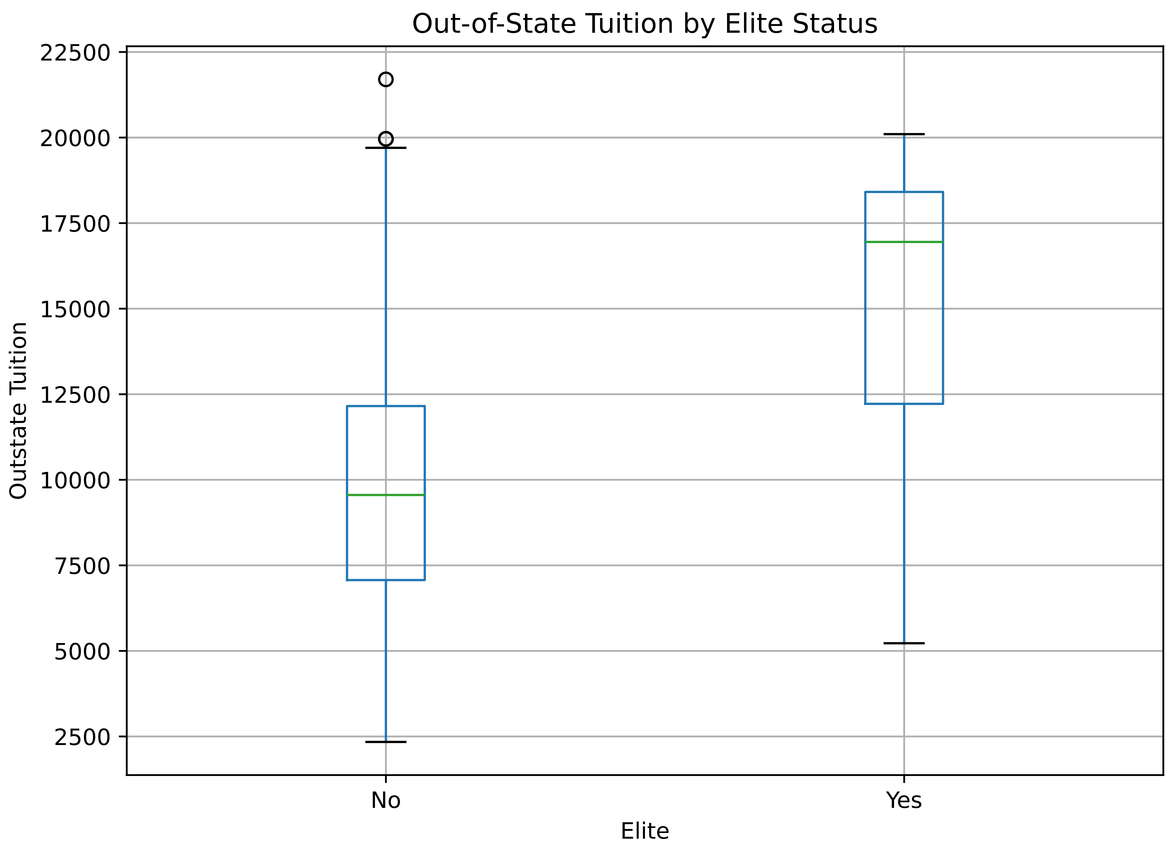


(f)

```
college['Elite'] = pd.cut(
    college['Top10perc'],
    bins=[0, 50, 100],
    labels=['No', 'Yes']
)

print(college['Elite'].value_counts())

college.boxplot(column='Outstate', by='Elite', figsize=(8,6))
```

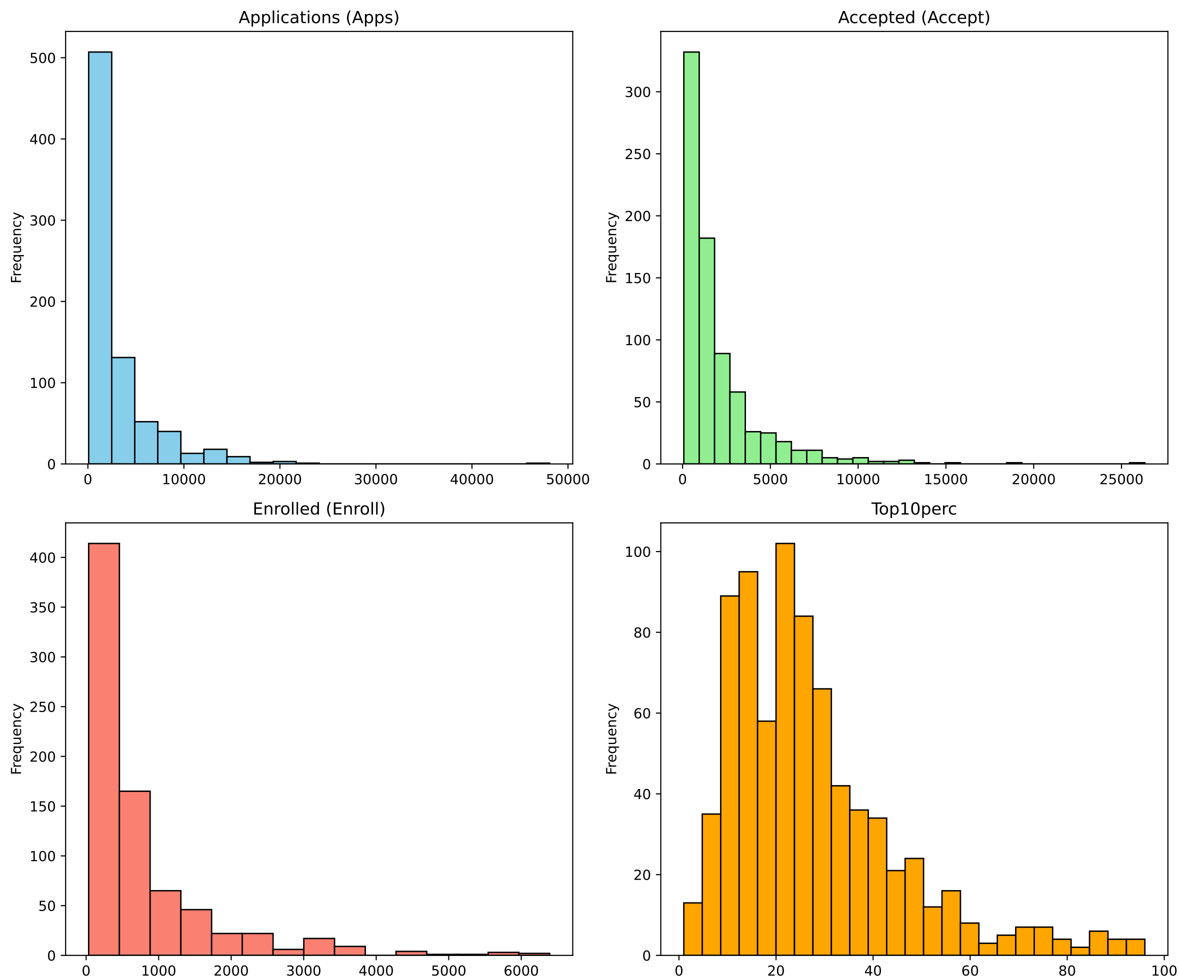
```python
plt.title("Out-of-State Tuition by Elite Status")
plt.suptitle("")
plt.xlabel("Elite")
plt.ylabel("Outstate Tuition")
plt.show()
```

```
Elite
No     699
Yes     78
Name: count, dtype: int64
```



(g)

```python
cols = ["Apps", "Accept", "Enroll", "Top10perc"]

fig, axes = plt.subplots(2, 2, figsize=(12, 10))

college["Apps"].plot.hist(bins=20, ax=axes[0,0], color="skyblue", edgecolor="black")
axes[0,0].set_title("Applications (Apps)")

college["Accept"].plot.hist(bins=30, ax=axes[0,1], color="lightgreen", edgecolor="black")
axes[0,1].set_title("Accepted (Accept)")

college["Enroll"].plot.hist(bins=15, ax=axes[1,0], color="salmon", edgecolor="black")
axes[1,0].set_title("Enrolled (Enroll)")

college["Top10perc"].plot.hist(bins=25, ax=axes[1,1], color="orange", edgecolor="black")
axes[1,1].set_title("Top10perc")

plt.tight_layout()
plt.show()
```



(h)

Summary:

Private schools generally have much higher Outstate tuition compared to public schools, with a clear separation in the boxplots. Only a small fraction of schools (78 of 777) are classified as Elite (Top10perc > 50), and these also tend to have higher tuition than non-Elite schools. The distribution of Top10perc shows that most colleges have 10–40% of freshmen from the top 10% of their high school class, but a small set of highly selective institutions reach above 80%. The number of applications, acceptances, and enrollments are highly right-skewed: while most schools have under 10,000 applications, a few receive over 40,000. Graduation rates are generally between 40–80%, though some data values exceed 100%. Overall, the dataset is heterogeneous, covering small private colleges, large state schools, and elite universities. Tuition and selectivity (Top10perc) appear positively related, and both the Private and Elite variables highlight systematic differences in costs and student profiles.

Exercise 4

1

The unit sphere in $\mathbb{R}^p$ fits inside the hypercube $[-1, 1]^p$, whose side length is 2.
In p dimensions, the volume of a hypercube is (side length)^p. Therefore,

$V_c(p) = 2^p$

2

The volume of the p-dimensional unit ball is

$V_s(p) = \dfrac{\pi^{p/2}}{\Gamma(\frac{p}{2}+1)}$

- If $p = 2m$ (even): $V_s(2m) = \dfrac{\pi^m}{m!}$
- If $p = 2m+1$ (odd): $V_s(2m+1) = \dfrac{2^{m+1}\pi^m}{(2m+1)!!}$

Examples: $V_s(2) = \pi$, $V_s(3) = \frac{4}{3}\pi$

3

The ratio is defined as

$\alpha(p) = \dfrac{V_s(p)}{V_c(p)}$

where $V_s(p) = \dfrac{\pi^{p/2}}{\Gamma(\frac{p}{2}+1)}$ and $V_c(p) = 2^p$.
Thus,

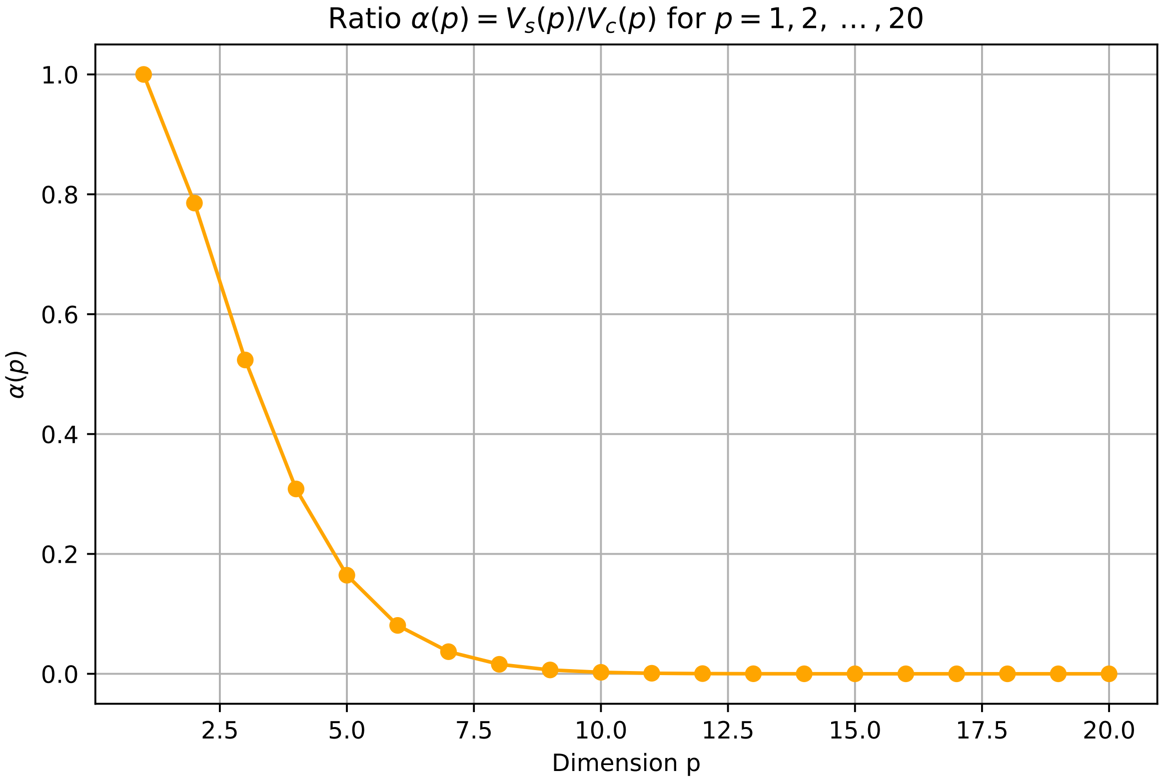
$\alpha(p) = \dfrac{\pi^{p/2}}{2^p\,\Gamma(\frac{p}{2}+1)}$

4

```
def V_s(p):
    return math.pi**(p/2) / math.gamma(p/2 + 1)

def V_c(p):
    return 2**p

p_vals = np.arange(1, 21)
alpha_vals = [V_s(p) / V_c(p) for p in p_vals]

df = pd.DataFrame({"p": p_vals, "alpha(p)": alpha_vals})
print(df)

plt.figure(figsize=(8, 5))
plt.plot(p_vals, alpha_vals, marker="o", color="orange")
plt.title(r"Ratio $\alpha(p) = V_s(p)/V_c(p)$ for $p=1,2,\dots,20$")
plt.xlabel("Dimension p")
plt.ylabel(r"$\alpha(p)$")
plt.grid(True)
plt.show()
```

```
     p      alpha(p)
0    1  1.000000e+00
1    2  7.853982e-01
2    3  5.235988e-01
3    4  3.084251e-01
4    5  1.644934e-01
5    6  8.074551e-02
6    7  3.691223e-02
7    8  1.585434e-02
8    9  6.442400e-03
9   10  2.490395e-03
10  11  9.199726e-04
11  12  3.259919e-04
12  13  1.111607e-04
13  14  3.657620e-05
14  15  1.164073e-05
15  16  3.590860e-06
16  17  1.075600e-06
17  18  3.133617e-07
```

```
18  19  8.892365e-08
19  20  2.461137e-08
```



5

The statement "In higher dimensions most of the volume is in the corners" is accurate. As dimension increases, the inscribed sphere occupies a vanishing fraction of the cube's volume. Most of the cube's volume lies near its corners and edges, far from the center. This is a direct manifestation of the curse of dimensionality.