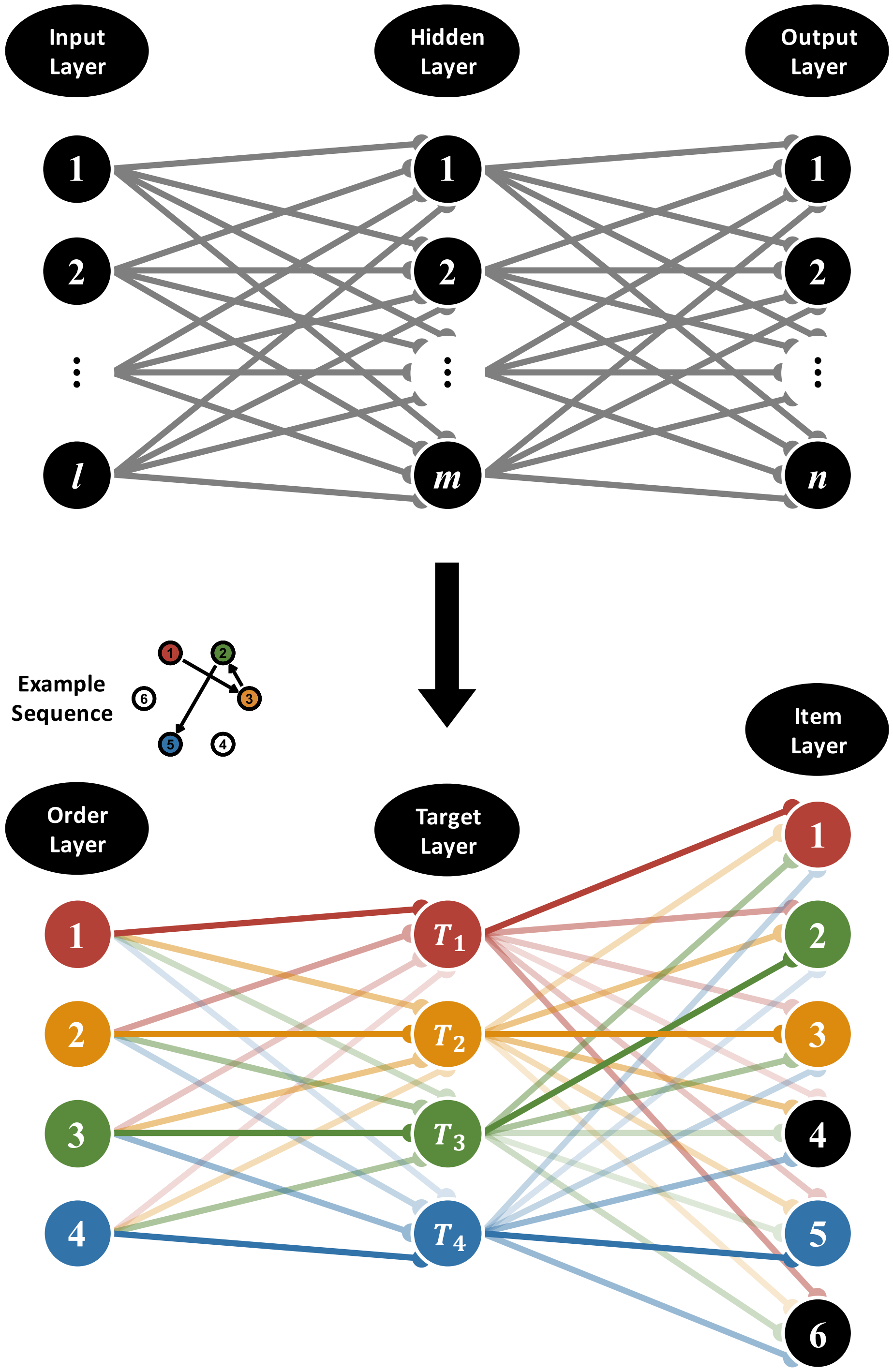
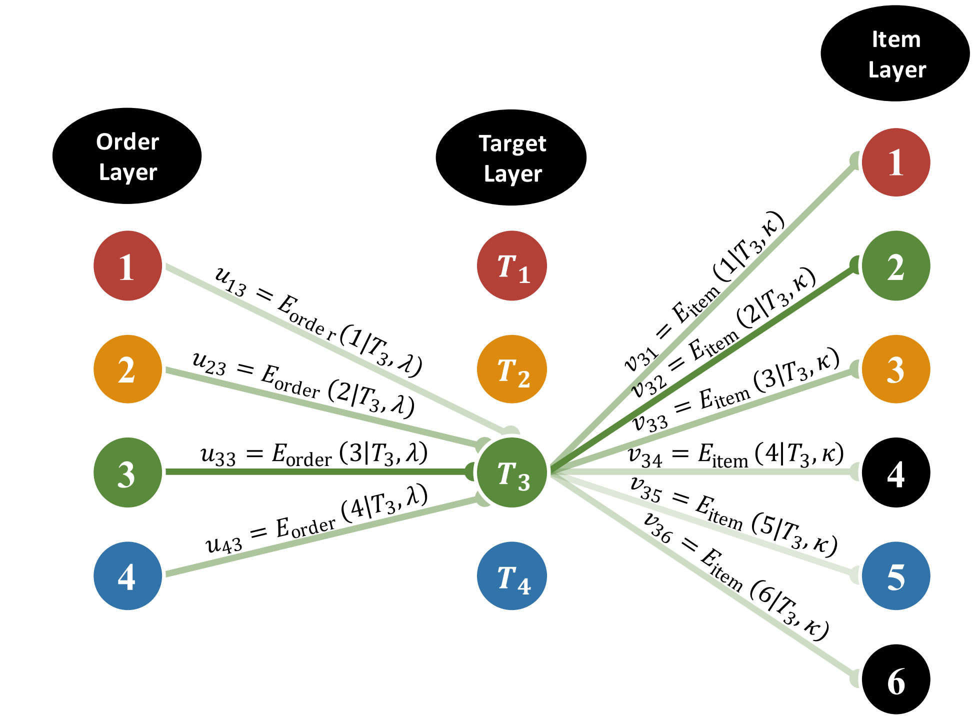
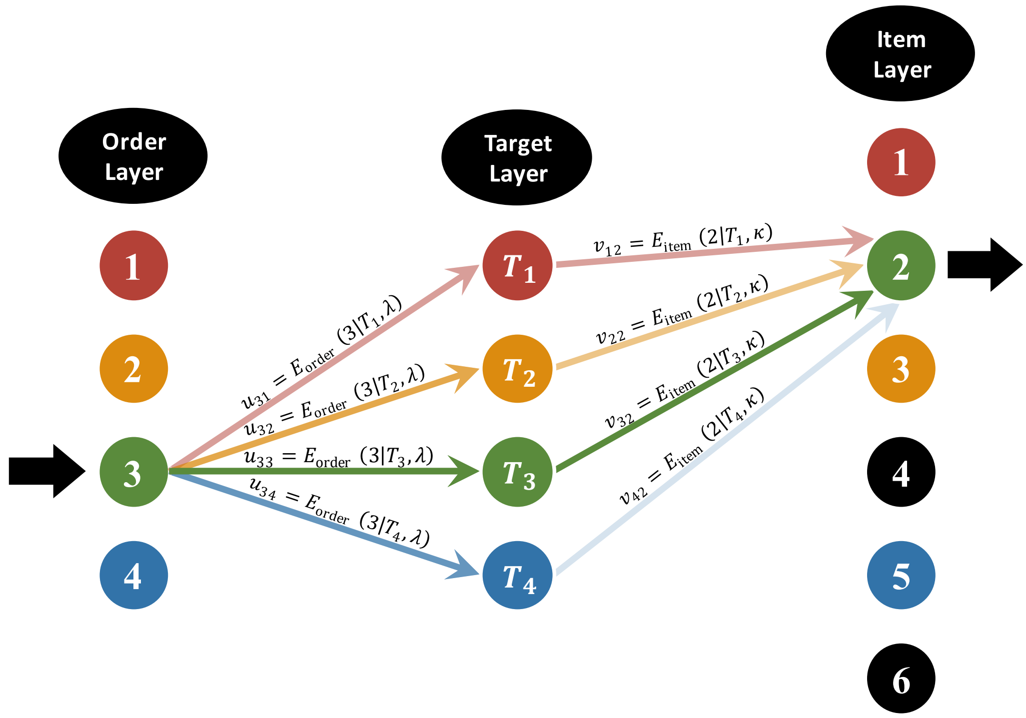
Indeed, as far as we know, the decoding (readout) process of spatial sequences in the brain has not been understood yet. To link our decoding model to any principled inference, we probably could compare such process with a simple feedforward neural network (see the below figures).



We can assume three functional layers in this network: the order layer (input layer), the target layer (hidden layer) and the item layer (output layer). Taking the sequence $S = (1, 3, 2, 5)$ as an example, our encoding model could be seen as the neural network illustrated above. The opacity of lines means the strength of connections between different units.



Taking the 3rd target unit as an example, the weighted connections from order units to it were calculated approximately as the Laplace distribution, and the weighted connections from it to item units were calculated approximately as the von Mises distribution.



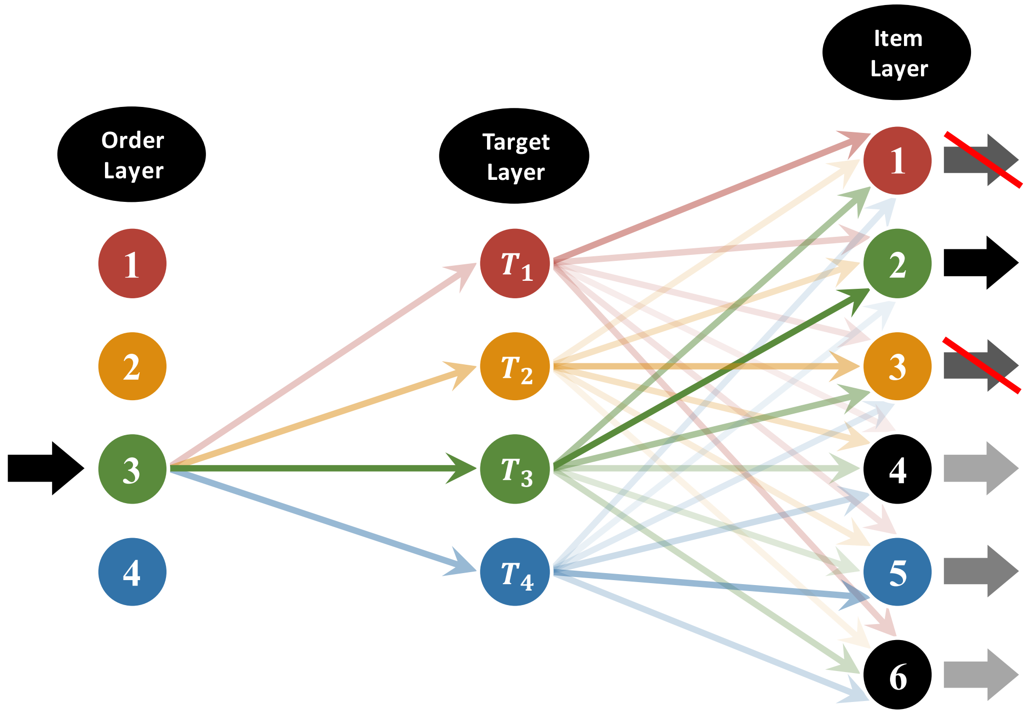
Then we could calculate the binding strength between the 3rd order unit and the 2nd item unit. We assumed that the F-I curve (relationship between input current and firing rate) of the units in the target layer were expressed approximately as the linear function, which means the activation function were ReLU (scaled by the weights $\{w_i\}_{i=1}^{4}$), and the bias were set to 0. And after adding the bias η (random choice noise) in the item units, we have:

$$FR\left(\mathrm{ID}_{\mathrm{output}}=2\middle|\mathrm{ID}_{\mathrm{input}}=3\right)=\sum_{i=1}^{4} w_i\cdot u_{3i}\cdot v_{i2}+\eta$$

$$=\sum_{i=1}^{4} w_i\cdot E_{\mathrm{order}}(3|T_i,\lambda)\cdot E_{\mathrm{item}}(2|T_i,\kappa)+\eta$$

$$=EM_S(3,\ 2)$$

That means the firing rate of 2nd item unit under the activation of the 3rd order unit is just the $EM_S(3,\ 2)$ we defined before.



Finally, we could calculate the probability of the retrieved item with the activation of the 3rd order unit. As illustrated above, the first two response items were both right and they should not be chosen again, so $FR(1|3)$ and $FR(3|3)$ would be removed or set to 0.

We could come up with two basic hypotheses in the retrieval process:

1. The retrieved item unit will be the first-firing one from some point in time (typically denoted t = 0).
2. $\forall$ item unit $Y\in\{1,\ 2,\ \ldots,\ 6\}$, $FR(Y|3)$ was denoted as λ_Y, and it would not change until an item being retrieved. That means the number of spikes in a fixed interval of time was $n_Y\sim\mathcal{P}(\lambda_Y)$, and the first spike's time of item Y was $t_Y\sim\mathcal{E}(\lambda_Y)$. The PDF of t_Y was $f_Y(t)=\lambda_Y e^{-\lambda_Y t}$, and the CDF of t_Y was $F_Y(t)=\mathrm{P}(t_Y\le t)=1-e^{-\lambda_Y t}$.

Then we could calculate the probability that the unit y_* is the first-firing one:

$$\mathrm{P}\left(\left\{t_{y_*}<t_Y\right\}_{Y\neq y_*}\middle|3\right)=\int_0^{+\infty} f_{y_*}(t)\prod_{Y\neq y_*}\mathrm{P}(t_Y>t)\,\mathrm{d}t=\frac{\lambda_{y_*}}{\sum_{Y=1}^{6}\lambda_Y}=\frac{FR(y_*|3)}{\sum_{Y=1}^{6}FR(Y|3)}$$

This may explain the 1-norm normalization (Luce's choice axiom) for the probability of retrieval items.

In short, we used the idea of a simple feedforward neural network to construct the conjunctive coding model for both encoding and decoding processes.