

DANCING LINKS 패키지와 그 활용

EXACT COVER PROBLEM



남수진

2018년 2월 3일 토요일

2018 한국텍학회 학술대회 및 정기총회
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1. Dancing links
2. Exact cover problem
 - Polyominoes
 - Sudoku
 - N-Queens puzzle

DANCING LINKS

- Exact cover 문제를 해결하는 알고리즘에 사용되는 기법
- 백트래킹을 효율적으로 구현하는 방법 (*do, undo* 연산)

EXACT COVER PROBLEM

EXACT COVER PROBLEM

0과 1로만 구성된 행렬에서 각 열이 정확히 하나의 1만 갖도록 하는 행들의 집합을 구하라.

$$\begin{pmatrix} 0 & 0 & 1 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 1 & 0 & 1 \end{pmatrix}$$

EXACT COVER PROBLEM

$$\begin{pmatrix} 0 & 0 & 1 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 1 & 0 & 1 \end{pmatrix}$$

EXACT COVER PROBLEM

$$\begin{pmatrix} 0 & 0 & 1 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 1 & 0 & 1 \end{pmatrix}$$

| A | B | C | D | E | F | G |
|---|---|---|---|---|---|---|
| C | E | | | | | |
| A | D | G | | | | |
| B | C | F | | | | |
| A | D | F | | | | |
| B | G | | | | | |
| D | E | G | | | | |

EXACT COVER PROBLEM

| <i>A</i> | <i>B</i> | <i>C</i> | <i>D</i> | <i>E</i> | <i>F</i> | <i>G</i> |
|----------|----------|----------|----------|----------|----------|----------|
| <hr/> | | | | | | |
| <i>C</i> | <i>E</i> | | | | | |
| <i>A</i> | <i>D</i> | <i>G</i> | | | | |
| <i>B</i> | <i>C</i> | <i>F</i> | | | | |
| <i>A</i> | <i>D</i> | <i>F</i> | | | | |
| <i>B</i> | <i>G</i> | | | | | |
| <i>D</i> | <i>E</i> | <i>G</i> | | | | |

EXACT COVER PROBLEM

| <i>A</i> | <i>B</i> | <i>C</i> | <i>D</i> | <i>E</i> | <i>F</i> | <i>G</i> |
|----------|----------|----------|----------|----------|----------|----------|
| <hr/> | | | | | | |
| <i>C</i> | <i>E</i> | | | | | |
| <i>A</i> | <i>D</i> | <i>G</i> | | | | |
| <i>B</i> | <i>C</i> | <i>F</i> | | | | |
| <i>A</i> | <i>D</i> | <i>F</i> | | | | |
| <i>B</i> | <i>G</i> | | | | | |
| <i>D</i> | <i>E</i> | <i>G</i> | | | | |

EXACT COVER PROBLEM

| <i>A</i> | <i>B</i> | <i>C</i> | <i>D</i> | <i>E</i> | <i>F</i> | <i>G</i> |
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| <i>C</i> | <i>E</i> | | | | | |
| <i>A</i> | <i>D</i> | <i>G</i> | | | | |
| <i>B</i> | <i>C</i> | <i>F</i> | | | | |

EXACT COVER PROBLEM

| <i>A</i> | <i>B</i> | <i>C</i> | <i>D</i> | <i>E</i> | <i>F</i> | <i>G</i> |
|----------|----------|----------|----------|----------|----------|----------|
| <hr/> | | | | | | |
| <i>C</i> | <i>E</i> | | | | | |
| <i>A</i> | <i>D</i> | <i>G</i> | | | | |
| <i>B</i> | <i>C</i> | <i>F</i> | | | | |

EXACT COVER PROBLEM

| <i>A</i> | <i>B</i> | <i>C</i> | <i>D</i> | <i>E</i> | <i>F</i> | <i>G</i> |
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| <hr/> | | | | | | |
| <i>C</i> | <i>E</i> | | | | | |
| <i>A</i> | <i>D</i> | <i>G</i> | | | | |

EXACT COVER PROBLEM

| <i>A</i> | <i>B</i> | <i>C</i> | <i>D</i> | <i>E</i> | <i>F</i> | <i>G</i> |
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| <hr/> | | | | | | |
| | <i>C</i> | <i>E</i> | | | | |
| <i>A</i> | | <i>D</i> | | <i>G</i> | | |
| | <i>B</i> | <i>C</i> | | <i>F</i> | | |
| <i>A</i> | | <i>D</i> | | <i>F</i> | | |
| | <i>B</i> | | | <i>G</i> | | |
| | <i>D</i> | <i>E</i> | | <i>G</i> | | |

EXACT COVER PROBLEM

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|----------|----------|----------|----------|----------|----------|----------|
| <i>A</i> | <i>B</i> | <i>C</i> | <i>D</i> | <i>E</i> | <i>F</i> | <i>G</i> |
| <hr/> | | | | | | |
| <i>C</i> | <i>E</i> | | | | | |

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|----------|----------|----------|
| <i>A</i> | <i>D</i> | <i>F</i> |
| <i>B</i> | <i>G</i> | |

EXACT COVER PROBLEM

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|----------|----------|----------|----------|----------|----------|----------|
| <i>A</i> | <i>B</i> | <i>C</i> | <i>D</i> | <i>E</i> | <i>F</i> | <i>G</i> |
| <hr/> | | | | | | |
| <i>C</i> | <i>E</i> | | | | | |

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|----------|----------|----------|
| <i>A</i> | <i>D</i> | <i>F</i> |
| <i>B</i> | <i>G</i> | |

EXACT COVER PROBLEM

| | | | | | | |
|----------|----------|----------|----------|----------|----------|----------|
| <i>A</i> | <i>B</i> | <i>C</i> | <i>D</i> | <i>E</i> | <i>F</i> | <i>G</i> |
| <hr/> | | | | | | |
| <i>C</i> | <i>E</i> | | | | | |

| | | |
|----------|----------|----------|
| <i>A</i> | <i>D</i> | <i>F</i> |
| <i>B</i> | <i>G</i> | |

github.com/sjnam/lua-dancing-links

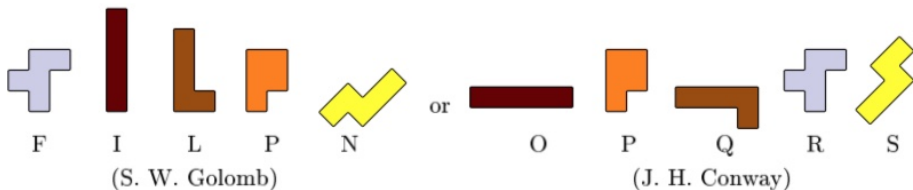
EXACT COVER PROBLEM

POLYOMINOES

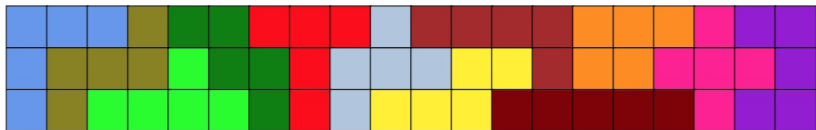
PENTOMINOES



But two different systems of nomenclature have been proposed for the other five:



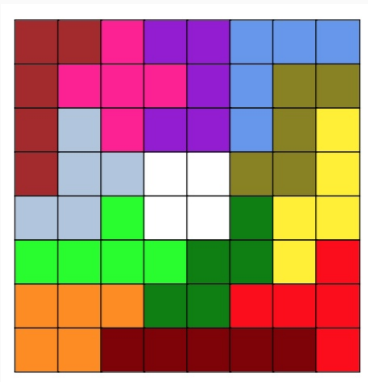
PENTOMINOES 3×20



```
\usepackage{pentominoes}
```

```
\pentominoes{3x20.dlx}{3}{20}.
```

PENTOMINOES 8×8 WITH CENTER HOLE



`\pentominoes{8x8_center_hole.dlx}{8}{8}.`

EXACT COVER PROBLEM

SUDOKU

SUDOKU

```
\usepackage{sudoku-dlx}  
\Sudoku{9.....6.3.4.....9...915.8..8.5..7..%  
..3.9.4.....2..1.9.32176.....6..1.2.3.8...5...1}
```

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| 9 | | | | | | | | 6 |
| | 3 | | 4 | | | | | 9 |
| | | | 9 | 1 | 5 | | 8 | |
| | 8 | | 5 | | | 7 | | |
| | | 3 | | 9 | | 4 | | |
| | | 2 | | | 1 | | 9 | |
| 3 | 2 | 1 | 7 | 6 | | | | |
| 6 | | | 1 | | 2 | | 3 | |
| 8 | | | | 5 | | | | 1 |

SUDOKU

```
\Sudoku*{9.....6.3.4....9...915.8..8.5..7..%  
..3.9.4....2..1.9.32176....6..1.2.3.8...5...1}
```

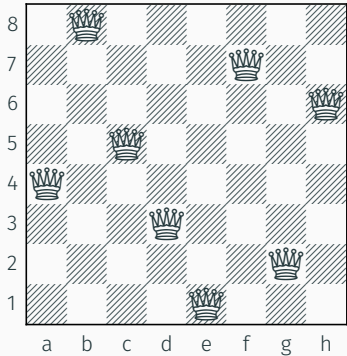
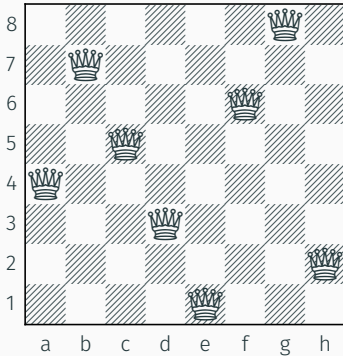
| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| 9 | 1 | 4 | 8 | 3 | 7 | 2 | 5 | 6 |
| 5 | 3 | 8 | 4 | 2 | 6 | 1 | 7 | 9 |
| 2 | 7 | 6 | 9 | 1 | 5 | 3 | 8 | 4 |
| 1 | 8 | 9 | 5 | 4 | 3 | 7 | 6 | 2 |
| 7 | 6 | 3 | 2 | 9 | 8 | 4 | 1 | 5 |
| 4 | 5 | 2 | 6 | 7 | 1 | 8 | 9 | 3 |
| 3 | 2 | 1 | 7 | 6 | 9 | 5 | 4 | 8 |
| 6 | 4 | 5 | 1 | 8 | 2 | 9 | 3 | 7 |
| 8 | 9 | 7 | 3 | 5 | 4 | 6 | 2 | 1 |

EXACT COVER PROBLEM

N-QUEENS PUZZLE

N-QUEENS

```
\usepackage{queens}  
\queens{8}{2}.
```



- The Art of Computer Programming PRE-FASCICLE 5C
- github.com/sjnam/lua-dancing-links

감사합니다