



DANCING LINKS 패키지와 그 활용

EXACT COVER PROBLEM

남수진

2018년 2월 3일 토요일

2018 한국텍학회 학술대회 및 정기총회
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1. Dancing links
2. Exact cover problem
 - Polyominoes
 - Sudoku
 - N-Queens puzzle

DANCING LINKS

- Exact cover 문제를 해결하는 알고리즘에 사용되는 기법
- 백트래킹을 효율적으로 구현하는 방법 (*do, undo* 연산)

EXACT COVER PROBLEM

EXACT COVER PROBLEM

0과 1로만 구성된 행렬에서 각 열이 정확히 하나의 1만 갖도록 하는 행들의 집합을 구하라.

$$\begin{pmatrix} 0 & 0 & 1 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 1 & 0 & 1 \end{pmatrix}$$

EXACT COVER PROBLEM

$$\begin{pmatrix} 0 & 0 & 1 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 1 & 0 & 1 \end{pmatrix}$$

EXACT COVER PROBLEM

$$\begin{pmatrix} 0 & 0 & 1 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 1 & 0 & 1 \end{pmatrix}$$

A	B	C	D	E	F	G
C	E					
A	D	G				
B	C	F				
A	D	F				
B	G					
D	E	G				

EXACT COVER PROBLEM

<i>A</i>	<i>B</i>	<i>C</i>	<i>D</i>	<i>E</i>	<i>F</i>	<i>G</i>
<hr/>						
<i>C</i>	<i>E</i>					
<i>A</i>	<i>D</i>	<i>G</i>				
<i>B</i>	<i>C</i>	<i>F</i>				
<i>A</i>	<i>D</i>	<i>F</i>				
<i>B</i>	<i>G</i>					
<i>D</i>	<i>E</i>	<i>G</i>				

EXACT COVER PROBLEM

<i>A</i>	<i>B</i>	<i>C</i>	<i>D</i>	<i>E</i>	<i>F</i>	<i>G</i>
<hr/>						
	<i>C</i>	<i>E</i>				
<i>A</i>	<i>D</i>	<i>G</i>				
<i>B</i>	<i>C</i>	<i>F</i>				
<i>A</i>	<i>D</i>	<i>F</i>				
<i>B</i>	<i>G</i>					
<i>D</i>	<i>E</i>	<i>G</i>				

EXACT COVER PROBLEM

<i>A</i>	<i>B</i>	<i>C</i>	<i>D</i>	<i>E</i>	<i>F</i>	<i>G</i>
<hr/>						
<i>C</i>	<i>E</i>					
<i>A</i>	<i>D</i>	<i>G</i>				
<i>B</i>	<i>C</i>	<i>F</i>				

EXACT COVER PROBLEM

<i>A</i>	<i>B</i>	<i>C</i>	<i>D</i>	<i>E</i>	<i>F</i>	<i>G</i>
<hr/>						
<i>C</i>	<i>E</i>					
<i>A</i>	<i>D</i>	<i>G</i>				
<i>B</i>	<i>C</i>	<i>F</i>				

EXACT COVER PROBLEM

<i>A</i>	<i>B</i>	<i>C</i>	<i>D</i>	<i>E</i>	<i>F</i>	<i>G</i>
<hr/>						
<i>C</i>	<i>E</i>					
<i>A</i>	<i>D</i>	<i>G</i>				

EXACT COVER PROBLEM

<i>A</i>	<i>B</i>	<i>C</i>	<i>D</i>	<i>E</i>	<i>F</i>	<i>G</i>
<hr/>						
<i>C</i>	<i>E</i>					
<i>A</i>	<i>D</i>	<i>G</i>				
<i>B</i>	<i>C</i>	<i>F</i>				
<i>A</i>	<i>D</i>	<i>F</i>				
<i>B</i>	<i>G</i>					
<i>D</i>	<i>E</i>	<i>G</i>				

EXACT COVER PROBLEM

<i>A</i>	<i>B</i>	<i>C</i>	<i>D</i>	<i>E</i>	<i>F</i>	<i>G</i>
<hr/>						
<i>C</i>	<i>E</i>					

<i>A</i>	<i>D</i>	<i>F</i>
<i>B</i>	<i>G</i>	

EXACT COVER PROBLEM

<i>A</i>	<i>B</i>	<i>C</i>	<i>D</i>	<i>E</i>	<i>F</i>	<i>G</i>
<hr/>						
<i>C</i>	<i>E</i>					

<i>A</i>	<i>D</i>	<i>F</i>
<i>B</i>	<i>G</i>	

EXACT COVER PROBLEM

<i>A</i>	<i>B</i>	<i>C</i>	<i>D</i>	<i>E</i>	<i>F</i>	<i>G</i>
<hr/>						
<i>C</i>	<i>E</i>					

<i>A</i>	<i>D</i>	<i>F</i>
<i>B</i>	<i>G</i>	

github.com/sjnam/lua-dancing-links

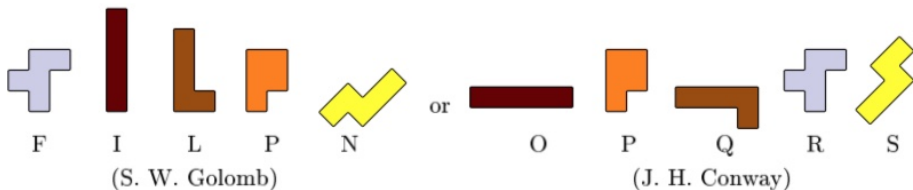
EXACT COVER PROBLEM

POLYOMINOES

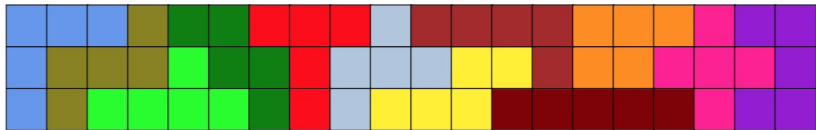
PENTOMINOES



But two different systems of nomenclature have been proposed for the other five:



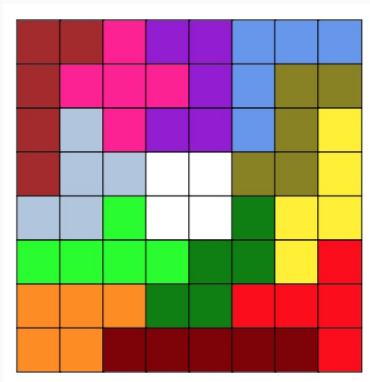
PENTOMINOES 3×20



```
\usepackage{pentominoes}
```

```
\pentominoes{3x20.dlx}{3}{20}.
```

PENTOMINOES 8×8 WITH CENTER HOLE



`\pentominoes{8x8_center_hole.dlx}{8}{8}.`

EXACT COVER PROBLEM

SUDOKU

SUDOKU

```
\usepackage{sudoku-dlx}  
\Sudoku{9.....6.3.4.....9...915.8..8.5..7..%  
..3.9.4.....2..1.9.32176.....6..1.2.3.8...5...1}
```

9								6
	3		4					9
			9	1	5		8	
	8		5			7		
		3		9		4		
		2			1		9	
3	2	1	7	6				
6			1		2		3	
8				5				1

SUDOKU

```
\Sudoku*{9.....6.3.4.....9...915.8..8.5..7..%  
..3.9.4.....2..1.9.32176.....6..1.2.3.8...5...1}
```

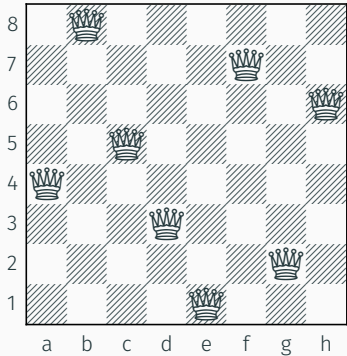
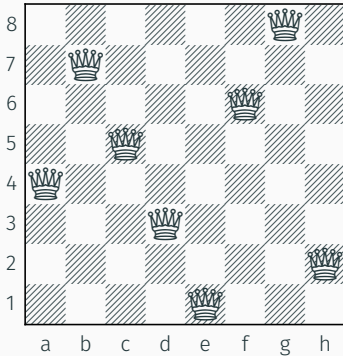
9	1	4	8	3	7	2	5	6
5	3	8	4	2	6	1	7	9
2	7	6	9	1	5	3	8	4
1	8	9	5	4	3	7	6	2
7	6	3	2	9	8	4	1	5
4	5	2	6	7	1	8	9	3
3	2	1	7	6	9	5	4	8
6	4	5	1	8	2	9	3	7
8	9	7	3	5	4	6	2	1

EXACT COVER PROBLEM

N-QUEENS PUZZLE

N-QUEENS

```
\usepackage{queens}  
\queens{8}{2}.
```



- The Art of Computer Programming PRE-FASCICLE 5C
- github.com/sjnam/lua-dancing-links

감사합니다