Definition. Let $\pi : P \to M$ be a principal G-bundle with group G acting on the left on a manifold F.

$$
\begin{aligned}
G : P \times F &\to P \times F \\
(u, f) &\mapsto (ug, g^{-1}f)
\end{aligned}
$$

The associated fiber bundle $E = P \times_\rho F$ is defined as the quotient of $P \times F$ by the relation $\sim$

$$E = P \times F / \sim$$

where

$$(u, f) \sim (ug, g^{-1}f)$$

This definition is very natural, since if we let F in the definition be the $\mathbb{R}^d$ with general linear group $GL(d, \mathbb{R})$ and let P be the frame bundle, then the associated bundle is isomorphic to tangent bundle. The ug can be considered as change of frame, and g^{-1} is the change of components of basis along the change of frame.