

$$y'' - 2y' + 2y = e^x \cos(x)$$

$$y(0) = 0$$

$$y'(0) = 0$$

VII

$$1) \lambda^2 - 2\lambda + 2 = 0$$

$$(\lambda - 1)^2 = -1$$

$$\lambda = 1 \pm i$$

$$Y_1 = e^x \cos(x)$$

$$Y_2 = e^x \sin(x)$$

$$y_{\text{hom}} = C_1 Y_1 + C_2 Y_2$$

$$2) 1. y_{u_1} = (A + Bx) \cos(x) + (C + Dx) \sin(x)$$

$$y'_{u_1} = (B + C + Dx) \cos(x) - (D - A - Bx) \sin(x)$$

$$y''_{u_1} = (2D - A - Bx) \cos(x) + (-2B - C - Dx) \sin(x)$$

$$(A - 2B - 2C + 2D + (B - 2D)x) \cos(x) + (2A - 2B + C - 2D + (2B + D)x) \sin(x) = x \cos(x)$$

$$\begin{cases} A - 2B - 2C + 2D = 0 \\ B - 2D = 1 \\ 2A - 2B + C - 2D = 0 \\ 2B + D = 0 \end{cases} \quad \begin{cases} A = \frac{2}{25} \\ B = \frac{1}{5} \\ C = -\frac{14}{25} \\ D = -\frac{2}{5} \end{cases}$$

$$2. y_{u_2} = A e^x$$

$$y'_{u_2} = A e^x$$

$$y''_{u_2} = A e^x$$

$$A e^x = e^x$$

$$A = 1$$

$$y(x) = C_1 e^x \cos(x) + C_2 e^x \sin(x) + \left(\frac{2}{25} + \frac{1}{5}x\right) \cos(x) + \left(-\frac{14}{25} - \frac{2}{5}x\right) \sin(x) + e^x$$

$$y(0) = C_1 + \frac{27}{25} = 0 \rightarrow C_1 = -\frac{27}{25}$$

$$y'(0) = C_1 + C_2 + \frac{46}{25} = 0 \rightarrow C_2 = \frac{11}{25}$$

$$y(x) = \cancel{\frac{11}{25}} \left( \frac{11}{25} e^x - \frac{2x}{5} - \frac{14}{25} \right) \sin(x) + \left( \frac{x}{5} - \frac{27e^x}{25} + \frac{2}{25} \right) \cos(x) + e^x$$