

EP 501 Homework 4: Least Squares and Interpolation

October 20, 2020

Instructions:

- Complete all listed steps to the problems.
- Submit all source code, which must be either MATLAB or Python, and output (results printed to the screen or plotted) via Canvas.
- Results must be compiled into a single .pdf file which contains descriptions of the calculations that you have done alongside the results.
- Source codes must run and produce the same essential output presented in your documents.
- Discussing the assignment with others is fine, but you must not copy the code of another student *verbatim*; this is considered an academic integrity violation.
- During submission on the canvas website compress all of your files for a given assignment into a single .zip file, e.g. `assignment4.zip` . I should be able to run your codes by unzipping the files and then opening them and running them in the appropriate developer environment (MATLAB or Spyder).
- You may use, *verbatim* or modified, any of the example codes from one of the course repositories contained on the GitHub organization website <https://github.com/Zettergren-Courses>.
- For demonstrating that your code is correct when you turn in the assignment, you must use the test problems referenced in the assignment, namely `test_lsq.mat` and `test_interp.mat`.

Purpose of this assignment:

- Learn principles behind data fitting and polynomial approximation.
- Develop good coding and documentation practices, such that your programs are easily understood by others.
- Hone skills of developing, debugging, and testing your own software
- Learn how to build programs on top of existing codes

1. Least squares and data fitting: this problem requires use of the example dataset from the repository, `test_lsq.mat`, which provides data for x_i, y_i, σ_{yi} .

- (a) Write a program that performs a linear least squares fit of a set of data to a polynomial of arbitrary order n .
- (b) Use this program to fit the data to a line and a quadratic form. Plot your results and the data on the same axis so they can be easily compared. Test your results against the built-in Matlab functions `polyfit` and `polyval`. Which fit looks better (linear or quadratic)?
- (c) A more rigorous way of deciding between preferred function forms in fits (e.g. linear, quadratic, cubic, quartic, etc.) is to define a *goodness-of-fit* statistic that quantifies how effective a particular form is at describing a given data set. The most commonly used goodness of fit statistic is the *reduced Chi-squared statistic* defined by:

$$\chi^2_\nu = \frac{1}{\nu} \sum_i \frac{(y_i - f(x_i))^2}{\sigma_{y,i}^2}, \quad (1)$$

where x_i are the points of the independent variable at which data are sampled, y_i are the data, $f(\cdot)$ is the function which is being fitted to the data (linear, quadratic, cubic, etc.), σ_i is the uncertainty of measurement y_i and ν is the number of degrees of freedom in your fit (number of data points minus number of parameters being estimated). Write a function that evaluates χ^2_ν for a polynomial fit that has been performed.

- (d) Use your goodness-of-fit statistic to determine whether is best (in the statistical sense) to fit these data with a linear, quadratic, or cubic polynomial.

2. Bilinear interpolation: this problem requires use of the grid and data from the repository.

- (a) Write a function that takes in a grid of points describing some independent variable (say x_i), and a point to which the data are to be interpolated x' and finds the index i into the array x_i such that: $x_i \leq x' \leq x_{i+1}$.
- (b) Use the function from part (a) to construct an additional function that works over a 2D grid x, y . I.e. given two grids x_i, y_j find the indices i, j such that: $x_i \leq x' \leq x_{i+1}$ and $y_j \leq y' \leq y_{j+1}$.
- (c) Use your results from parts a and b to create a bilinear interpolation function that takes in a sequence of data points $\{x'_k, y'_k\}$ to which data are being interpolated, a grid x_i, y_j , and a dataset f_{ij} that is defined over this grid and produces bilinearly interpolated values of f at the points $\{x'_k, y'_k\}$. Write your program so that the input points are simply a flat list and not necessarily a 2D grid of points.
- (d) Test your results against Matlab's bilinear interpolation function (`interp2`) and show that you get the same result. Use the test data from the repository (`test_interp.mat`). The source grid data are stored in `xg, yg`, while the value of the function at those points is in `f2D`. `xgi, ygi` are the densely sampled grid point to which the data are to be interpolated for this test.