Haverford Problem Solving Group
October 20, 2021

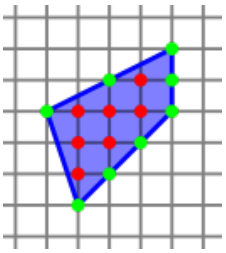
Problem 3. Find the 2000^{th} digit in the square root of $N = 11\ldots1$, where N contains 1998 digits, all of them 1's.

Problem 5. Can you show how to express any positive fraction as a sum of distinct positive reciprocal whole numbers? For example, $7/3 = 1/1 + 1/2 + 1/3 + 1/4 + 1/5 + 1/20$.

Problem 6. Can the portion of any parabola inside a circle of radius 1 have a length greater than 4?

Problem 10. Suppose that a polygon has integer coordinates for all of its vertices. Let i be the number of integer points that are interior to the polygon, and let b be the number of integer points on its boundary (including vertices as well as points along the sides of the polygon). Then the area of this polygon is

$$i + \frac{b}{2} - 1.$$



Problem 11. Determine whether there exist non-constant polynomials $P(x)$ and $Q(x)$ with real coefficients satisfying

$$P(x)^{10} + P(x)^9 = Q(x)^{21} + Q(x)^{20}.$$

Problem 12. Ann and Bob play a game on an infinite checkered plane making moves in turn. A move consists in orienting any unit grid-segment that has not been oriented before. If at some stage some oriented segments form an oriented cycle, Bob wins.

(a) Bob makes the first move. Does Bob have a strategy that guarantees him to win?

(b) Ann makes the first move. Does Bob have a strategy that guarantees him to win?

If you are not in our Discord server, you should definitely join. We will post there handouts, resources, solutions, room/time changes, and (most important of all) pictures whatever food we will have in the meeting. Point you phone camera to the QR code to join it.