Haverford Problem Solving Group
September 29, 2021

Problem 1. When 4444^{4444} is written in decimal notation, the sum of its digits is A. Let B be the sum of the digits of A. Find the sum of the digits of B.

Problem 2 (Modified after last week's progress)**.** Prove that

$$\sin\left(\frac{\pi}{11}\right)\sin\left(\frac{2\pi}{11}\right)\cdots\sin\left(\frac{10\pi}{11}\right)=\frac{11}{2^{10}},$$

or more generally, prove that

$$\sin\left(\frac{\pi}{n}\right)\sin\left(\frac{2\pi}{n}\right)\cdots\sin\left(\frac{(n-1)\pi}{n}\right)=\frac{n}{2^{n-1}}.$$

Problem 3. Find the 2000^{th} digit in the square root of $N = 11\ldots1$, where N contains 1998 digits, all of them 1's.

Problem 5. Can you show how to express any positive fraction as a sum of distinct positive reciprocal whole numbers? For example, $7/3 = 1/1 + 1/2 + 1/3 + 1/4 + 1/5 + 1/20$.

Problem 6. Can the portion of any parabola inside a circle of radius 1 have a length greater than 4?

Problem 7. Show that for each positive integer n,

$$n! = \prod_{i=1}^{n} \operatorname{lcm}\{1, 2, \ldots, \lfloor n/i \rfloor\}.$$

(Here lcm denotes the least common multiple, and $\lfloor x \rfloor$ denotes the greatest integer $\leq x$.)

If you are not in our Discord server, you should definitely join. We will post there handouts, resources, solutions, room/time changes, and (most important of all) pictures whatever food we will have in the meeting. Point you phone camera to the QR code to join it.