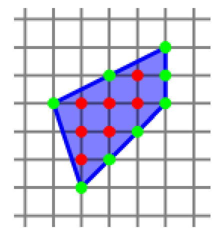
Haverford Problem Solving Group
November 10, 2021

Register for Putnam!
https://artofproblemsolving.com/contests/putnam/student

Problem 6. Can the portion of any parabola inside a circle of radius 1 have a length greater than 4?

Problem 10. Suppose that a polygon has integer coordinates for all of its vertices. Let i be the number of integer points that are interior to the polygon, and let b be the number of integer points on its boundary (including vertices as well as points along the sides of the polygon). Then the area of this polygon is

$$i + \frac{b}{2} - 1.$$



Problem 11. Determine whether there exist non-constant polynomials $P(x)$ and $Q(x)$ with real coefficients satisfying

$$P(x)^{10} + P(x)^{9} = Q(x)^{21} + Q(x)^{20}.$$

Problem 12. Ann and Bob play a game on an infinite checkered plane making moves in turn. A move consists in orienting any unit grid-segment that has not been oriented before. If at some stage some oriented segments form an oriented cycle, Bob wins.

(a) Bob makes the first move. Does Bob have a strategy that guarantees him to win?

(b) Ann makes the first move. Does Bob have a strategy that guarantees him to win?

Problem 14. Suppose that f is a function on the interval $[1,3]$ such that $-1 \le f(x) \le 1$ for all x and $\displaystyle\int_1^3 f(x)\,dx = 0$. How large can $\displaystyle\int_1^3 \frac{f(x)}{x}\,dx$ be?

Problem 16. Find a real number c and a positive number L for which

$$\lim_{r\to\infty} \frac{r^c \int_0^{\pi/2} x^r \sin x\,dx}{\int_0^{\pi/2} x^r \cos x\,dx} = L.$$

Problem 17. Let p be an odd prime number. Determine how many p-element subsets A of $\{1, 2, \ldots, 2p\}$ are such that the sum of elements of A is divisible by p.

Problem 18. Let S be the set of all ordered triples (p, q, r) of prime numbers for which at least one rational number x satisfies $px^2 + qx + r = 0$. Which primes appear in seven or more elements of S?

Previous Problems

Problem 3. Find the 2000^{th} digit in the square root of $N = 11\ldots1$, where N contains 1998 digits, all of them 1's.

Problem 15. Let $a_0 = 5/2$ and $a_k = a_{k-1}^2 - 2$ for $k \geq 1$. Compute

$$\prod_{k=0}^{\infty} \left(1 - \frac{1}{a_k}\right)$$

in closed form.

If you are not in our Discord server, you should definitely join. We will post there handouts, resources, solutions, room/time changes, and (most important of all) pictures whatever food we will have in the meeting. Point your phone camera to the QR code to join it.