HAVERFORD PROBLEM SOLVING GROUP
OCTOBER 6, 2021

Problem 2 (Modified after last week's progress)**.** Prove that

$$\sin\left(\frac{\pi}{11}\right)\sin\left(\frac{2\pi}{11}\right)\cdots\sin\left(\frac{10\pi}{11}\right)=\frac{11}{2^{10}},$$

or more generally, prove that

$$\sin\left(\frac{\pi}{n}\right)\sin\left(\frac{2\pi}{n}\right)\cdots\sin\left(\frac{(n-1)\pi}{n}\right)=\frac{n}{2^{n-1}}.$$

Problem 3. Find the 2000^{th} digit in the square root of $N = 11\ldots1$, where N contains 1998 digits, all of them 1's.

Problem 5. Can you show how to express any positive fraction as a sum of distinct positive reciprocal whole numbers? For example, $7/3 = 1/1 + 1/2 + 1/3 + 1/4 + 1/5 + 1/20$.

Problem 6. Can the portion of any parabola inside a circle of radius 1 have a length greater than 4?

Problem 8. Define a sequence $(u_n)_{n=0}^{\infty}$ by $u_0 = u_1 = u_2 = 1$ and thereafter by the condition that

$$\det\begin{pmatrix} u_n & u_{n+1} \\ u_{n+2} & u_{n+3}\end{pmatrix} = n!$$

for all $n \geq 0$. Show that u_n is an integer for all n. (By convention, $0! = 1$.)

Problem 9. Show that every positive integer is a sum of one or more numbers of the form 2^r3^s, where r and s are nonnegative integers and no summand divides another. (For example, $23 = 9 + 8 + 6$.)

If you are not in our Discord server, you should definitely join. We will post there handouts, resources, solutions, room/time changes, and (most important of all) pictures whatever food we will have in the meeting. Point you phone camera to the QR code to join it.