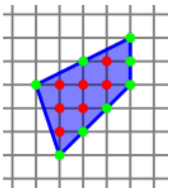
Haverford Problem Solving Group
October 27, 2021

Problem 3. Find the 2000^{th} digit in the square root of $N = 11\ldots1$, where N contains 1998 digits, all of them 1's.

Problem 6. Can the portion of any parabola inside a circle of radius 1 have a length greater than 4?

Problem 10. Suppose that a polygon has integer coordinates for all of its vertices. Let i be the number of integer points that are interior to the polygon, and let b be the number of integer points on its boundary (including vertices as well as points along the sides of the polygon). Then the area of this polygon is



$$i + \frac{b}{2} - 1.$$

Problem 11. Determine whether there exist non-constant polynomials $P(x)$ and $Q(x)$ with real coefficients satisfying

$$P(x)^{10} + P(x)^{9} = Q(x)^{21} + Q(x)^{20}.$$

Problem 12. Ann and Bob play a game on an infinite checkered plane making moves in turn. A move consists in orienting any unit grid-segment that has not been oriented before. If at some stage some oriented segments form an oriented cycle, Bob wins.

(a) Bob makes the first move. Does Bob have a strategy that guarantees him to win?

(b) Ann makes the first move. Does Bob have a strategy that guarantees him to win?

Problem 13. Given a list of the positive integers $1, 2, 3, 4, \ldots$, take the first three numbers $1, 2, 3$ and their sum 6 and cross all four numbers off the list. Repeat with the three smallest remaining numbers $4, 5, 7$ and their sum 16. Continue in this way, crossing off the three smallest remaining numbers and their sum and consider the sequence of sums produced: $6, 16, 27, 36, \ldots$. Prove or disprove that there is some number in this sequence whose base 10 representation ends with 2015.

Problem 14. Suppose that f is a function on the interval $[1, 3]$ such that $-1 \leq f(x) \leq 1$ for all x and $\displaystyle\int_1^3 f(x)\,dx = 0$. How large can $\displaystyle\int_1^3 \frac{f(x)}{x}\,dx$ be?

If you are not in our Discord server, you should definitely join. We will post there handouts, resources, solutions, room/time changes, and (most important of all) pictures whatever food we will have in the meeting. Point your phone camera to the QR code to join it.