MATH 317 FALL 2021: IN-CLASS ACTIVITY FOR MON. AUG. 30 (COMPLETENESS)

GUILHERME ZEUS DANTAS E MOURA

Our first goal is to define the set $\mathbb{R}$ of real numbers. We will *assume* we know definitions of:

- the set of natural numbers $= \mathbb{N} = \{1, 2, 3, \dots\}$
- the set of integers $= \mathbb{Z} = \{\dots, -3, -2, -1, 0, 1, 2, 3, \dots\}$
- the set of rational numbers $= \mathbb{Q} =$ the set of all ratios m/n where m, n are integers and $n \neq 0$

To make the jump from $\mathbb{Q}$ to $\mathbb{R}$ turns out to include a subtle new idea called "completeness." To explore this idea (before we define it later this week), try the following. For each of the five situations described below, provide an example as requested, or say "NO SUCH EXAMPLE EXISTS."

Give an example of:

(1) a sequence of rational numbers that has a finite limit, with that limit being a rational number.

The sequence $1, \frac{1}{2}, \frac{1}{3}, \dots, \frac{1}{n}, \dots$ has limit 0.

(2) a sequence of rational numbers that has a finite limit, with that limit being an irrational number.

The sequence $\left\lfloor \sqrt{2} \right\rfloor, \frac{\lfloor 2\sqrt{2} \rfloor}{2}, \frac{\lfloor 3\sqrt{2} \rfloor}{3}, \dots, \frac{\lfloor n\sqrt{2} \rfloor}{n}, \dots$ has limit $\sqrt{2}$.

(3) a sequence of irrational numbers that has a finite limit, with that limit being a rational number.

The sequence $\left\lfloor \sqrt{2} \right\rfloor - \sqrt{2}, \frac{\lfloor 2\sqrt{2} \rfloor}{2} - \sqrt{2}, \frac{\lfloor 3\sqrt{2} \rfloor}{3} - \sqrt{2}, \dots, \frac{\lfloor n\sqrt{2} \rfloor}{n} - \sqrt{2}, \dots$ has limit 0.

(4) a sequence of irrational numbers that has a finite limit, with that limit being an irrational number.

The sequence $1 + \sqrt{2}, \frac{1}{2} + \sqrt{2}, \frac{1}{3} + \sqrt{2}, \dots, \frac{1}{n} + \sqrt{2}, \dots$ has limit 0.

(5) a sequence of irrational numbers that has a finite limit, with that limit being an irrational number.

NO SUCH EXAMPLE EXISTS.

Date: August 30, 2021.