PROPOSAL FOR TERM PAPER: BOUNDS ON CODING THEORY FROM ALGEBRAIC GEOMETRY

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1. Proposal of Topic

Coding theory is concerned with finding efficient ways to encode a message so that one may discover errors in the message — and perhaps even correct. In algebraic coding theory, we study efficient codes generated from algebraic geometric methods.

In my proposed paper, I plan on constructing the Reed-Solomon codes, generalizing them using projective curves, and understand the results from [**TVZ82**] on finding a bound better than the well-known Gilbert–Varshamov bound.

2. Outline

2.1. **Coding theory.** First, we need to define what a *code* is, as well as what does it mean for a code to be good. We shall define length n, dimension k, minimum distance d, code rate $R = n/k$, and relative minimum distance $\delta = d/n$.

2.2. **Singleton bound and a promising example.** We shall state and prove the Singleton Bound for linear codes (state and maybe prove for non-linear codes) and define maximum distance separable (MDS) codes. Then, define Reed–Solomon codes and show they meet the Singleton Bound, but have a limited length in comparison to the alphabet size.

2.3. **Generalized Reed-Solomon codes.** We shall redefine the Reed–Solomon codes using language related to a projective line. There is a way to replace the "projective line" with a "projective plane curve" and create other codes, called *Generalized Reed-Solomon codes* or simply *algebraic geometric codes*. We want large R and δ, and these codes yield

$$R + \delta \geq 1 + 1/n - g/n, \tag{1}$$

where n is the number of rational points of a curve X, with genus g.

2.4. **Final thoughts.** On equation (**??**), we observe that good algebraic geometric codes are generated by curves with a large ratio between n and g. On [**TVZ82**], the authors present a sequence of such curves, with n/g large enough to create a better bound than the Gilbert–Varshamov one.

Date: March 19, 2021.

3. Annotated Bibliography

[**TVZ82**] **TVZ82**.

My proposed paper is aimed towards understanding the results of this article, as we see in subsection **??**. However, its language is rather complicated and heavy on algebraic geometry, so I will need to use other sources to understand it.

[**TVN07**] **TVN07**.

This is a book written by the authors of the previous article, that starts Algebraic Coding Theory from the scratch; thus it is an amazing resource to understand the language used in [**TVZ82**]. However, it is quite long and has a lot of information not directly related to [**TVZ82**]; so I'll primarily use this source to search for definitions and details about terms I encounter elsewhere.

[**Wal00**] **Wal00**.

Aimed for undergraduates, this book explains definitions and motivations from coding theory (which correspond to subsections **??** and **??**); explains facts from algebraic geometry; describes the algebraic geometric codes; and discusses the results from [**TVZ82**].

[**LS87**] **LS87**.

This is an expository article that aims to simplify the methods from algebraic geometry used in [**TVZ82**]. Their approach is similar to the one found in [**Wal00**], but it is much shorter and concise. It will be a great complement to the other sources.