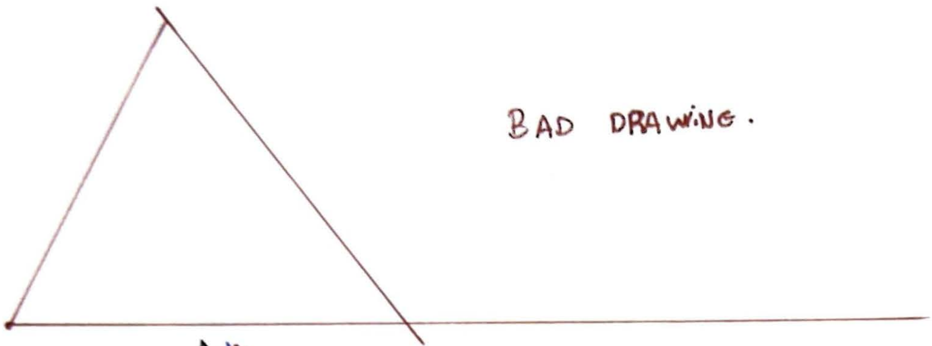
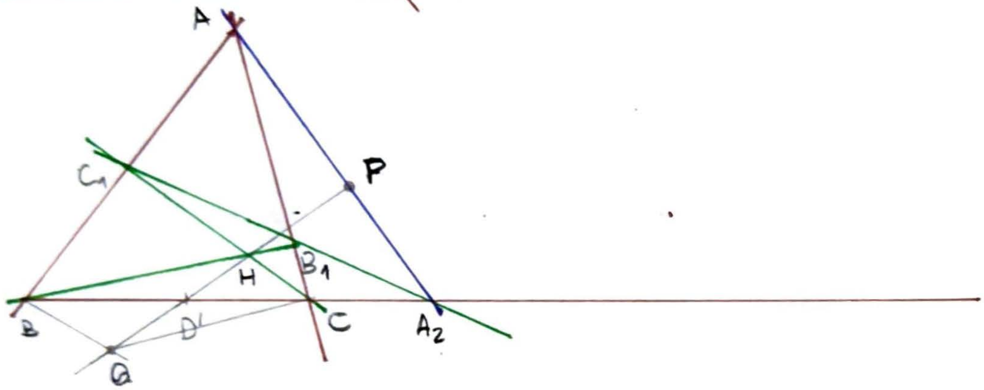
(USA TSTST 2012) - P4

Let us prove that H is the point of concurrence



BAD DRAWING.



Let P be the projection of H in AA_2 and $D' = HP \cap BC$.

It is enough to prove that D' is the midpoint of BC, thus proving that H is in all three lines. (analougously)

(B, C_1, B_1, C) is ciclic by 90's

$$A_2P \cdot A_2A = A_2B_1 \cdot A_2C_1 = A_2B \cdot A_2C \Rightarrow$$

(A, P, B_1, H, C_1) is ciclic, by 90°'s

$\Rightarrow (A, P, C, B)$ is ciclic. Let ω be this circle.

Let Q be the other intersection of HP with ω.

$90° = \angle APQ = \angle ABQ = \angle ACQ \Rightarrow BQ \parallel HC$ and $CQ \parallel HB \Rightarrow$

$\Rightarrow Q, B, C, H$ is a parallelgram and their diagonals meet in their midpoints $\Rightarrow$ D' is midpoint of BC.

$\square$