Problemas Sortidos V

Guilherme Zeus Dantas e Moura
zeusdanmou@gmail.com

Problema 1

We build a tower of 2×1 dominoes in the following way. First, we place 55 dominoes on the table such that they cover a 10×11 rectangle; this is the first story of the tower. We then build every new level with 55 domioes above the exact same 10×11 rectangle. The tower is called *stable* if for every non-lattice point of the 10×11 rectangle, we can find a domino that has an inner point above it. How many stories is the lowest *stable* tower?

Problema 2

Let ω_1 and ω_2 be two circles such that $\omega_1 \cap \omega_2 = \{A, B\}$. Let X be a point on ω_2 and Y be a point on ω_1 such that $BY \perp BX$. Suppose that O is the center of ω_1 and $X' = \omega_2 \cap OX$.

If $K = \omega_2 \cap X'Y$, prove that X is the midpoint of the arc AK.

Problema 3

Serafina and Florência play tennis. The player to first win at least four points and at least two more than the other player wins. We know that Serafina gets a point each time with probability $p \leq \frac{1}{2}$, independent of the game so far. Prove that the probability that Serafina wins is at most $2p^2$.

Problema 4

An acute triangle ABC with $AB < AC < BC$ is inscribed in a circle $c(O, R)$. The circle $c_1(A, AC)$ intersects the circle c at point D and intersects CB at E. If the line AE intersects c at F and G lies in BC such that $EB = BG$, prove that F, E, D, G are concyclic.

Problema 5

Let $N > 1$ and let $a_1, a_2, \ldots, a_N$ be nonnegative reals with sum at most 500. Prove that there exist integers $k \geq 1$ and $1 = n_0 < n_1 < \cdots < n_k = N$ such that

$$\sum_{i=1}^{k} n_i a_{n_{i-1}} < 2005.$$

Problema 6

Em Tumbólia existem n times de futebol. Deseja-se organizar um campeonato em que cada time joga exatamente uma vez com cada um dos outros. Todos os jogos ocorrem aos domingos, e um time não pode jogar mais de uma vez no mesmo dia. Determine o menor inteiro positivo m para o qual é possível realizar um tal campeonato em m domingos.