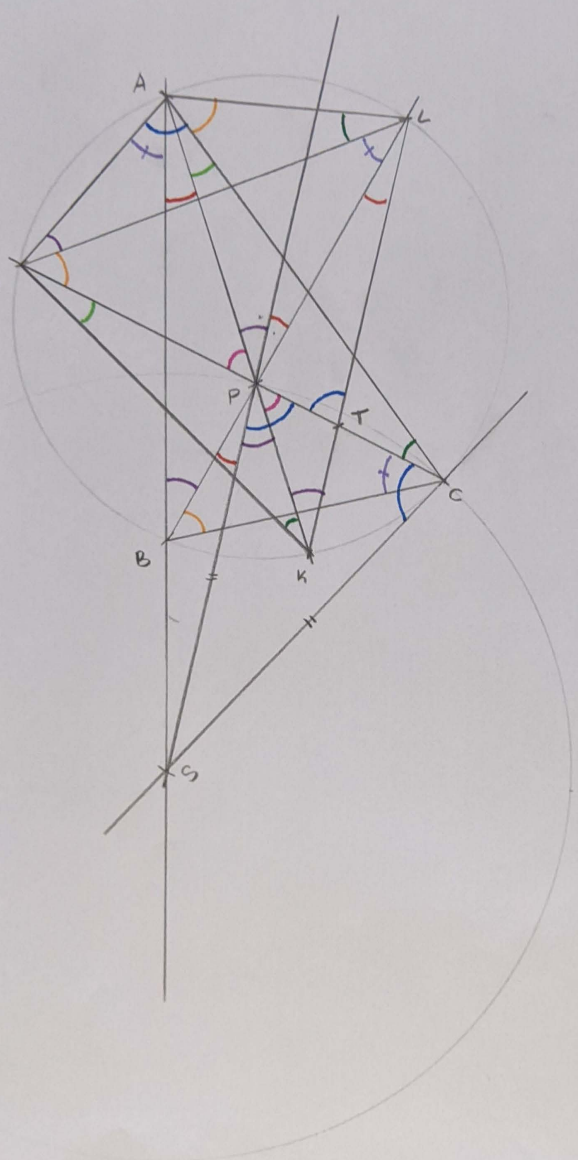
Problem 1 (IMO 2010/4). *Let P be a point interior to triangle ABC (with $CA \neq CB$). The lines AP, BP and CP meet again its circumcircle Γ at K, L, respectively M. The tangent line at C to Γ meets the line AB at S. Show that from $SC = SP$ follows $MK = ML$.*

IMO 2010 - P4



• $SA \cdot SB = SC^2 = SP^2 \Rightarrow$

$\Rightarrow \Delta SAP \sim \Delta SPB \Rightarrow$

$\Rightarrow \angle SAP = \angle SPB = \measuredangle$
$\quad \angle SAP = \angle BLK$

• $\angle SCP = \angle SPC = \measuredangle$
$\quad \angle SCP = \angle CAM$

Como $\angle BPS = \angle BLK = \measuredangle \Rightarrow$
$\Rightarrow PS \parallel KL$

Seja $T = PC \cap KT$

$\angle SPC = \angle PTL = \measuredangle$.

Logo, pelo $\angle PTL = \measuredangle$,
$\overset{\frown}{KC} + \overset{\frown}{ML} = 2 \cdot \measuredangle$

Mas, pelo $\angle MCS = \measuredangle$,
$\overset{\frown}{MC} = 2 \cdot \measuredangle$

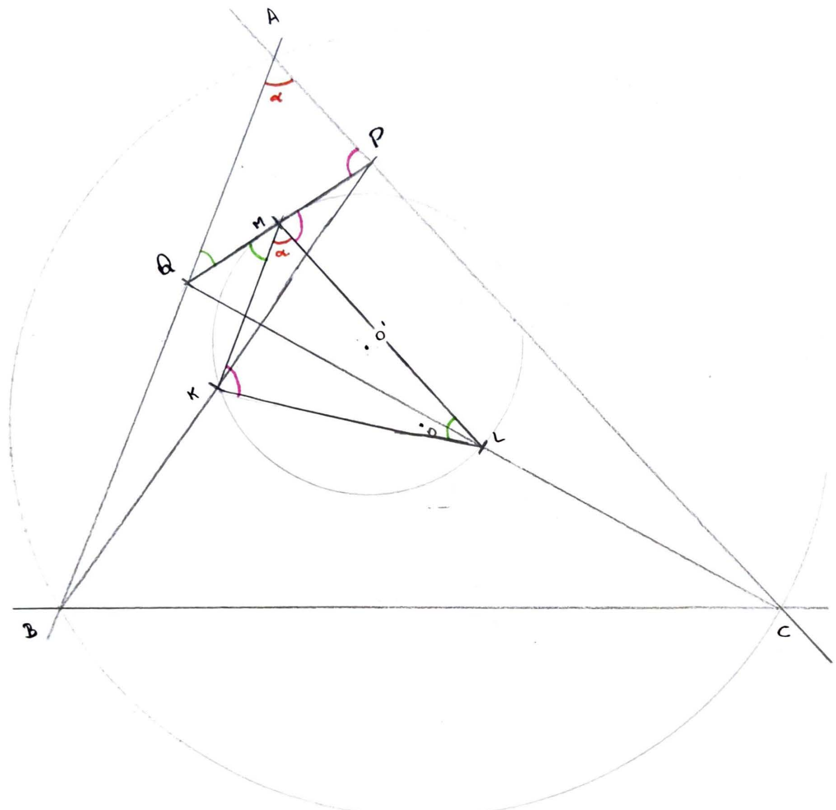
$\Rightarrow \overset{\frown}{KC} + \overset{\frown}{ML} = \overset{\frown}{MC} \Rightarrow$

$\Rightarrow \overset{\frown}{ML} = \overset{\frown}{MK} \Rightarrow$

$\Rightarrow ML = MK$. □

Problem 2 (IMO 2009/2). *Let ABC be a triangle with circumcentre O. The points P and Q are interior points of the sides CA and AB respectively. Let K, L and M be the midpoints of the segments BP, CQ and PQ. respectively, and let Γ be the circle passing through K, L and M. Suppose that the line PQ is tangent to the circle Γ. Prove that $OP = OQ$.*

IMO 2009 _ P2



Quero $OQ = OP \iff Pot_\Gamma Q = Pot_\Gamma P \iff AQ \cdot QB = AP \cdot PC \iff$

$\iff \dfrac{AQ}{AP} = \dfrac{PC}{QB}$. Mas $\dfrac{AQ}{AP} \underset{\uparrow}{=} \dfrac{ML}{MK} \underset{\uparrow}{=} \dfrac{PC}{QB}$.

semelhança

base média

$\square$

Problem 3 (EGMO 2020/5)**.** *Consider the triangle ABC with $\angle BCA > 90°$. The circumcircle Γ of ABC has radius R. There is a point P in the interior of the line segment AB such that $PB = PC$ and the length of PA is R. The perpendicular bisector of PB intersects Γ at the points D and E.*

Prove P is the incentre of triangle CDE.

Problem 4 (IMO 2014/3). *Seja $ABCD$ um quadrilátero convexo com $\angle ABC = \angle CDA = 90°$. O ponto H é o pé da perperndicular de A sobre BD. Os pontos S e T são escolhidos sobre os lados AB e AD, respectivamente, de modo que H esteja no interior do triângulo SCT e*

$$\angle CHS - \angle CSB = 90°, \quad \angle THC - \angle DTC = 90°.$$

Prove que a reta BD é tangente à circunferência circunscrita ao triângulo TSH.

Problem 5 (IMO 2008/1)**.** *Seja H o ortocentro do triângulo acutângulo ABC. O círculo Γ_A, centrado no ponto médio de BC que passa por H intersecta a reta BC nos pontos A_1 e A_2. Da mesma maneira, defina os pontos B_1, B_2, C_1 e C_2.*

Prove que os seis pontos A_1, A_2, B_1, B_2, C_1 e C_2 são concíclicos.