Problema 1 Positive integers between 1 to 100 inclusive are written on a blackboard, each exactly once. One operation involves choosing two numbers a and b on the blackboard and erasing them, then writing the greatest common divisor of $a^2 + b^2 + 2$ and $a^2b^2 + 3$.

After a number of operations, there is only one positive integer left on the blackboard. Prove this number cannot be a perfect square.

Problema 2 Given a scalene triangle $\triangle ABC$, D, E lie on segments AB, AC respectively such that $CA = CD, BA = BE$. Let ω be the circumcircle of $\triangle ADE$. P is the reflection of A across BC, and PD, PE meets ω again at X, Y respectively. Prove that BX and CY intersect on ω.

Problema 3 Let $S = \{1, 2, \ldots, 999\}$. Consider a function $f : S \to S$, such that for any $n \in S$,

$$f^{n+f(n)+1}(n) = f^{nf(n)}(n) = n.$$

Prove that there exists $a \in S$, such that $f(a) = a$. Here $f^k(n) = \underbrace{f(f(\ldots f}_{k}(n)\ldots))$.

Problema 4 Let n be an odd positive integer, and consider an infinite square grid. Prove that it is impossible to fill in one of $1, 2$ or 3 in every cell, which simultaneously satisfies the following conditions:

(1) Any two cells which share a common side does not have the same number filled in them.

(2) For any 1×3 or 3×1 subgrid, the numbers filled does not contain $1, 2, 3$ in that order be it reading from top to bottom, bottom to top, or left to right, or right to left.

(3) The sum of numbers of any $n \times n$ subgrid is the same.

Problema 5 Let T be a positive integer. Find all functions $f : \mathbb{Z}^+ \times \mathbb{Z}^+ \to \mathbb{Z}^+$, such that there exists integers $C_0, C_1, \ldots, C_T$ satisfying:

(1) For any positive integer n, the number of positive integer pairs (k, l) such that $f(k, l) = n$ is exactly n.

(2) For any $t = 0, 1, \ldots, T$, as well as for any positive integer pair (k, l), the equality $f(k+t, l+T-t) - f(k, l) = C_t$ holds.

Problema 1 Let a, b, c be positive integers. Prove that $lcm(a, b) \neq lcm(a + c, b + c)$.

Problema 2 Let N be a positive integer. There are positive integers $a_1, a_2, \cdots, a_N$ and all of them are not multiples of 2^{N+1}. For each integer $n \geq N + 1$, set a_n as below:

If the remainder of a_k divided by 2^n is the smallest of the remainder of $a_1, \cdots, a_{n-1}$ divided by 2^n, set $a_n = 2a_k$. If there are several integers k which satisfy the above condition, put the biggest one.

Prove the existence of a positive integer M which satisfies $a_n = a_M$ for $n \geq M$.

Problema 3 Let ABC be an acute-angled triangle with the circumcenter O. Let D, E and F be the feet of the altitudes from A, B and C, respectively, and let M be the midpoint of BC. AD and EF meet at X, AO and BC meet at Y, and let Z be the midpoint of XY. Prove that A, Z, M are collinear.

Problema 4 Let $n \geq 3$ be an integer. There are n people, and a meeting which at least 3 people attend is held everyday. Each attendant shake hands with the rest attendants at every meeting. After the nth meeting, every pair of the n people shook hands exactly once. Prove that every meeting was attended by the same number of attendants.

Problema 5 Let $x_1, x_2, \cdots, x_{1000}$ be integers, and $\sum_{i=1}^{1000} x_i^k$ are all multiples of 2017 for any positive integers $k \leq 672$. Prove that $x_1, x_2, \cdots, x_{1000}$ are all multiples of 2017.

Note: 2017 is a prime number.

Problema 1 Let p be an odd prime number. For positive integer k satisfying $1 \leq k \leq p-1$, the number of divisors of $kp+1$ between k and p exclusive is a_k. Find the value of $a_1 + a_2 + \ldots + a_{p-1}$.

Problema 2 Let $ABCD$ be a concyclic quadrilateral such that $AB : AD = CD : CB$. The line AD intersects the line BC at X, and the line AB intersects the line CD at Y. Let E, F, G and H are the midpoints of the edges AB, BC, CD and DA respectively. The bisector of angle AXB intersects the segment EG at S, and that of angle AYD intersects the segment FH at T. Prove that the lines ST and BD are pararell.

Problema 3 Let n be a positive integer. In JMO kingdom there are 2^n citizens and a king. In terms of currency, the kingdom uses paper bills with value $\$2^n$ and coins with value $\$2^a (a = 0, 1 \ldots, n-1)$. Every citizen has infinitely many paper bills. Let the total number of coins in the kingdom be S. One fine day, the king decided to implement a policy which is to be carried out every night:

Each citizen must decide on a finite amount of money based on the coins that he currently has, and he must pass that amount to either another citizen or the king;Each citizen must pass exactly \$1 more than the amount he received from other citizens.

Find the minimum value of S such that the king will be able to collect money every night eternally.

Problema 4 Find all functions $f : \mathbb{R} \to \mathbb{R}$ such that

$$f(yf(x) - x) = f(x)f(y) + 2x$$

for all $x,\ y \in \mathbb{R}$.

Problema 5 m, n are positive integers such that $m \geq 2$, $n < \frac{3}{2}(m-1)$. In a country there are m cities and n roads, each road connect two different cities, and there can be multiple roads between two cities. Prove that there exist a way to separate the cities into two groups α and β, where all roads connecting a city in α to a city in β is converted to a highway, and satisfies the following conditions:

Both groups have at least one city, andfor each city, the number of highways coming out from that city does not exceed 1.

Problema 1 Find all positive integers n such that $\frac{10^n}{n^3+n^2+n+1}$ is an integer.

Problema 2 Problema `math/japan/2015/2` não encontrado!

Problema 3 Problema `math/japan/2015/3` não encontrado!

Problema 4 Scalene triangle ABC has circumcircle Γ and incenter I. The incircle of triangle ABC touches side AB, AC at D, E respectively. Circumcircle of triangle BEI intersects Γ again at P distinct from B, circumcircle of triangle CDI intersects Γ again at Q distinct from C. Prove that the 4 points D, E, P, Q are concyclic.

Problema 5 Let a be a fixed positive integer. For a given positive integer n, consider the following assertion.

In an infinite two-dimensional grid of squares, n different cells are colored black. Let K denote the number of a by a squares in the grid containing exactly a black cells. Then over all possible choices of the n black cells, the maximum possible K is $a(n+1-a)$.

Prove that there exists a positive integer N such that for all $n \geq N$, this assertion is true.

Problema 1 Let O be the circumcenter of triangle ABC, and let l be the line passing through the midpoint of segment BC which is also perpendicular to the bisector of angle $\angle BAC$. Suppose that the midpoint of segment AO lies on l. Find $\angle BAC$.

Problema 2 Find all ordered triplets of positive integers $(a,\ b,\ c)$ such that $2^a + 3^b + 1 = 6^c$.

Problema 3 In a school, there are n students and some of them are friends each other. (Friendship is mutual.) Define a, b the minimum value which satisfies the following conditions:

(1) We can divide students into a teams such that two students in the same team are always friends.

(2) We can divide students into b teams such that two students in the same team are never friends.

Find the maximum value of $N = a + b$ in terms of n.

Problema 4 Let Γ be the circumcircle of triangle ABC, and let l be the tangent line of Γ passing A. Let D, E be the points each on side AB, AC such that $BD : DA = AE : EC$. Line DE meets Γ at points F, G. The line parallel to AC passing D meets l at H, the line parallel to AB passing E meets l at I. Prove that there exists a circle passing four points F, G, H, I and tangent to line BC.

Problema 5 Find the maximum value of real number k such that

$$\frac{a}{1 + 9bc + k(b-c)^2} + \frac{b}{1 + 9ca + k(c-a)^2} + \frac{c}{1 + 9ab + k(a-b)^2} \geq \frac{1}{2}$$

holds for all non-negative real numbers $a,\ b,\ c$ satisfying $a + b + c = 1$.

Problema 1 Let n, k be positive integers with $n \geq k$. There are n persons, each person belongs to exactly one of group 1, group 2, $\ldots$, group k and more than or equal to one person belong to any groups.

Show that n^2 sweets can be delivered to n persons in such way that all of the following condition are satisfied:

- At least one sweet are delivered to each person.
- a_i sweet are delivered to each person belonging to group i $(1 \leq i \leq k)$.
- If $1 \leq i < j \leq k$, then $a_i \geq a_j$.

Problema 2 Find all functions $f : \mathbb{Z} \to \mathbb{R}$ such that the equality

$$f(m) + f(n) = f(mn) + f(m + n + mn)$$

holds for all $m, n \in \mathbb{Z}$.

Problema 3 Let $n \geq 2$ be a positive integer. Find the minimum value of positive integer m for which there exist positive integers $a_1,\ a_2,\ \cdots, a_n$ such that:

- $a_1 < a_2 < \cdots < a_n = m$
- $\frac{a_1^2+a_2^2}{2},\ \frac{a_2^2+a_3^2}{2},\ \cdots,\ \frac{a_{n-1}^2+a_n^2}{2}$ are all square numbers.

Problema 4 Given an acute-angled triangle ABC, let H be the orthocenter. A cirlcle passing through the points B, C and a cirlcle with a diameter AH intersect at two distinct points X, Y. Let D be the foot of the perpendicular drawn from A to line BC, and let K be the foot of the perpendicular drawn from D to line XY. Show that $\angle BKD = \angle CKD$.

Problema 5 Let n be a positive integer. Given are points $P_1,\ P_2,\ \cdots,\ P_{4n}$ of which any three points are not collinear. For $i = 1,\ 2,\ \cdots,\ 4n$, rotating half-line P_iP_{i-1} clockwise in 90° about the pivot P_i gives half-line P_iP_{i+1}. Find the maximum value of the number of the pairs of $(i,\ j)$ such that line segments P_iP_{i+1} and P_jP_{j+1} intersect at except endpoints.

Note that: $P_0 = P_{4n},\ P_{4n+1} = P_1$ and $1 \leq i < j \leq 4n$.

Problema 1 Given a triangle ABC, the tangent of the circumcircle at A intersects with the line BC at P. Let $Q,\ R$ be the points of symmetry for P across the lines $AB,\ AC$ respectively. Prove that the line BC intersects orthogonally with the line QR.

Problema 2 Find all functions $f:\mathbb{R}\mapsto\mathbb{R}$ such that $f(f(x+y)f(x-y)) = x^2 - yf(y)$ for all $x, y \in \mathbb{R}$.

Problema 3 Let p be prime. Find all possible integers n such that for all integers x, if $x^n - 1$ is divisible by p, then $x^n - 1$ is divisible by p^2 as well.

Problema 4 Given two triangles PAB and PCD such that $PA = PB,\ PC = PD,\ P,\ A,\ C$ and $B,\ P,\ D$ are collinear in this order respectively.

The circle S_1 passing through $A,\ C$ intersects with the circle S_2 passing through $B,\ D$ at distinct points $X,\ Y$.

Prove that the circumcenter of the triangle PXY is the midpoint of the centers of $S_1,\ S_2$.

Problema 5 Given is a piece at the origin of the coordinate plane. Two persons $A,\ B$ act as following.

First, A marks on a lattice point, on which the piece cannot be anymore put. Then B moves the piece from the point $(x,\ y)$ to the point $(x+1,\ y)$ or $(x,\ y+1)$, a number of m times $(1 \leq m \leq k)$. Note that we may not move the piece to a marked point. If A wins when B can't move any pieces, then find all possible integers k such that A will win in a finite number of moves, regardless of how B moves the piece.

Problema 1 Given an acute triangle ABC with the midpoint M of BC. Draw the perpendicular HP from the orthocenter H of ABC to AM.

Show that $AM \cdot PM = BM^2$.

Problema 2 Find all of quintuple of positive integers (a, n, p, q, r) such that $a^n - 1 = (a^p - 1)(a^q - 1)(a^r - 1)$.

Problema 3 Person A writes down non negative integers in each N grid running in a line horizontally. When A says one non negative integer,

Person B replaces some number in N grid by the number that A said. Repeat this procedure, when these numbers are arranged in the order of monotone increasing in the wider sense, the procedure is over.

Is it possible that B can finish in regard less of A?

Problema 4 Find all functions $f : \mathbb{R} \to \mathbb{R}$ such that $f(f(x) - f(y)) = f(f(x)) - 2x^2 f(y) + f(y^2)$ for all $x, y \in \mathbb{R}$.

Problema 5 Given 4 points on a plane.

Suppose radii of 4 incircles of the triangles, which can be formed by any 3 points taken from the 4 points, are equal.

Prove that all of the triangles are congruent.

Problema 1 Given an acute-angled triangle ABC such that $AB \neq AC$. Draw the perpendicular AH from A to BC. Suppose that if we take points P, Q in such a way that three points A, B, P and three points A, C, Q are collinear in this order respectively, then we have four points B, C, P, Q are concyclic and $HP = HQ$. Prove that H is the circumcenter of $\triangle APQ$.

Problema 2 Let k be positive integer and m be odd number. Prove that there exists positive integer n such that $n^n - m$ is divisible by 2^k.

Problema 3 There are 2010 islands and 2009
bridges connecting them. Suppose that any bridges are connected by one
bridge or not the endpoints are connected to 2 distinct islands and we
can travel a few times by crossing bridges from each island to any other
islands.
Now a letter from each island was sent to some island, note that, some
letter may sent to same island, then the following fact was proved that:
In case of connecting island A and island B by bridge, the habitant of island A and that of island B are mutually connected by bridge or the same island (itself).
Prove that at least one of the following statements (1) or (2) hold.
(1) There exists island for which a letter was sent to the same island.
(2) There exist 2 islands, connecting bridge, whose letter are exchanged each other.

Problema 4 Let x, y, z be positive real numbers.
Prove that
$$\frac{1+yz+zx}{(1+x+y)^2}+\frac{1+zx+xy}{(1+y+z)^2}+\frac{1+xy+yz}{(1+z+x)^2}\geq 1$$

Problema 5 Given a convex 2010 polygonal whose any 3 diagonals have no intersection points except vertices. Consider closed broken lines which have 2010
diagonals (not including edges) and they pass through each vertex
exactly one time. Find the possible maximum value of the number of
self-crossing. Note that we call closed broken lines such that broken line $P_1P_2\cdots P_nP_{n+1}$ has the property $P_1 = P_{n+1}$.

Problema 1 Find all positive integers n such that $8^n + n$ is divisible by $2^n + n$.

Problema 2 Let N be postive integer. Some integers are written in a black board and those satisfy the following conditions.

1. Any numbers written are integers which are from 1 to N.
2. More than one integer which is from 1 to N is written.
3. The sum of numbers written is even.

If we mark X to some numbers written and mark Y to all remaining numbers, then prove that we can set the sum of numbers marked X are equal to that of numbers marked Y.

Problema 3 Let $k \geq 2$ be integer, $n_1,\ n_2,\ n_3$ be positive integers and $a_1,\ a_2,\ a_3$ be integers from $1,\ 2,\ \cdots,\ k-1$.

Let $b_i = a_i \sum_{j=0}^{n_i} k^j$ $(i = 1,\ 2,\ 3)$. Find all possible pairs of integers $(n_1,\ n_2,\ n_3)$ such that $b_1 b_2 = b_3$.

Problema 4 Let Γ be a circumcircle. A circle with center O touches to line segment BC at P and touches the arc BC of Γ which doesn't have A at Q. If $\angle BAO = \angle CAO$, then prove that $\angle PAO = \angle QAO$.

Problema 5 Find all functions f, defined on the non negative real numbers and taking non negative real numbers such that $f(x^2) + f(y) = f(x^2 + y + xf(4y))$ for any non negative real numbers $x,\ y$.

Problema 1 Let $P(x)$ be a polynomial with integer coefficients such that $P(n^2) = 0$ for some non zero integers n. Prove that $P(a^2) \neq 1$ for all non zero rational numbers $a \neq 0$.

Problema 2 There are 2008 red cards and 2008 white cards. 2008 players sit down in circular toward the inside of the circle in situation that 2 red cards and 2 white cards from each card are delivered to each person. Each person conducts the following procedure in one turn as follows.($*$) If you have more than one red card, then you will pass one red card to the left-neighbouring player.

If you have no red card, then you will pass one white card to the left -neighbouring player.

Find the maximum value of the number of turn required for the state such that all person will have one red card and one white card first.

Problema 3 Given an acute-angled triangle ABC with circumcenter O. The circle passing through two points $A,\ O$ intersects with the line AB and AC at $P,\ Q$ other than A respectively. If the lengths of the line segments $PQ,\ BC$ are equal, then find the angle $\leq 90^\circ$ that the lines PQ and BC make.

Problema 4 Find all functions $f : \mathbb{R} \to \mathbb{R}$ such that

$$f(x+y)f(f(x)-y) = xf(x) - yf(y)$$

for all $x, y \in \mathbb{R}$.

Problema 5 Can it be existed postive integers n such that there are integers b and non zero integers a_i $(i = 1,\ 2,\ \cdots n)$ for rational numbers r which satisfies $r = b + \sum_{i=1}^{n} \frac{1}{a_i}$?

Problema 1 Let n be positive integers. Two persons play a game in which they are calling a integer m $(1 \leq m \leq n)$ alternately. Note that you may not call the number which have already said. The game is over when no one can call numbers, if the sum of the numbers that the lead have said is divisible by 3, then the lead wins, otherwise the the second move wins. Find n for which there exists the way of forestalling.

Problema 2 Find all functions f, defined on the positive real numbers and taking real numbers such that

$$f(x)+f(y) \leq \frac{f(x+y)}{2}, \quad \frac{f(x)}{x}+\frac{f(y)}{y} \geq \frac{f(x+y)}{x+y}$$

for all $x,\ y>0$.

Problema 3 Let Γ be the circumcircle of triangle ABC. Denote the circle which touches to the sides $AB,\ AC$ and touches to Γ internally at P by Γ_A, and the circle which touches to the sides $AB,\ BC$ and touches to Γ internally at Q by Γ_B, and the circle which touches to the sides $AC,\ BC$ and touches to Γ internally at R by Γ_C. Prove that the lines $AP,\ BQ,\ CR$ are concurrent.

Problema 4 On a plane, call the band with width d be the set of all points which are away the distance of less than or equal to $\frac{d}{2}$ from a line.Given four points $A,\ B,\ C,\ D$ on a plane.If you take three points among them, there exists the band with width 1 containing them. Prove that there exist the band with width $\sqrt{2}$ containing all four points .

Problema 5 For real positive numbers x, the set $A(x)$ is defined by

$$A(x)=\{[nx] \mid n \in \mathbb{N}\},$$

where $[r]$ denotes the greatest integer not exceeding real numbers r. Find all irrational numbers $\alpha>1$ satisfying the following condition.

Condition: If positive real number β satisfies $A(\alpha) \supset A(\beta)$, then $\frac{\beta}{\alpha}$ is integer.

Problema 1 Given five distinct points A, M, B, C, D in this order on the circumference of the circle O such that $MA = MB$.

Let P, Q be the intersection points of the line AC and MD, and that of the line BD and MC, respectively.

If two intersection points of the line PQ and the circumference of the circle O are X, Y, then prove that $MX = MY$.

Problema 2 Determine all integers k for which there exist infinitely the pairs of integers (a, b, c) satisfying the following equation.

$$(a^2 - k)(b^2 - k) = c^2 - k.$$

Problema 3 Find all functions f, defined on real numbers and taking real values such that $\{f(x)\}^2 + 2yf(x) + f(y) = f(y + f(x))$ for all real numbers x, y.

Problema 4 Let m, n be integers such that $2 \leq m \leq n$ and let a, a' be integers which are less than or equal to m and let b, b' be integers which are less than or equal to n such that $(a, b) \neq (a'\ b')$. Given a town of the rectangular shaped chessboard which is made up of $m's$ road running north and south which is called Line and $n's$ road running west and east which is called Street. Denote the intersection point of the a th Line from the west and b th Street from the north by A, and a' th Line from the west and b' th Street from the north by B, including the edge for both cases.Find all pair of (m, n, a, b, a', b') such that by passing through each crossroads of the town exactly one time, you can reach the point B from the point A including in the start point and goal one.

Problema 5 For any positive real numbers x_1, x_2, x_3, y_1, y_2, y_3, z_1, z_2, z_3, find the maximum value of real number A such that if

$$M = (x_1^3 + x_2^3 + x_3^3 + 1)(y_1^3 + y_2^3 + y_3^3 + 1)(z_1^3 + z_2^3 + z_3^3 + 1)$$

and

$$N = A(x_1 + y_1 + z_1)(x_2 + y_2 + z_2)(x_3 + y_3 + z_3),$$

then $M \geq N$ always holds, then find the condition that the equality holds.

Problema 1 Double-faced coins are arranged with all the heads facing upward in the shape of 17×17.At one operation, you turn over 5 consecutive coins in longitudinal or 5 ones in transversal or 5 ones in oblique at the same time.Now can you make all those reverses face upward when you repeat this operation?

Problema 2 Let $P(x, y), Q(x, y)$ be two-variable polynomials with the coefficients of integer.Supposed that when a_n, b_n are determined for certain integers $a_0,\ b_0$ by $a_{n+1} = P(a_n, b_n),\quad b_{n+1} = Q(a_n, b_n)\quad (n = 0, 1, 2, \cdots)$ there existed positive integer k such that $(a_1, b_1) \neq (a_0, b_0)$ and $(a_k, b_k) = (a_0, b_0)$.Prove that the number of the lattice points on the segment with end points of (a_n, b_n) and (a_{n+1}, b_{n+1}) is indepedent of n.

Problema 3 Let a, b, c be positive real numbers such that $a + b + c = 1$.Prove the following inequality.

$$a\sqrt[3]{1+b-c} + b\sqrt[3]{1+c-a} + c\sqrt[3]{1+a-b} \leqq 1$$

Problema 4 Given two points A and B on a circle Γ. Let the tangents to this circle Γ at the points A and B meet at a point X. Let C and D be two points on the circle Γ such that the points C, D, X are collinear in this order and such that the lines CA and BD are perpendicular.

Let the line CA intersect the line BD at a point F.

Let the line CD intersect the line AB at a point G.

Let H be the point of intersection of the segment BD and the perpendicular bisector of the segment GX.

Prove that the four points X, F, G, H lie on one circle.

Problema 5 You are the boss. You have ten men and there are ten tasks. Your men have two parameters to each task, one is enthusiasm, the other is ability.Now you are to assign the tasks to your men one by one.When man A has more enthusiasm about task v than about task u, and man A has better ability in task v than man B does, though if you assign task u to man A and task v to man B, man A

feel unsatisfied.Or, if there is a more efficient way than yours that you can assign each task to men with better ability, you will be brought to account by your employer.Prove that there is a way to assign tasks without any dissatisfaction or disadvantage.

Problema 1 Problema `math/japan/2004/1` não encontrado!

Problema 2 Find all functions $f : \mathbb{R} \mapsto \mathbb{R}$ such that $f(xf(x) + f(y)) = (f(x))^2 + y$ for all $x, y \in \mathbb{R}$.

Problema 3 Given two planes $\pi_1,\ \pi_2$ intersecting orthogonally in space. Let A, B be two distinct points on the line of intersection of π_1 and π_2, and C be the point which is on π_2 but not on π_1. Denote by P the intersection point of the bisector of $\angle BCA$ and AB, and denote S by the circumference on π_1 with a diameter AB. For an arbiterary plane π_3 which contains CP, if D, E are the intersection points of π_3 and S, then prove that CP is the bisector of $\angle DCE$.

Problema 4 For positive real numbers a, b, c satisfying $a + b + c = 1$,
Prove that we have $\frac{1+a}{1-a} + \frac{1+b}{1-b} + \frac{1+c}{1-c} \leqq 2\left(\frac{b}{a} + \frac{c}{b} + \frac{a}{c}\right)$. Note that you don't need to state for the condition for which the equality holds.

Problema 5 In a land any towns are connected by roads with exactly other three towns. Last year we made a trip starting from a town coming back to the town by visiting exactly one time all towns in land. This year we are plan to make a trip in the same way as last year's trip. Note that you can't take such order completely same as last year's one or trace the order only reversely. Prove that this is possible.

Problema 1 A point P lies in $\triangle ABC$. The lines BP, CP meet AC, AB at Q, R respectively. Given that $AR = RB = CP, CQ = PQ$, find $\angle BRC$.

Problema 2 We have two distinct positive integers a, b with $a|b$. Each of a, b consists of $2n$ decimal digits. The first n digits of a are identical to the last n digits of b, and vice versa. Determine a, b.

Problema 3 Find the greatest real number k such that, for any positive a, b, c with $a^2 > bc$, $(a^2 - bc)^2 > k(b^2 - ca)(c^2 - ab)$.

Problema 4 Let $p, q \geq 2$ be coprime integers. A list of integers $(r, a_1, a_2, ..., a_n)$ with $|a_i| \geq 2$ for all i is said to be an expansion of p/q if $\frac{p}{q} = r + \frac{1}{a_1 + \frac{1}{a_2 + \frac{1}{... + \frac{1}{a_n}}}}$.

Now define the weight of an expansion $(r, a_1, a_2, ..., a_n)$ to be the product$(|a_1| - 1)(|a_2| - 1)...(|a_n| - 1)$.

Show that the sum of the weights of all expansions of p/q is q.

Problema 5 Find the greatest possible integer n such that one can place n points in a plane with no three on a line, and color each of them either red, green, or yellow so that:

(i) inside each triangle with all vertices red there is a green point.
(ii) inside each triangle with all vertices green there is a yellow point.
(iii) inside each triangle with all vertices yellow there is a red point.

Problema 1 Distinct points A, M, B with $AM = MB$ are given on circle (C_0) in this order. Let P be a point on the arc AB not containing M. Circle (C_1) is internally tangent to (C_0) at P and tangent to AB at Q. Prove that the product $MP \cdot MQ$ is independent of the position of P.

Problema 2 There are $n \geq 3$

coins on a circle. Consider a coin and the two coins adjacent to it; if

there are an odd number of heads among the three, we call it good. An operation consists of turning over all good coins simultaneously. Initially, exactly one of the n coins is a head. The operation is repeatedly performed.

(a) Prove that if n is odd, the coins will never be all-tails.

(b) For which values of n is it possible to make the coins all-tails after several operations? Find, in terms of n, the number of operations needed for this to occur.

Problema 3 Denote by $S(n)$ the sum of decimal digits of a positive integer n. Show that there exist 2002 distinct positive integers $n_1, n_2, \cdots, n_{2002}$ such that $n_1 + S(n_1) = n_2 + S(n_2) = \cdots = n_{2002} + S(n_{2002})$.

Problema 4 Let $n \geq 3$ be natural numbers, and let $a_1,\ a_2,\ \cdots,\ a_n,\ \ b_1,\ b_2,\ \cdots,\ b_n$ be positive numbers such that $a_1 + a_2 + \cdots + a_n = 1,\ b_1^2 + b_2^2 + \cdots + b_n^2 = 1$. Prove that $a_1(b_1 + a_2) + a_2(b_2 + a_3) + \cdots + a_n(b_n + a_1) < 1$.

Problema 5 Let S be a set of 2002 points in the coordinate plane, no two of which have the same $x-$ or $y-$coordinate. For any two points $P, Q \in S$, consider the rectangle with one diagonal PQ and the sides parallel to the axes. Denote by W_{PQ} the number of points of S lying in the interior of this rectangle. Determine the maximum N such that, no matter how the points of S are distributed, there always exist points P, Q in S with $W_{PQ} \geq N$.

Problema 1 Each square of an $m \times n$ chessboard is painted black or white in such a way that for every black square, the number of black squares adjacent to it is odd (two squares are adjacent if they share one edge). Prove that the number of black squares is even.

Problema 2 An integer $n > 0$ is written in decimal system as $\overline{a_m a_{m-1} \ldots a_1}$. Find all n such that

$$n = (a_m + 1)(a_{m-1} + 1) \cdots (a_1 + 1)$$

Problema 3 Three nonnegative real numbers satisfy a, b, c satisfy $a^2 \le b^2 + c^2, b^2 \le c^2 + a^2$ and $c^2 \le a^2 + b^2$. Prove the inequality

$$(a + b + c)(a^2 + b^2 + c^2)(a^3 + b^3 + c^3) \ge 4(a^6 + b^6 + c^6).$$

When does equality hold?

Problema 4 Let p be a prime number and m a positive integer. Show that there exists a positive integer n such that the decimal representation of p^n contains a string of m consecutive zeros.

Problema 5 Suppose that ABC and PQR are triangles such that A, P are the midpoints of QR, BC respectively, and QR, BC are the bisectors of $\angle BAC, \angle QPR$. Prove that $AB + AC = PQ + PR$.

Problema 1 Consider the points $O(0,0)$ and $A(0,1/2)$ on the coordinate plane. Prove that there is no finite sequence of rational points $P_1, P_2, \dots, P_n$ in the plane such that

$$OP_1 = P_1P_2 = \dots = P_{n-1}P_n = P_nA = 1$$

Problema 2 Let $3n$ cards, denoted by distinct letters $a_1, a_2, \dots, a_{3n}$, be put in line in this order from left to right. After each shuffle, the sequence $a_1, a_2, \dots, a_{3n}$ is replaced by the sequence $a_3, a_6, \dots, a_{3n}, a_2, a_5, \dots, a_{3n-1}, a_1, a_4, \dots, a_{3n-2}$. Is it possible to replace the sequence of cards $1, 2, \dots, 192$ by the reverse sequence $192, 191, \dots, 1$ by a finite number of shuffles?

Problema 3 Given five points A, B, C, D, E in a plane, no three of which are collinear, prove the inequality

$$AB + BC + CA + DE \leq AD + AE + BD + BE + CD + CE$$

Problema 4 Given a natural number $n \geq 3$, prove that there exists a set A_n with the following two properties:

1) A_n consists of n distinct natural numbers

2) For any $a \in A$, the remainder of the product of all elements of $A_n \backslash \{a\}$ divided by a is 1.

Problema 5 Finitely many lines are given in a plane. We call an intersection point a point that belongs to at least two of the given lines, and a good intersection point

a point that belongs to exactly two lines. Assuming there at least two intersection points, find the minimum number of good intersection points.

Problema 1 One can place a stone on each of the squares of a 1999×1999 board. Find the minimum number of stones that must be placed so that, for any blank square on the board, the total number of stones placed in the corresponding row and column is at least 1999.

Problema 2 Let $f(x) = x^3 + 17$. Prove that for every integer $n \geq 2$ there exists a natural number x for which $f(x)$ is divisible by 3^n but not by 3^{n+1}.

Problema 3 From a group of $2n + 1$ weights, if we remove any weight, the remaining $2n$, can be divided in two groups of n elements, such that they have the same total weight. Prove all weights are equal.

Problema 4 Prove that the polynomial $f(x) = (x^2 + 1)(x^2 + 2^2) \cdots (x^2 + n^2) + 1$ cannot be expressed as a product of two polynomials with integer coefficients with degree greater than 1.

Problema 5 All sides of a convex hexagon $ABCDEF$ are 1. Let M, m be the maximum and minimum possible values of three diagonals AD, BE, CF. Find the range of M, m.

Problema 1 Let $p \geq 3$ be a prime, and let p points $A_0, \ldots, A_{p-1}$ lie on a circle in that order. Above the point $A_{1+\cdots+k-1}$ we write the number k for $k = 1, \ldots, p$ (so 1 is written above A_0). How many points have at least one number written above them?

Problema 2 A country has 1998 airports connected by some direct flights. For any three airports, some two are not connected by a direct flight. What is the maximum number of direct flights that can be offered?

Problema 3 Let $P_1, \ldots P_n$ be the sequence of vertices of a closed polygons whose sides may properly intersect each other at points other than the vertices. The external angle at P_i is defined as $180°$ minus the angle of rotation about P_i required to bring the ray P_iP_{i-1} onto the ray P_iP_{i+1}, taken in the range $(0°, 360°)$. (Here $P_0 = P_n$ and $P_1 = P_{n+1}$). Prove that if the sum of the external angles is a multiple of $720°$, then the number of self-intersections is odd.

Problema 4 Let $c_{n,m}$ be the number of permutations of $\{1, \ldots, n\}$ which can be written as the product of m transpositions of the form $(i, i+1)$ for some $i = 1, \ldots, n-1$ but not of $m-1$ suct transpositions. Prove that for all $n \in \mathbb{N}$,

$$\sum_{m=0}^{\infty} c_{n,m} t^m = \prod_{i=1}^{n} (1 + t + \cdots + t^{i-1}).$$

Problema 5 On each of 12 points around a circle we place a disk with one white side and one black side. We may perform the following move: select a black disk, and reverse its two neighbors. Find all initial configurations from which some sequence of such moves leads to the position where all disks but one are white.

Problema 1 Take 10 points inside the circle
with diameter 5. Prove that for any these 10 points there exist two
points whose distance is less than 2.

Problema 2 Prove that $\frac{(b+c-a)^2}{(b+c)^2+a^2} + \frac{(c+a-b)^2}{(c+a)^2+b^2} + \frac{(a+b-c)^2}{(a+b)^2+c^2} \geq \frac{3}{5}$
for any positive real numbers a, b, c.

Problema 3 Call the Graph the set which composed of several vertices $P_1, \cdots P_2$ and several edges (segments) connecting two points among these vertices. Now let G be a graph with 9 vertices and satisfies the following condition.

Condition: Even if we select any five points from the vertices in G, there exist at least two edges whose endpoints are included in the set of 5 points.

What is the minimum possible numbers of edges satisfying the condition?

Problema 4 Let A, B, C, D be points in space which are not on a same plane and no any 3 points are not colinear.

Suppose that the sum of the segments $AX + BX + CX + DX$ is minimized at $X = X_0$ which is different from A, B, C, D. Prove that $\angle AX_0B = \angle CX_0D$.

Problema 5 The letters A or B are assigned on the points divided equally into 2^n $(n = 1, 2, \cdots)$ parts of a circumference.If you choose n
letters from any succesively arranging points directed clockwise, prove
that there exists the way of assignning for which the line of letters
are mutually distinct.

Problema 1 A plane is partitioned into triangles. Let $\mathcal{T}_0$ denote the set of vertices of triangles in the partition. Let ABC be a triangle with $A, B, C \in \mathcal{T}_0$ and θ be its smallest angle. Assume that no point of $\mathcal{T}_0$ lies inside the circumcircle of $\triangle ABC$. Prove that there exists a triangle σ in the partition such that its intersection with $\triangle ABC$ is nonempty and whose every angle is greater than θ.

Problema 2 Let m, n be positive integers with $(m, n) = 1$. Find $(5^m + 7^m, 5^n + 7^n)$.

Problema 3 Let $x > 1$ be a real number which is not an integer. For each $n \in \mathbb{N}$, let $a_n = \lfloor x^{n+1} \rfloor - x\lfloor x^n \rfloor$. Prove that the sequence (a_n) is not periodic.

Problema 4 Let θ
be the maximum of the six angles between six edges of a regular
tetrahedron in space and a fixed plane. When the tetrahedron is rotated
in space, find the maximum of θ.

Problema 5 Let q be a real number with $\frac{1+\sqrt{5}}{2} < q < 2$. If a positive integer n is represented in binary system as $2^k + a_{k-1}2^{k-1} + \cdots + 2a_1 + a_0$, where $a_i \in \{0, 1\}$, define

$$p_n = q^k + a_{k-1}q^{k-1} + \cdots + qa_1 + a_0.$$

Prove that there exist infinitely many positive integers k with the property that there is no $l \in \mathbb{N}$ such that $p_{2k} < p_l < p_{2k+1}$.

Problema 1 Let $n \geq 2$ be integers and r be positive integers such that r is not the multiple of n, and let g be the greatest common measure of n and r. Prove that

$$\sum_{i=1}^{n-1} \left\{ \frac{ri}{n} \right\} = \frac{1}{2}(n-g).$$

where $\{x\}$ is the fractional part, that is to say, which means the value by subtracting x from the maximum integer value which is equal or less than x.

Problema 2 Find all the non-constant rational function of $f(x)$ and real numbers a satisfying $\{f(x)\}^2 - a = f(x^2)$.

Here a rational function of x is the equation expressed by the ratio of two polynominals of x.

Problema 3 Given a convex pentagon $ABCDE$. Let S, R be the intersection points of AC or AD and BE respectively, and let the intersection points T, P of CA or CE and BD respectively.Let Q be the intersection point of CE and AD. If all of $\triangle ASR$, $\triangle BTS$, $\triangle CPT$, $\triangle DQP$, and $\triangle ERQ$ have the area of 1, then find the area of the following pentagons.

(1) The pentagon $PQRST$.

(2) The pentagon $ABCDE$.

Problema 4 The sequence $\{a_1,\ a_2,\ \cdots\}$ is defined by $a_{2n} = a_n,\ a_{2n+1} = (-1)^n$. A point P moves on the coordinate plane as follows.

(1) Let P_0 be the origin, P moves in a distance of 1 from P_0 toward in the positive direction of x-axis, Denote this point by P_i.

(2) After P has moved to P_i, it turns 90° to the left and moves in a distance of 1 when $a_i = 1$, and turns 90° to the right and moves in a distance of 1 when $a_i = -1$. Denote this point by P_{i+1},where $i = 1, 2, \cdots$. Prove that P can't pass on the same segment more than two times.

Problema 5 Let k, n be integers such that $1 \leq k \leq n$,and let $a_1,\ a_2,\ \cdots,\ a_k$ be numbers satisfying the following equations.

$$\begin{cases} a_1 + \cdots\cdots + a_k = n \\ a_1^2 + \cdots\cdots + a_k^2 = n \\ \qquad\quad \vdots \\ a_1^k + \cdots\cdots + a_k^k = n \end{cases}$$

Prove that

$$(x+a_1)(x+a_2)\cdots(x+a_k) = x^k + {}_nC_1\ x^{k-1} + {}_nC_2\ x^{k-2} + \cdots + {}_nC_k.$$

where ${}_iC_j$ is a binomial coefficient which means $\frac{i\cdot(i-1)\cdots(i-j+1)}{j\cdot(j-1)\cdots2\cdot1}$.

Problema 1 For any positive integer n, let a_n denote the closest integer to $\sqrt{n}$, and let $b_n = n + a_n$. Determine the increasing sequence (c_n) of positive integers which do not occur in the sequence (b_n).

Problema 2 Five points, no three collinear, are given on the plane. Let $l_1, l_2, \ldots, l_{10}$ be the lengths of the ten segments joining any two of the given points. Prove that if $l_1^2, \ldots, l_9^2$ are rational numbers, then l_{10}^2 is also a rational number.

Problema 3 Let P_0 be a point in the plane of triangle $A_0A_1A_2$. Define P_i $(i = 1, \ldots, 6)$ inductively as the point symmetric to P_{i-1} with respect to A_k, where k is the remainder when i is divided by 3.

a) Prove that $P_6 \equiv P_1$.

b) Find the locus of points P_0 for which P_iP_{i+1} does not meet the interior of $\triangle A_0A_1A_2$ for $0 \le i \le 5$.

Problema 4 In a triangle ABC, M is the midpoint of BC. Given that $\angle MAC = 15°$, find the maximum possible value of $\angle ABC$.

Problema 5 In a deck of N cards, the cards are denoted by 1 to N. These cards are dealt to N people twice. A person X wins a prize if there is no person Y who got a card with a smaller number than X both times. Determine the expected number of prize winners.

Problema 1 Call a word forming by alphabetical small letters $a, b, c, \cdots x, y, z$ and a periodic word arranging by a certain word more than two times repeatedly.For example kyonkyon is eight-letter periodic word. Given two words W_1, W_2 which have the same number of letters and have a different first letter,

if you will remove the letter, W_1 and W_2 will be same word.Prove that either W_1 or W_2 is not periodic.

Problema 2 Denote by $d(n)$ the largest odd divisor of positive integers n and define $D(n)$, $T(n)$ as follows.

$$D(n) = d(1) + d(2) + \cdots + d(n), \quad T(n) = 1 + 2 + \cdots + n.$$

Prove that there existed infinitely positive integers n such that $3D(n) = 2T(n)$.

Problema 3 x students took an exam with y
problems. Each student solved a half of the problem and the number of
person those who solved the problem is equal for each problem.For any
two students exactly three problems could be solved by the two persons.

Find all pairs of (x, y) satisfying this condition, then show an example to prove that it is possiible the case in terms of a, b as follows:

Note that every student solve (a) or don't solve (b).

Problema 4 Given five radii $l_1, \cdots l_5$ of a sphere S, no three of these radii are on a same plane.

Choose a pair to endpoint from each radius $l_1, \cdots l_5$. Find the number of choices such that five points are in a hemisphere among 32 choices of an endpoint.

Problema 5 Prove that there existed a positive number C, irrelevant to n and $a_1, a_2, \cdots a_n$, satisfying the following condition.

Condition: For arbiterary positive numbers n and arbiterary real numbers $a_1, \cdots a_n$, the following inequality holds.

$$\max_{0\leq x\leq 2} \prod_{j=1}^{n} |x - a_j| \leq C^n \max_{0\leq x\leq 1} \prod_{j=1}^{n} |x - a_j|.$$

Problema 1 Let $x,\ y$ be relatively prime numbers such that $xy \neq 1$. For positive even integer n, prove that $x+y$ isn't a divisor of $x^n + y^n$.

Problema 2 Suppose that $D,\ E$ are points on $AB,\ AC$ of $\triangle ABC$ with area 1 respectively and P is $BE \cap CD$. When $D,\ E$ move on $AB,\ AC$ with satisfying the condition $[BCED] = 2\triangle PBC$, find the maximum area of $\triangle PDE$.

Problema 3 Prove the inequality $\sum_{k=1}^{n-1} \frac{n}{n-k} \cdot \frac{1}{2^{k-1}} < 4$ $(n \geq 2)$.

Problema 4 Let A be a $m \times n$ $(m \neq n)$ matrix with the entries 0 and 1. Suppose that if f is injective such that $f : \{1,\ 2,\ \cdots,\ m\} \longrightarrow \{1,\ 2,\ \cdots,\ n\}$, then there exists $1 \leq i \leq m$ such that $(i,\ f(i))$ element is 0.

Prove that there exist $S \subseteq \{1,\ 2,\ \cdots,\ m\}$ and $T \subseteq \{1,\ 2,\ \cdots,\ n\}$ satisfying the condition:

- if $i \in S,\ j \in T$, then $(i,\ j)$ entry is 0.
- $\text{card } S + \text{card } T > n$.

Problema 5 Suppose that $n \geq 2$ be integer and $a_1,\ a_2,\ a_3,\ a_4$ satisfy the following condition:

i) n and a_i $(i = 1,\ 2,\ 3,\ 4)$ are relatively prime.

ii) $(ka_1)_n + (ka_2)_n + (ka_3)_n + (ka_4)_n = 2n$ for $k = 1,\ 2,\ \cdots,\ n-1$.

Note that $(a)_n$ expresses the divisor when a is divided by n.

Prove that $(a_1)_n,\ (a_2)_n,\ (a_3)_n,\ (a_4)_n$ can be divided into two pair with sum n.

Problema 1 Let $P,\ Q,\ R$ be the points such that $BP : PC = CQ : QA = AR : RB = t : 1-t\ (0 < t < 1)$ for a triangle ABC.

Denote K by the area of the triangle with segments $AP,\ BQ,\ CR$ as side lengths and L by triangle ABC, find $\frac{K}{L}$ in terms of t.

Problema 2 Let N be the set of the whole of positive integers. The mapping from N to N is defined as follows: $p(1) = 2,\ p(2) = 3,\ p(3) = 4,\ p(4) = 1, \quad p(n) = n\ (n \geq 5)$,$q(1) = 3,\ q(2) = 4,\ q(3) = 2,\ q(4) = 1, \quad p(n) = n\ (n \geq 5)$. Answer the following questions.

(1) If you make a mapping $f : N \to N$ sucessfully, we have f such that $f(f(n)) = p(n) + 2$. Give an example.

(2) Prove that it is impossible that $f(f(n)) = q(n) + 2$ holds in regardless of any definition for $f : N \to N$.

Problema 3 Let A be a positive 16 digit integer.If you take out some consecutive digits integers among A, prove that we can make the product of the numbers be square number. For example if some digit of A is 4, you may take out only the digit.

Problema 4 A rectangular of a $10 * 14$ is divided into small 140 unit squares and painted in red and white like chess board as below.

We put 0 or 1 in the square such that each row and column has an odd numbers of 1.

Prove that the number of 1 contained in red-painted square is even.

The pattern arranged by a red and a white square alternatively.

RWRWRWRW.....
WRWRWRWR.....
RWRWRWRW.....
WRWRWRWR.....

Problema 5 Let A be a set of $n \geq 2$ points on a plane. Prove that there exists a circle which contains at least $\left[\frac{n}{3}\right]$ points of A among circles (involving perimeter) with some end points taken from A as the diameter, where $[x]$ is the greatest integer which is less than or equal to x.