Problemas Sortidos de Geomeria – #2

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1 Questões Interessantes

Problema 1.1. Let ABC be a triangle and H its orthocenter. Let D be a point lying on the segment AC and let E be the point on the line BC such that $BC \perp DE$. Prove that $EH \perp BD$ if and only if BD bisects AE.

Problema 1.2. Let D be the footpoint of the altitude from B in the triangle ABC , where $AB = 1$. The incircle of triangle BCD coincides with the centroid of triangle ABC. Find the lengths of AC and BC.

Problema 1.3. Let P be a point inside the acute angle $\angle BAC$. Suppose that $\angle ABP = \angle ACP = 90°$. The points D and E are on the segments BA and CA, respectively, such that $BD = BP$ and $CP = CE$. The points F and G are on the segments AC and AB, respectively, such that DF is perpendicular to AB and EG is perpendicular to AC. Show that $PF = PG$.

Problema 1.4. In the non-isosceles triangle ABC an altitude from A meets side BC in D . Let M be the midpoint of BC and let N be the reflection of M in D . The circumcirle of triangle AMN intersects the side AB in $P \neq A$ and the side AC in $Q \neq A$. Prove that AN, BQ and CP are concurrent.

2 Problemas Divertidos

Problema 2.1. Os círculos ω_1 e ω_2 se intersectam nos pontos A e B. O ponto C está na reta tangente por A de ω_1 tal que $\angle ABC = 90°$. Uma reta ℓ passa por C e corta ω_2 nos pontos P e Q. As retas AP e AQ cortam ω_1 novamente em X e Z, respectivamente. Seja Y o pé da altitude de A na reta ℓ. Prove que os pontos X, Y e Z são colineares.

Problema 2.2. Quadrilateral $XABY$ is inscribed in the semicircle ω with diameter XY. Segments AY and BX meet at P. Point Z is the foot of the perpendicular from P to line XY. Point C lies on ω such that line XC is perpendicular to line AZ. Let Q be the intersection of segments AY and XC. Prove that

$$\frac{BY}{XP} + \frac{CY}{XQ} = \frac{AY}{AX}.$$

Problema 2.3. In acute triangle $ABC, \angle A = 45^o$. Points O, H are the circumcenter and the orthocenter of ABC, respectively. D is the foot of altitude from B. Point X is the midpoint of arc AH of the circumcircle of triangle ADH that contains D. Prove that $DX = DO$.

Problema 2.4. Quadrilateral $APBQ$ is inscribed in circle ω with $\angle P = \angle Q = 90°$ and $AP = AQ < BP$. Let X be a variable point on segment $\overline{PQ}$. Line AX meets ω again at S (other than A). Point T lies on arc AQB of ω such that $\overline{XT}$ is perpendicular to $\overline{AX}$. Let M denote the midpoint of chord $\overline{ST}$. As X varies on segment $\overline{PQ}$, show that M moves along a circle.