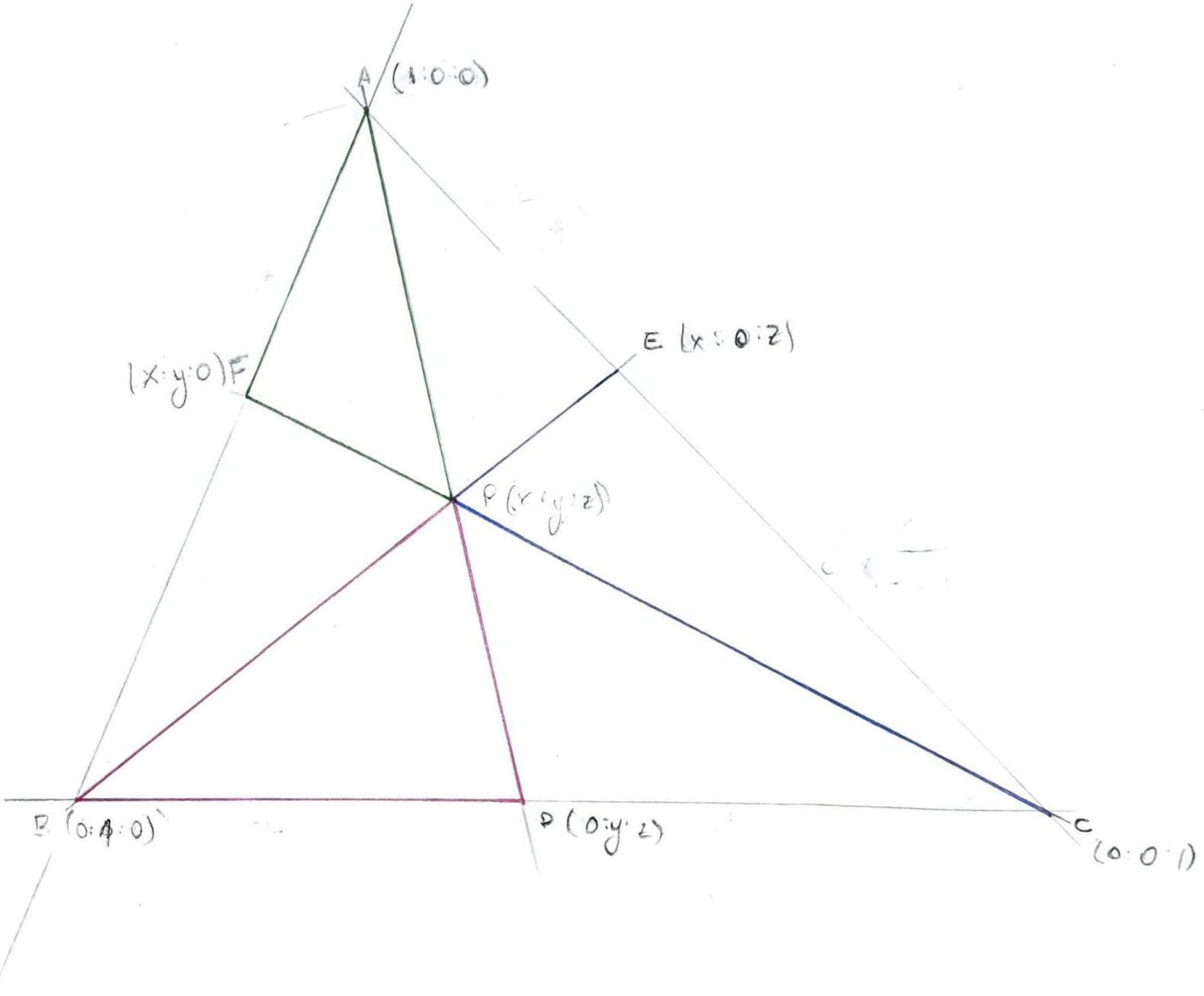
A (1:0:0)
E (x:0:z)
(x:y:0) F
P (x:y:z)
B (0:1:0)
D (0:y:z)
C (0:0:1)

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$$\frac{[AFP]}{[ABC]} = \begin{vmatrix} 1 & 0 & 0 \\ x & y & 0 \\ x & y & z \end{vmatrix} \cdot \frac{1}{(x+y+z)} \cdot \frac{1}{(x+y)}$$

$$K = \frac{zy}{(x+y+z)(x+y)}$$

$$\frac{[BDP]}{[ABC]} = \begin{vmatrix} 0 & 1 & 0 \\ 0 & y & z \\ x & y & z \end{vmatrix} \cdot \frac{1}{(x+y+z)} \cdot \frac{1}{y+z}$$

$$K = \frac{xz}{(x+y+z)(y+z)}$$

$$K = \frac{yx}{(x+y+z)(x+z)}$$

W.LO.G: $x+y+z=1$.

$$K = \frac{zy}{1-z} = \frac{xz}{1-x} = \frac{yx}{1-y}$$

$$\begin{aligned} yz &= K - Kz \\ xz &= K - Kx \\ xy &= K - Ky \end{aligned} \quad \left| \begin{aligned} K &= z(y+K) \\ &= x(z+K) \\ &= y(x+K) \end{aligned} \right.$$

$$\rightarrow z = \frac{K}{(y+K)}, \quad x = \frac{K}{(z+K)}, \quad y = \frac{K}{(x+K)}$$

$$x = \frac{K}{\frac{K}{y+K} + K} = \frac{y+K}{1+y+K} = 1 - \frac{1}{1+y+K}$$

$$y = \frac{K}{1 - \frac{1}{1+y+K} + K}$$

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$$y = \frac{K(1+y+K)}{y+K+K+Ky+K^2} \Rightarrow y^2+2Ky+Ky^2+K^2y = K+Ky+K^2$$

$$y^2+Ky+Ky^2+K^2y = K+K^2$$

$K \neq 1$: $\quad y^2(\cancel{1+K}) + yK(\cancel{1+K}) = K(\cancel{1+K})$

$$y^2+yK = K$$

$$y(y+K) = K$$

$$y = \frac{K}{y+K} = z \Rightarrow y = z$$

Analogamente, $x = y$, $z = x$

$$\Rightarrow x = y = z = 1/3 \Rightarrow$$

$$\Rightarrow K = \frac{1}{6} \Rightarrow [ABC] = 6 \quad \blacksquare$$