

**PROBLEM 1**

Let  $n \geq 1$  and  $x_1, \dots, x_n \geq 0$ . Prove that

$$(x_1 + \frac{x_2}{2} + \dots + \frac{x_n}{n})(x_1 + 2x_2 + \dots + nx_n) \leq \frac{(n+1)^2}{4n}(x_1 + x_2 + \dots + x_n)^2.$$

**PROBLEM 2**

Let  $N$  be a positive integer. Consider a  $N \times N$  array of square unit cells. Two corner cells that lie on the same longest diagonal are colored black, and the rest of the array is white. A move consists of choosing a row or a column and changing the color of every cell in the chosen row or column.

What is the minimal number of additional cells that one has to color black such that, after a finite number of moves, a completely black board can be reached?

**PROBLEM 3**

Let  $P$  be a point inside a triangle  $ABC$  such that

$$\frac{AP + BP}{AB} = \frac{BP + CP}{BC} = \frac{CP + AP}{CA}.$$

Lines  $AP$ ,  $BP$ ,  $CP$  intersect the circumcircle of triangle  $ABC$  again in  $A'$ ,  $B'$ ,  $C'$ . Prove that the triangles  $ABC$  and  $A'B'C'$  have a common incircle.

**PROBLEM 4**

Find all pairs  $(p, q)$  of prime numbers such that

$$p(p^2 - p - 1) = q(2q + 3).$$