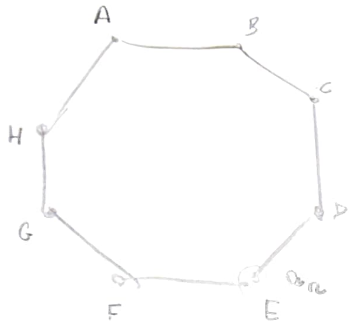
①



$a_{2n} = \#(\text{caminhos } A \to E)_n$

$b_{2n} = \#(\text{caminhos } A \to C)_n$
$= \#(\text{caminhos } A \to G)_n$

$c_{2n} = \#(\text{caminhos } A \to A)_n$

$a_0 = 0; \quad b_0 = 0; \quad c_0 = 1.$

$$c_{2n} = 2 \cdot b_{2(n-1)} + 2 \cdot c_{2(n-1)} \qquad (I)$$

$$b_{2n} = c_{2(n-1)} + 2 b_{2(n-1)} \qquad (II)$$

$$a_{2n} = 2 b_{2(n-1)} \qquad (III)$$

(II) − (I):

$$b_{2n} = c_{2n} - c_{2(n-1)}$$

$$c_{2n} = 2c_{2(n-1)} + 2b_{2(n-1)}$$

$$= 2c_{2(n-1)} + 2c_{2(n-2)} + 4b_{2(n-2)}$$

$$= 2c_{2(n-1)} + 2c_{2(n-2)} + 4c_{2(n-3)} + 8b_{2(n-3)}$$

$$= \dots$$

$$c_{2n} = 2c_{2(n-1)} + 2c_{2(n-2)} + 4c_{2(n-3)} + 8c_{2(n-4)} + 16c_{2(n-5)} + \dots$$

Mas,

$$c_{2(n-1)} = 2c_{2(n-2)} + 2c_{2(n-3)} + 4c_{2(n-4)} + 8c_{2(n-5)} + 16c_{2(n-6)} + \dots$$

$$\Rightarrow 2c_{2(n-1)} - 2c_{2(n-2)} = 2c_{2(n-2)} + 4c_{2(n-3)} + 8c_{2(n-4)} + 16c_{2(n-5)} + \dots$$

Logo:

$$c_{2n} = 2c_{2(n-1)} + \left(2c_{2(n-1)} - 2c_{2(n-2)}\right)$$

$$\Rightarrow \boxed{c_{2n} = 4c_{2(n-1)} - 2c_{2(n-2)}}$$

$\Rightarrow$ (Por indução) $c_{2n} = \frac{1}{2}\left((2-\sqrt{2})^n + (2+\sqrt{2})^n\right)$

$$b_{2n} = \frac{1}{2}\left((2-\sqrt{2})^{n} + (2+\sqrt{2})^{n} - (2-\sqrt{2})^{n-1} - (2+\sqrt{2})^{n-1} \right)$$

$$= \frac{1}{2}\left((2-\sqrt{2})^{n-1} \cdot \underbrace{\cancel{(2-\sqrt{2}-1)}}_{(1-\sqrt{2})} + (2+\sqrt{2})^{n-1} \cdot \underbrace{\cancel{(2+\sqrt{2}-1)}}_{(1+\sqrt{2})} \right)$$

$$= \frac{1}{2\sqrt{2}}\left((2-\sqrt{2})^{n-1}(\sqrt{2}-2) + (2+\sqrt{2})^{n-1}(\sqrt{2}+2) \right)$$

$$b_{2n} = \frac{1}{2\sqrt{2}}\left((2+\sqrt{2})^{n} - (2-\sqrt{2})^{n} \right)$$

$$a_{2n} = 2 \cdot b_{2(n-1)} = \frac{1}{\sqrt{2}}\left((2+\sqrt{2})^{n-1} - (2-\sqrt{2})^{n-1} \right)$$