$$2a_{n+2} = a_{n+1} + 4a_n, \quad \forall n = 0, \dots, 3028$$

$$\Rightarrow a_n \equiv 0 \pmod 2, \quad \forall n = 1, \dots, 3029.$$

Let $b_n = \frac{a_n}{2}$, for $n = 1, \dots, 3029$.

We know that:

$$2a_{n+2} = 2b_{n+1} + 4a_n, \quad \forall n = 0, \dots, 3028$$

$$a_{n+2} = b_{n+1} + 2a_n,$$

$$a_{n+1} \equiv b_n \pmod 2, \quad \forall n = 1, \dots, 3029.$$

However, $a_{n+1} \equiv 0 \pmod 2$, $\forall n = 0, \dots, 3028$.

$$\Rightarrow b_n \equiv 0 \pmod 2, \quad \forall n = 1, \dots, 3028.$$

Let $c_n := \frac{b_n}{2}$, for $n = 1, \dots, 3028$.

Summarizing:

- $a_1, a_2, \dots, a_{3029}$ are multiple of 2.
- $a_1, a_2, \dots, a_{3028}$ are multiple of 4.

Lemma: In a sequence of size $3K+1$, $x_0, \dots, x_{3K}$, such that

$$2x_{n+2} = x_{n+1} + 4x_n, \quad \forall n = 0, \dots, 3K-2 \qquad (*)$$

the sequence of size $3(K-1)+1$ $x_1, \dots, x_{3(K-1)+1}$ has only multiples of 4.

Proof: • We know that $x_{n+1} \equiv 0 \pmod 2$, $\forall n = 0, \dots, 3K-2$.

$$\Rightarrow x_n \equiv 0 \pmod 2, \quad \forall n = 1, \dots, 3K-1$$

$$\Rightarrow x_{n+2} \equiv 0 \pmod 2, \quad \forall n = 0, \dots, 3K-3$$

Thus, $2x_{n+2} \equiv x_{n+1} \pmod 4$, $\forall n = 0, \dots, 3K-3$.

$$\Rightarrow 0 \equiv x_{n+1} \pmod 4, \quad \forall n = 0, \dots, 3K-3$$

$$\Rightarrow x_n \equiv 0 \pmod 4, \quad \forall n = 1, \dots, 3(K-1)+1$$

$\square$

As (a_n), $n=0, \dots, 3030$, has size $3(1010)+1$ and suffices $(*)$ $\Rightarrow$

$\Rightarrow (a_{n+1})$, $n=0, \dots, 3(1009)$ has only multiples of 4 $\Rightarrow$

$\Rightarrow \left(\frac{a_{n+1}}{2^2}\right)$, $n=0, \dots, 3(1009)$ has size $3(1009)+1$ and suffices $(*)$ $\Rightarrow$

$\Rightarrow \left(\frac{a_{n+2}}{2^2}\right)$, $n=0, \dots, 3(1008)$ has only multiples of 4 $\Rightarrow$

...

$\Rightarrow \left(\frac{a_{n+K}}{2^{2K}}\right)$, $n=0, \dots, 3(1010-K)$ has size $3(1010-K)+1$ and suffices $(*)$ $\Rightarrow$

$\Rightarrow \left(\dfrac{a_{n+(k+1)}}{2^{2k}} \right), n = 0, \ldots, 3(1010-(k+1))$ has only multiples of 4 $\Rightarrow$

$\Rightarrow \left(\dfrac{a_{n+(k+1)}}{2^{2(k+1)}} \right), n = 0, \ldots, 3(1010-(k+1))$ has size $3(1010-(k+1))+1$ and suffices $(*)$ $\Rightarrow$

...

$\Rightarrow \left(\dfrac{a_{n+1010}}{2^{2020}} \right), n = 0$ is a sequence of integers

$\Rightarrow$ a_{1010} is a multiple of 2^{2020}

$\blacksquare$

Remark: all sequences written are integer sequences.