
Problemas com ∞

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1. **(MODS July 2020, 4)** Ache o menor inteiro n tal que existe uma grade infinita de números inteiros que contenha uma ocorrência do número k em toda subgrade quadrada k por k , para todo $k \geq n$.
2. **(Andrei Neguț, Problems for the Mathematical Olympiads, C3)** There is an ant which walks on the infinite lattice, but it can only walk upwards or to the right. There is an infinite sequence of points (which the ant knows) $(A_n)_{n>0}$ such that immediately after the ant takes its i -th step, the point A_i becomes poisoned. Prove that, given any sequence of points $(A_n)_{n>0}$, there exists an infinite path that the ant can follow such that the ant never steps on a previously poisoned point.