$A, B, C \in \mathbb{Z}_{\geq 0}$

$$\frac{A^3+B^3+C^3}{3} \geq \sqrt[3]{A^3B^3C^3} \Rightarrow \boxed{A^3+B^3+C^3-3ABC \geq 0.}$$

$$A^3+B^3+C^3-3ABC = (A+B+C)\cdot\left(A^2+B^2+C^2-AB-AC-BC\right)$$

$$f(n,n,n) = n^3+n^3+n^3-3n\cdot n\cdot n.$$
$$= 0,$$

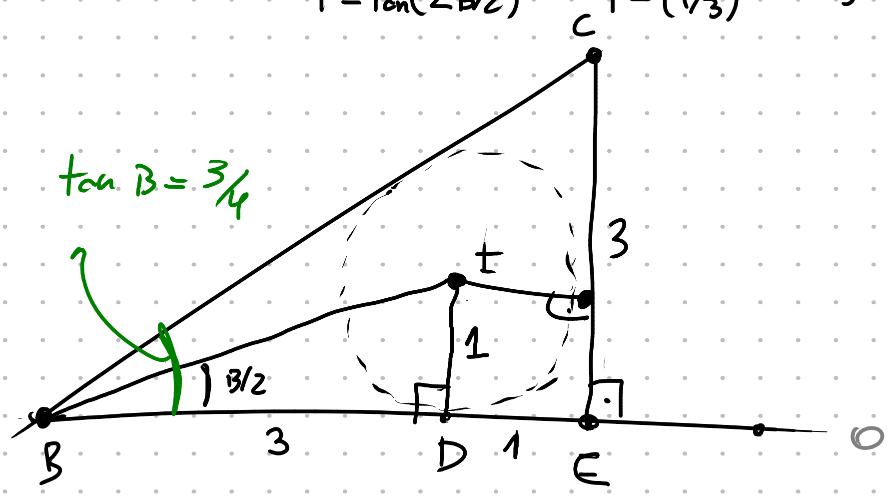
corta os cubos!

mas não **só** os cubos !!

$$f(n,n,n\pm1) = n^3+n^3+(n\pm1)^3-3n^2(n\pm1)$$
$$= \cancel{2n^3}+\cancel{n^3}\pm\cancel{3n^2}+3n\pm1-\cancel{3n^3}\mp\cancel{3n^2}$$
$$= 3n\pm1$$

$$f(A,B,C) = (A+B+C)(A^2+B^2+C^2-AB-BC-CA)$$
$$=(A+B+C)\left((A+B+C)^2-3(m)\right)$$

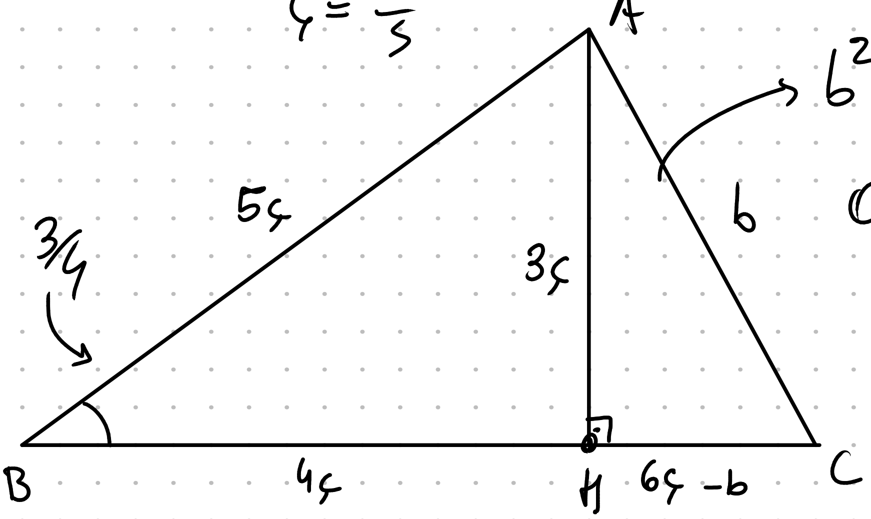
$$\tan(\angle B) = \frac{2\tan(\angle B/2)}{1-\tan^2(\angle B/2)} = \frac{2(1/3)}{1-(1/3)^2} = \frac{2}{3}\cdot\frac{9}{8} = \frac{3}{4}$$



$\underline{IG} // \underline{AB}.$

$\frac{A+B+C}{3}$

$\varsigma = \frac{c}{5}$



$b^2 = (6\varsigma - b)^2 + (3\varsigma)^2$

$0 = 45\varsigma^2 - 12\varsigma b$

$12b = 45\varsigma = 9c$

$\frac{b}{c} = \frac{3}{4}$

$$q(x)=\sum_{K=1}^{p-1} a_K \cdot x^K = x^1 - x^2 = -x(x-1).$$ $p=3 \Rightarrow n=1$

$$q(x)=\sum_{K=1}^{p-1} c_K x^K = x^1-x^2-x^3+x^4 = x(x+1)(x-1)^2$$ $p=5 \to n=2$

$$q(x)=\sum_{K=1}^{p-1} a_K x^K = x^1+x^2-x^3+x^4-x^5-x^6$$ ✓ raíz

$$1+2x-3x^2+4x^3-5x^4-6x^5$$

$q'(1) \longrightarrow 1+2-3+4-5-6 \equiv 0$ ✓ raíz dupl.

$$2-6x+12x^2-20x^3-30x^4$$

$q''(1) \longrightarrow 2-6+12-20-30 \equiv 0$ ✓ raíz tripl.

$$-6+24x-60x^2-120x^3$$ ✗ raíz 4ª.

$q^{(3)}(1) \rightsquigarrow -6+24-60-120 \equiv -1$

$p=7 \Rightarrow n=3.$

Conjectura: $n=\frac{p-1}{2}.$

$$q(x)=\sum_{K=1}^{p-1} a_K \cdot x^K.$$

$$q(1)=\boxed{\sum_{K=1}^{p-1} K^{\frac{p-1}{2}}}$$

g^{impar} ñ r.q. $\to -1$

g^{par} r.q. $\longrightarrow 1$

$$q'(x)=\sum_{K=1}^{p-1} K\cdot a_K \cdot x^{K-1}$$

$$q'(1)=\sum_{K=1}^{p-1} K\cdot c_K = \boxed{\sum_{K=1}^{p-1} K^{\frac{p+1}{2}}}$$? $\to \not\equiv 0$ $p=3$; $\to \equiv 0$ $p\geq 5$

$$q''(x)=\sum_{K=1}^{p-1} K(K-1)a_K \cdot x^{K-2}$$

$$g''(1) = \sum' k(k-1)a_k = \sum_1 k^{\frac{p+3}{2}} - \sum_1 k^{\frac{p+1}{2}}$$ → vai ser 0 por indução.

$$\equiv \boxed{\sum k^{\frac{p+3}{2}}}$$

Se $g^{(t)}(1) \equiv 0, \ \forall t < n$

$$g^{(n)} \equiv \sum_1 k^{\frac{p-1}{2}+n}$$ → $n = \frac{p-1}{2}$ é o 1° que dá errado.

$$\not\equiv 0$$

1 é raiz $\frac{p-1}{2}$-ésima.

1 não é raiz $\frac{p+1}{2}$ ésima

Logo

$$(x-1)^{\frac{p-1}{2}} \| g(x).$$

■

O que queremos estudar?

$$\sum_{k=1}^{p-1} k^t \equiv 0$$

se $p-1 \mid t$, $\displaystyle\sum_{k=1}^{p-1} k^t = \sum_{k=1}^{p-1} 1 \equiv -1$

se $p-1 \nmid t$,

$$\sum_{i=1}^{p-1} g^{it} = g^t \frac{\left(1-g^{(p-1)t}\right)}{1-g^t} = 0.$$

é 0

não é 0.

$n \geq 2m \geq 4$

$$\sum_{k=1}^{n} x_k = 0 \quad ; \quad \sum_{k=1}^{n} x_k^2 = n(n-1).$$

Ache o mínimo p/ $\sum_{k=1}^{m} x_k$.

$$\Big(\underbrace{a, a, \ldots, a}_{t}, \underbrace{b, b, \ldots, b}_{n-t} \Big)$$

(i) $at + b(n-t) = 0$ $\quad \left\{ \begin{array}{l} a = (n-t)\lambda \\ b = -t\lambda \end{array} \right.$

(ii) $t(n-t)^2\lambda^2 + (n-t)t^2\lambda^2 = \lambda^2 t(n-t)n = n(n-1)$

$$\lambda^2 = n-1 / t(n-t)$$

- Se $m \leq t$:

$$\sum_{k=1}^{m} a_k = ma = m\sqrt{n-1}\sqrt{\frac{n}{t}-1}$$

t maior possível

$t \mapsto n-1$

$$= m$$

Ex: $(1, 1, 1, \ldots, 1, 1-n)$

- se $m > t$:

$$\sum_{k=1}^{m} a_k = ta + (m-t)b$$

$$= \lambda t(n-m)$$

$$= (n-m)\sqrt{n-1} \cdot \sqrt{\frac{n}{n-t} - 1}$$

t mínimo:

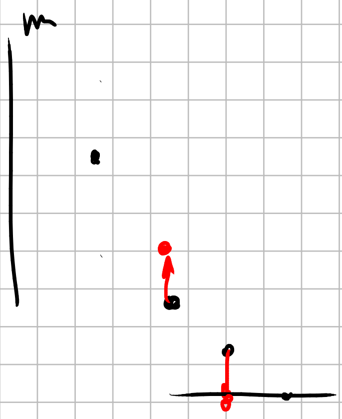
$t \mapsto 1$.

$$= n-m.$$

Ex: $(n-1, -1, -1, -1, \ldots, -1)$

Conjectura: $\sum_{k=1}^{m} x_k \geq m$

$$\sum_{k=1}^{n} x_k^2 = n(n-1)$$



$$\sum_{i=1}^{n} x_k = 0$$

$$\sum_{k=1}^{n} x_k^2 = C \sum_{k=1}^{n} x_k^2$$

$C > 1.$

Multiplicar todo mundo por $\frac{1}{\sqrt{C}} < 1$

$$\sum_{i=1}^{n} x_k = 0$$

$$\sum_{i=1}^{n} x_k^2 = \sum_{i=1}^{n} x_k^2$$

$$\sum_{i=1}^{m} x_k = \frac{1}{\sqrt{C}} \cdot \sum_{k=1}^{m} x_k$$

m=1 $\quad (2, 1, -1, -2) \rightsquigarrow (2, 1, 1, -4) \xrightarrow{\times\sqrt{\frac{10}{22}}} (\sim\sim\sim)$

$\sum^n x_k = 0$	$\sum^n x_k = 0$	$\sum^n x_k = 0$
$\sum^n x_k^2 = 10$	$\sum^n x_k^2 = 22$	$\sum^n x_k^2 = 10$
$\sum^m x_k = 2$	$\sum^m x_k = 2$	$\sum^m x_k = \sqrt{\frac{10}{22}} \cdot 2$

$$\left(x_1, x_2, \dots, x_m, x_{m+1}, \dots, x_n\right)$$

$x_m \quad x_m$

Ajeitar a soma = 0.

$$\left(a, \overset{a}{b}, \overset{a}{b}, \overset{a}{b}, \overset{a}{b}, \dots, \overset{a}{b}, c\right)$$

$a - b = d$

$c - (n-2)(a-b)$

$$\sum_1^n x_k^2 = a^2 + (n-2)b^2 + c^2$$

$$\sum^n x_k^2 = (n-1)a^2 + (c-(n-2)(a-b))^2$$
$$= a^2 + c^2 + (n-2)a^2 + (n-2)^2 d^2$$
$$-2(n-2)dc$$

$$\sum_1^m x_k = a + (m-1)b$$

$$\sum^m x_k = a + (m-1)b + (m-1)d$$

será se:

$$\frac{a+(m-1)b}{a+(m-1)b+(m-1)d} \geqslant \sqrt{\frac{a^2+c^2+(n-2)b^2}{a^2+c^2+(n-2)\left[a^2+(n-2)d^2-2dc\right]}}$$

$$1 + \frac{(m-1)d}{a+(m-1)b} \leq \sqrt{1 + \frac{(n-2)\left[a^2+(n-2)d^2-2dc-b^2\right]}{a^2+c^2+}}$$

$$a + (n-2)b + c = 0$$
$$a^2 + (n-2)b^2 + c^2 = n(n-1)$$

$$n c^2 \leq n(n-1)(n-1)$$
$$c^2 \leq (n-1)^2$$
$$\boxed{c \geqslant 1-n}$$

$$a^2 + (n-2)b^2 \geqslant \frac{(\overbrace{a+(n-2)b}^{-c})^2}{n-1}$$
$$\geqslant n-1$$
$$\boxed{a \geqslant 1}$$

$$a + (n-2)b \leq n-1$$

$$0 = a + (n-2)b + c \geqslant 1 + (n-2)b + 1-n = (n-2)(b-1)$$

$$\boxed{b \leq 1}$$