$\mathcal{S}$ = anti-cadeia

cadeia = $\mathcal{C} = \{T_0, T_1, T_2, \dots, T_\ell\}$

$$T_0 \underset{\neq}{\subset} T_1 \underset{\neq}{\subset} T_2 \cdots \underset{\neq}{\subset} T_\ell .$$

$n \quad n-1 \quad \dots \quad 1$

$$0 \le \underset{\substack{\shortparallel \\ 0}}{|T_0|} < \underset{\substack{\shortparallel \\ 1}}{|T_1|} < \underset{\substack{\shortparallel \\ 2}}{|T_2|} < \cdots < \underset{\substack{\shortparallel \\ n}}{|T_\ell|} \le n$$

Quantas cadeias maximais existem? $n!$

<u>Teo</u>: $|\mathcal{C} \cap \mathcal{S}| = 0$ ou 1.

$\mathcal{S}$ é fixo.

Contagem dupla: $(A, \mathcal{C})$, $A \in \mathcal{S}$, $A \in \mathcal{C}$, $\mathcal{C}$ cadeia maximal

Para cada $\mathcal{C}$, existem no máximo 1 par $(A, \mathcal{C})$

Para cada $A \in \mathcal{S}$, existem $|A|!\,(n-|A|)!$

$k-2 \quad k-1 \quad k \quad n-k \quad n-k-1 \quad n-k-2$

$$\dots \; n \quad T_{k-1} \quad \underset{\substack{\shortparallel \\ T_k}}{A} \; \dots$$

$$\sum_{A \in \mathcal{S}} \boxed{|A|!\,(n-|A|)!} = \#(A, \mathcal{C}) \le n!$$

$$\left(\begin{matrix} n \\ k \end{matrix}\right) \le \left(\begin{matrix} n \\ \lfloor n/2 \rfloor \end{matrix}\right)$$

$$\underset{\substack{\shortparallel \\ |S| \cdot 1/\binom{n}{\lfloor n/2 \rfloor}}}{\sum_{A \in S} \frac{1}{\binom{n}{\lfloor n/2 \rfloor}}} \le \sum_{A \in S} \frac{1}{\binom{n}{|A|}} \le 1 .$$

$$\Rightarrow \quad |S| \le \binom{n}{\lfloor n/2 \rfloor} .$$

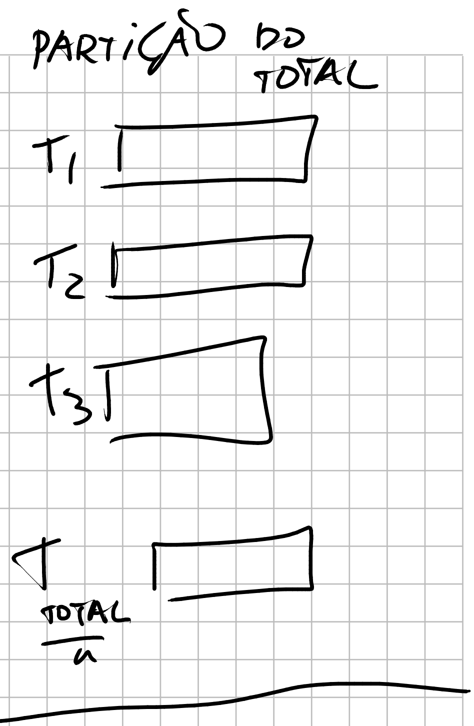
$$\binom{n}{0} \le \binom{n}{1} \le \binom{n}{2} \cdots \le \binom{n}{\lfloor n/2 \rfloor} = \binom{n}{\lceil n/2 \rceil} \ge \cdots \ge \binom{n}{n} .$$

$$|S| \le \frac{k}{n} \cdot (\text{TOTAL})$$

Quero achar T t.q.

$$T \subseteq \binom{[n]}{k} \qquad |T| = n$$

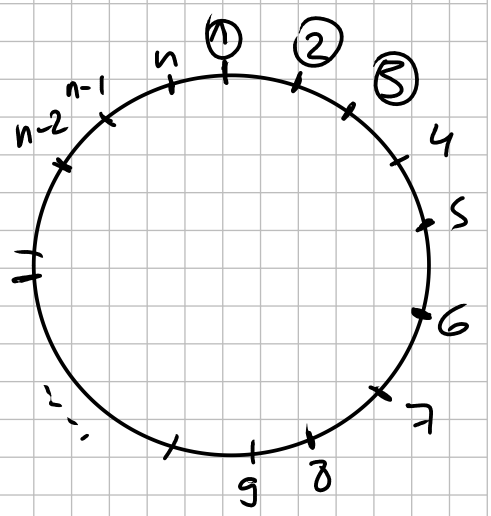
$$\to \quad |T \cap S| \le k$$



$[n] = \{1, 2, 3, 4, 5, 6, \dots, n-2, n-1, n\}$

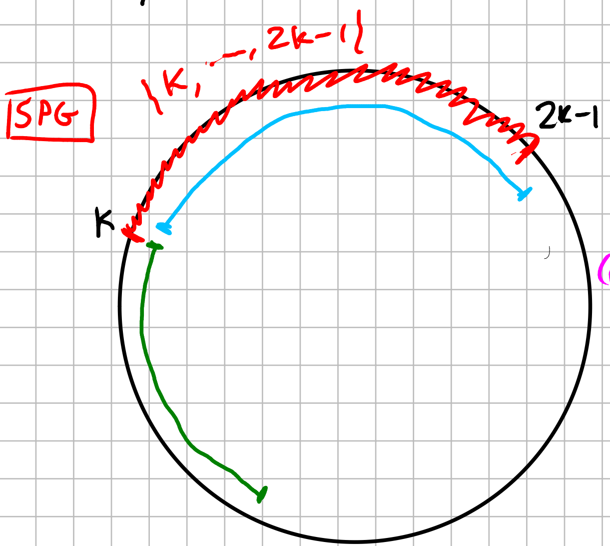
$$T = \Big\{ \boxed{\{1, 2, \dots, k-1, k\},} = I_1$$
$$\boxed{\{2, 3, \dots, k, k+1\}} = I_2$$
$$\vdots$$
$$\{n-k+1, \dots, n-1, n\},$$
$$\{n-k+2, \dots, n, 1\}$$
$$\vdots$$
$$\boxed{\{n, 1, 2, \dots, k-1\} \Big\}} = I_n.$$

S é int.
$\Downarrow$
$S \cap T$ é int.



$I_i = \{i, i+1, \dots, i+(k-1)\}$ (mod n).

i é pivot de I_i.



1° $\{1, 2, \dots, k\} \leftrightarrow \{k+1, \dots, 2k\}$

2° $\{2, \dots, k+1\} \leftrightarrow \{k+2, \dots, 2k+1\}$

$\vdots$

$(k-1)°$ $\{k-1, \dots, 2k-2\} \leftrightarrow \{2k-1, \dots, 3k-2\}$

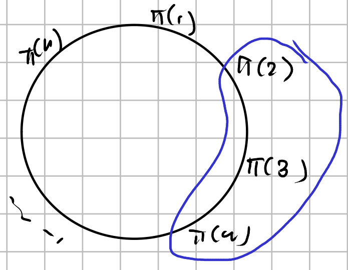
(π, A), π é permutação cíclica

TOTAL $= (n-1)!$

$A \in S$

$A = \{\pi(t), \pi(t+1), \dots, \pi(t+k-1)\}$ para algum t.

considerando mod n



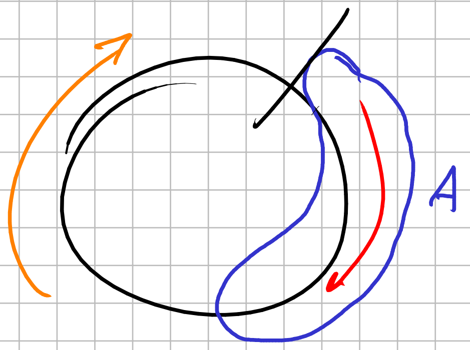
Contando por perm:

Pelo Lema, tem no máximo $\boxed{k}$. $(n-1)!$

Contando por A:

Tem $= |S| \cdot \boxed{(n-k)!\,k!}$

$A = \{a_1, a_2, a_3, \dots, a_k\}$



$\boxed{(n-k)!\;k!}$

$$|S| \leq \frac{k \cdot (n-1)!}{(n-k)! \cdot k!} = \binom{n-1}{k-1}.$$

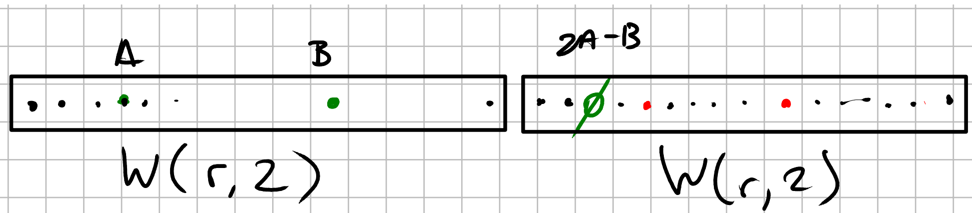
Problema 4 $c: \mathbb{N} \to [r]$

Sejam k e r inteiros positivos. Todo inteiro positivo é pintado com uma de r cores. Prove que existe uma progressão aritmética monocromática com k termos.

Definition 6.6

Let $W(r, k)$ be the minimal n such that (for all c: $[n] \to [r]$, there exists a monochromatic arithmetic progression of size k.)

$\boxed{K=3}$

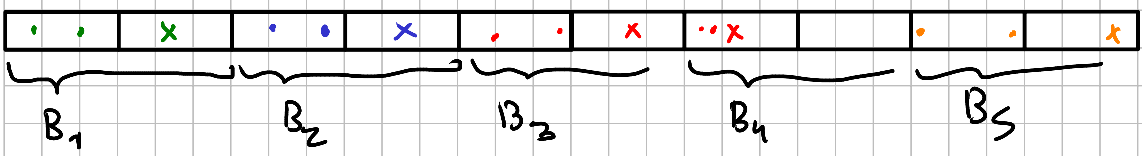


DEF: $PA_k(a, d) = \{a, a+d, \dots, a+(k-1)d\}$.

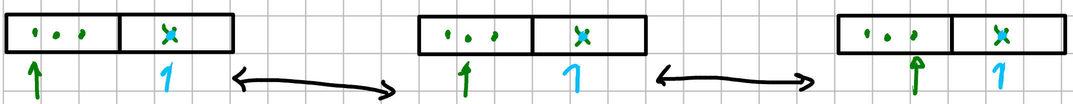
$PA_k(a,d)$ tem foco $a+kd$.

DEF: $PA_k(a_1, d_1), \dots, PA_k(a_s, d_s)$ são ω-focados se:

- têm cores distintos
- têm o mesmo foco.



$N = 2\,W(r,2)$. Pege r^N+1 blocos. $n(2,r) = 2 \cdot W(r,2) \cdot (r^N+1)$



$W(r,k) := \min\{n : \forall c:[n]\to[r], \text{ existe } k\text{-PA MC}\}$

Prove que $W(r,k) < \infty$.
Indução em K. $\longrightarrow$

DEF: $PA_k(a,d) = \{a, a+d, \dots, a+(k-1)d\}$.
$PA_k(a,d)$ tem foco $a+kd$.

DEF: $PA_k(a_1,d_1), \dots, PA_k(a_s,d_s)$ são co-focados se:

- têm cores distintas
- têm o mesmo foco.

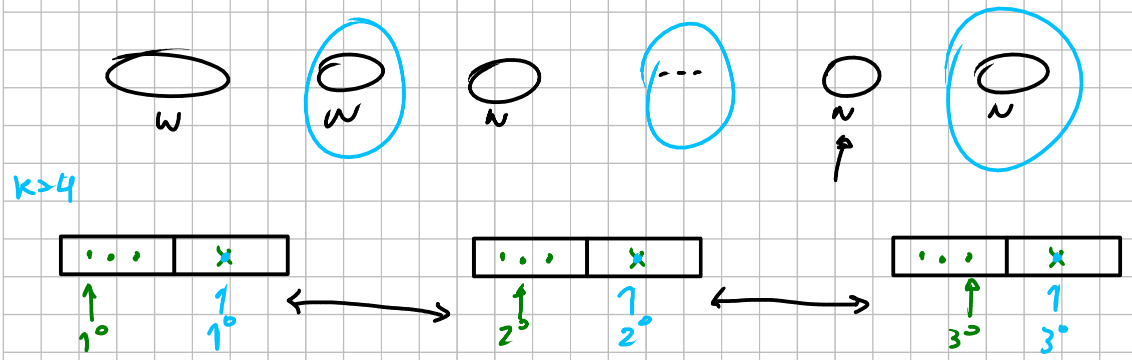
Lema: existe $n = \boxed{n(s,r)}$ tal que, $\forall c:[n]\to[r]$, existe K-PA MC ou existem s $(k-1)$-PA's co-focados.

Prova: indução em s.

$\underline{s=1}$: $n(1,r) = \boxed{W(r,k-1)} \Rightarrow \exists$ $(k-1)$-PA MC.

$\underline{s=1} \Rightarrow \underline{s=2}$: $N = 2\cdot n(1,r)$.

Vamos pegar $W(r^N, k-1)$ blocos de tamanho N.



$\underline{s-1 \to s}$ $N = 2\cdot n(s-1,r)$. $\leftarrow$

Vamos pegar $W(r^N, k-1)$ blocos de tamanho N.