$1, 2, 4 \mid 4K.$

Temos $4-2=2$.

Mas se $3 \mid K$? !!

Se $3 \nmid K$, bom!

↳ $6 \mid 2K$, $12 \mid 4K$

Ebo! $6-4=2$.

Mas se $5 \mid K$? !!

Se $5 \nmid K$, bom!

↳ $10 \mid 2K$, $20 \mid 4K$

Ebo! $12-10=2$.

Mas se $11 \mid K$? !!

Se $11 \nmid K$ bom!

↳ $22 \mid 2K$, $44 \mid 4K$

Ebo! $22-20=2$

Mas se $21 \mid K$? !!

Se $21 \nmid K$ bom!

↳ $42 \mid 2K$, $84 \mid 4K$

Ebo! $44-42=2$

$a_0 = 1 \to 2 \; +1$

$a_1 = 3 \to 6 \; -1$

$a_2 = 5 \to 10 \; +1$

$a_3 = 11 \to 22 \; -1$

$a_4 = 21 \to 42 \; +1$

$a_5 = 43$

$$a_{n+1} = 2a_n + (-1)^n$$

Seja m o menor índice t.q.

$$a_m \nmid K.$$

Queremos $a_m - 1, a_m + 1 \mid 4K$

$$c_n = 2c_{n-1} + (-1)^{n-1} \iff c_n + (-1)^n = 2c_{n-1}$$

$$c_{m-1} \mid K \Rightarrow 2c_{m-1} \mid 2K \Rightarrow a_m + (-1)^m \mid 2K \mid 4K$$

$$4c_{m-2} \mid 4K \Rightarrow 2\left(a_{m-1} + (-1)^{m-1}\right) \mid 4K$$

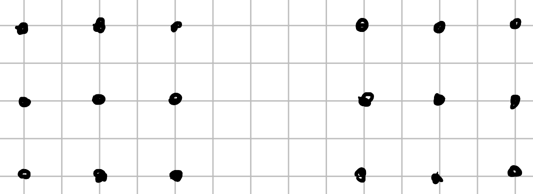
$$\Rightarrow \underbrace{c_m + (-1)^m + 2(-1)^{m-1}}_{a_m - (-1)^m} \mid 4K$$

Ache todos $v \in P$ t.q. é possível particionar $P/\{v\}$ em conjuntos A e B t.q.:

- $|A| = |B|$
- $\sum_{a \in A} a = \sum_{b \in B} b.$

Obs: $\begin{pmatrix} x \\ y \end{pmatrix} + \begin{pmatrix} z \\ w \end{pmatrix} = \begin{pmatrix} x+z \\ y+w \end{pmatrix}$

$P = \left\{ \begin{pmatrix} a \\ b \end{pmatrix} : a \in \{0,1,2\},\ b = \{0,1,2\} \right\}$

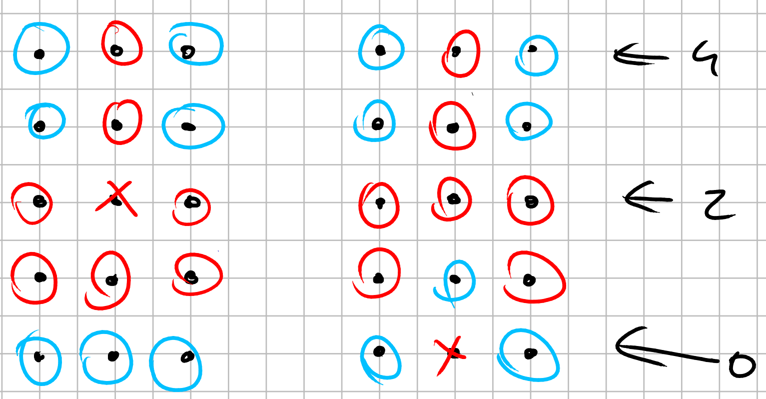


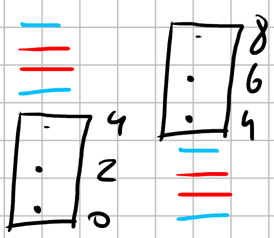
$\sum_{p \in P} p = \begin{pmatrix} 1 \\ 50 \end{pmatrix} \cdot 3 \cdot 101 = \begin{pmatrix} \text{ímpar} \\ \text{par} \end{pmatrix}$

$\sum_{p \in P/\{v\}} = \begin{pmatrix} \text{ímpar} \\ \text{par} \end{pmatrix} - \begin{pmatrix} x_v \\ y_v \end{pmatrix} = \begin{pmatrix} \text{par} \\ \text{par} \end{pmatrix}$

$\hookrightarrow \begin{pmatrix} 1 \\ \text{par} \end{pmatrix}$

$P = \left\{ \begin{pmatrix} a \\ b \end{pmatrix} : a \in \{0, \dots, 2\},\ b \in \{0, \dots, 4\} \right\}$





Achar propriedade útil dos pontos $(x,y) \in \mathbb{Z}^2$ t.q.

$$x^2+y^2=2^k.$$

$v_2(0) := \infty.$

se $v_2(x^2) \neq v_2(y^2)$

$$K = v_2(x^2+y^2) = \min(v_2(x^2), v_2(y^2))$$

spg $K = v_2(x^2)$ $\quad x^2 \geq 2^k \Rightarrow y=0$

$\hookrightarrow x=0$ ou $y=0$

$2t = \underline{v_2(x^2) = v_2(y^2)}$ ou $\begin{pmatrix} x=0 \\ y^2=2^k \end{pmatrix}$ ou $\begin{pmatrix} y=0 \\ x^2=2^k \end{pmatrix}$

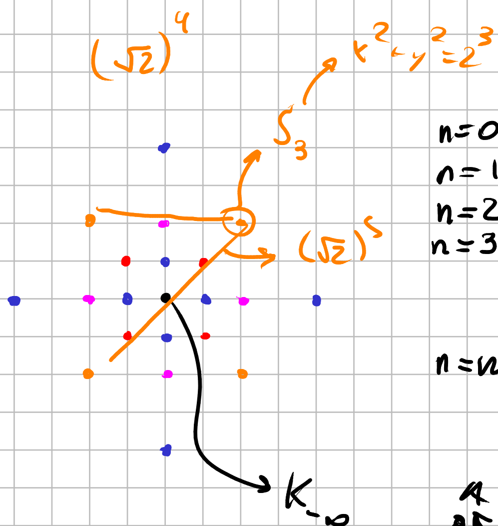
$\hookrightarrow 2^{2t}(x_0^2+y_0^2) = 2^k$

$$x_0^2+y_0^2 = 2^{k-2t} = 2 \Rightarrow x_0^2, y_0^2 = 1$$

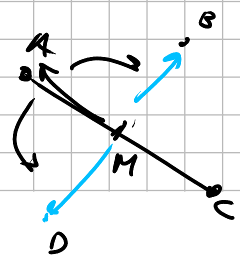
$$2 \equiv 1 + 1 \underset{(4)}{\equiv} 2^{k-2t} \Rightarrow \underline{k = 2t+1} \qquad x^2, y^2 = 2^{k-1}$$

As soluções de $x^2+y^2=2^k$ são:

$$\begin{cases} (0, \pm 2^{k/2}), (\pm 2^{k/2}, 0) & \text{se } k \text{ par.} \\ (\pm 2^{(k-1)/2}, \pm 2^{(k-1)/2}) & \text{se } k \text{ ímpar.} \end{cases}$$



$n=0$	$1+0$
$n=1$	$2+4$
$n=2$	$3+8$
$n=3$	$4+12$
$n=n$	$5n+1$



ABCD formando □. $A \in K_a$, $B \in K_b$, $C \in K_c$, $D \in K_d$

$A=(a_x,a_y)$; $C=(c_x,c_y)$

$$M=\left(\frac{a_x+c_x}{2}, \frac{a_y+c_y}{2}\right) \qquad \vec{MA}=\left(\frac{a_x-c_x}{2}, \frac{a_y-c_y}{2}\right)$$

$$\xrightarrow{90^\circ} \pm\left(\frac{a_y-c_y}{2}, \frac{c_x-a_x}{2}\right)$$

$$B,D=\left(\frac{a_x+c_x}{2}, \frac{a_y+c_y}{2}\right) \pm \left(\frac{a_y-c_y}{2}, \frac{c_x-a_x}{2}\right)$$

$$=\left(\frac{a_x \pm a_y}{2}+\frac{c_x \mp c_y}{2}, \frac{a_y \mp a_x}{2}+\frac{c_y \pm c_x}{2}\right) ?$$

$$\left(\frac{a_x \pm a_y}{2}+\frac{c_x \mp c_y}{2}\right)^2+\left(\frac{a_y \mp a_x}{2}+\frac{c_y \pm c_x}{2}\right)^2=$$

$$=\frac{1}{4}\left(\underbrace{2a_x^2+2a_y^2}_{2\cdot(2^a)}+\underbrace{2c_x^2+2c_y^2}_{2\cdot(2^c)}+\cancel{4(\mp a_x a_y \pm c_y c_x)}\right) \to 2^b,\ 2^d$$

$$2^a+2^c=2^b+2^d$$

$\Rightarrow$ S.P.G $\begin{pmatrix} a=b \\ c=d \end{pmatrix}$ ou $\begin{pmatrix} a=c, & b=a+1 \\ d=-\infty & \end{pmatrix}$

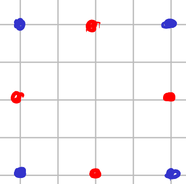
$$d(A,B)=(\sqrt{2})^{a+1} \text{ ou } (\sqrt{2})^{c+2}$$

$$d(C,D)=(\sqrt{2})^{c+1} \text{ ou } (\sqrt{2})^{c+2}$$

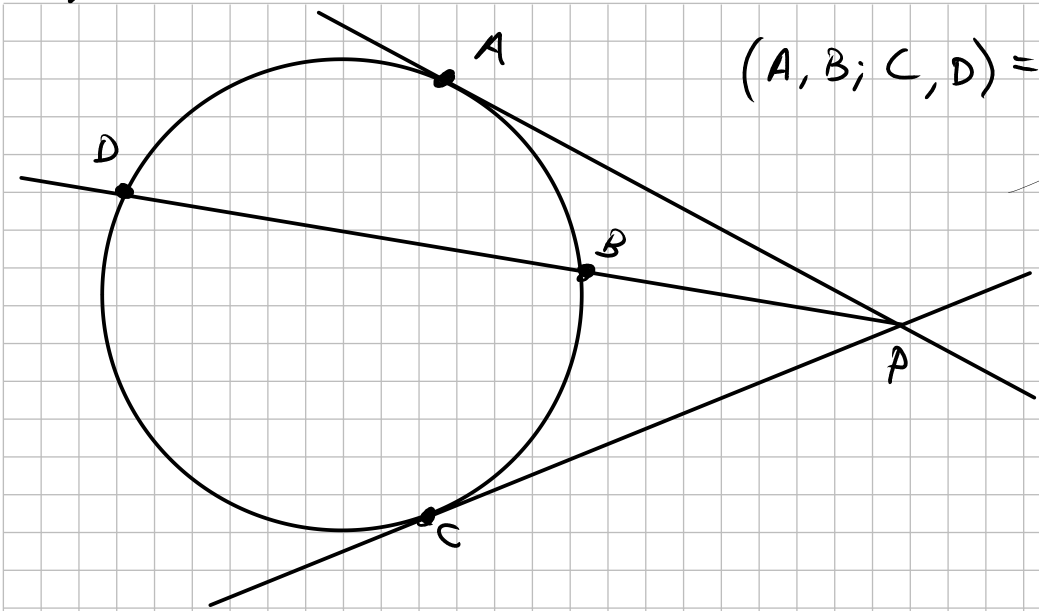
$a=c \to$ ✓

ou

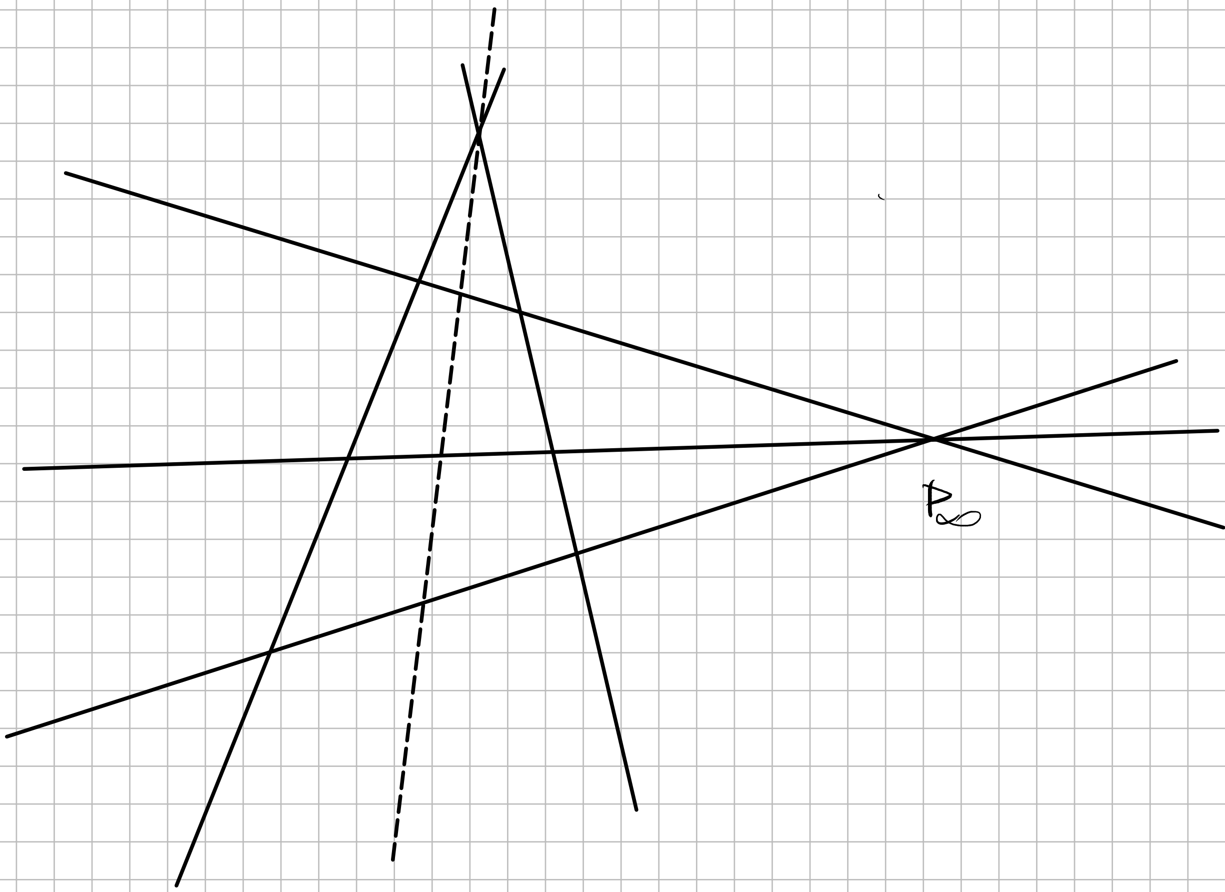
s.p.g $a=c+1 \to$ testar na mão e ver q. ñ dá'

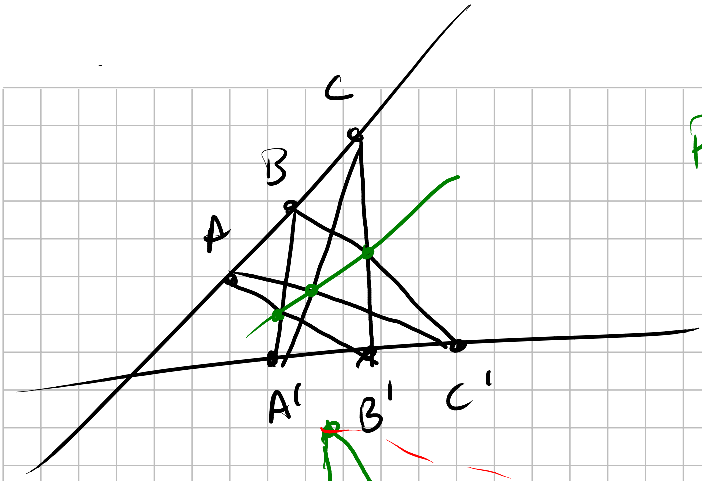


Lemma:



$(A, B; C, D) = -1$





Pappus