$$x^3 - 3x + 1 = 0$$
$$1 = 3x - x^3$$

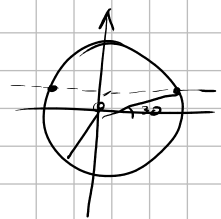
trigonometria: $\operatorname{sen}(3\theta) = 3\operatorname{sen}\theta - 4\operatorname{sen}^3\theta \leftarrow \operatorname{sen}(\alpha+\beta) = \operatorname{sen}(\alpha)\cos\beta + \cos\alpha\operatorname{sen}\beta$

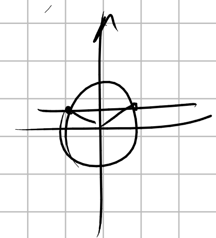
$x \leftarrow 2\operatorname{sen}\theta$.

$$= \frac{3x - x^3}{2} = 1$$

$$\operatorname{sen}(3\theta) = 1/2$$

$$3\theta = \begin{cases} 30^\circ + k \cdot 360^\circ \\ 150^\circ + k \cdot 360^\circ \end{cases} \qquad 390^\circ$$

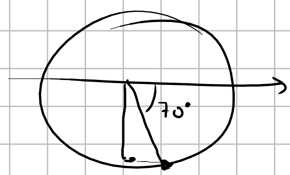




$\theta = -70^\circ$ $\theta = 10^\circ$; $\theta = 50^\circ$; $\theta = 130^\circ$; $\theta = 170^\circ$; $\theta = 250^\circ$; $\theta = 290^\circ$

$2\operatorname{sen}(10^\circ)$; $2\operatorname{sen}50^\circ$; $2\operatorname{sen}(-70^\circ)$

$2\cos(80^\circ)$; $2\cos(40^\circ)$; $2\cos(160^\circ)$



$x^3 - 3x + 1$:

$$\cos(40^\circ) + \cos(80^\circ) + \cos(160^\circ) = 0$$
$$\cos(40^\circ)\cdot\cos(80^\circ)\cdot\cos(160^\circ) = -1/8$$

$$\phi = \frac{1+\sqrt{5}}{2}.$$

Ache $\cos(40^\circ)\cdot\cos(80^\circ)\cos(160^\circ) = -1/8$.

$$\phi^2 - \phi - 1 = 0$$

$p = 2$ $a^3 - 3a + 1 \equiv 0 \pmod 2$

$a = 0 \to 1$
$a = 1 \to 2$

Falso

$p = 7$
$a=0$ 1
$a=1$ −1
$a=2$ 3
$a=3$ 5
$a=4$ 4
$a=5$ −1
$a=6$ 3

$p = 3$
$a = 0 \to 1$
$a = 1 \to 2$
$a = 2 \to 0$

3 funciona!

$p = 19$: $a = 3 \to 0$.
$a = 7 \to 0$
$a = 9 \to 0$

$p = 5$
$a = 0 \to 1$ $a = 3 \to 4$
$a = 1 \to -1$ $a = 4 \to 3$
$a = 2 \to 3$

$3 \to 3^2 - 2 = 7, \to 7^2 - 2 = 47 \equiv 9 \pmod{19}$

$9^2 - 2 = 79 \equiv 3 \pmod{19}$

Se $x = \cancel{2\cos\theta}$ é raiz de x^3-3x+1

$\Downarrow$

$\cancel{2\cos 2\theta}$ é raiz de x^3-3x+1

$\cancel{2(2\cos^2\theta - 1) = (2\cos\theta)^2 - 2}$

$= x^2 - 2$

} raízes em $\mathbb{R}$.

Conj.

Se α é raiz de x^3-3x+1
então α^2-2 é raiz de x^3-3x+1

} raízes mod p

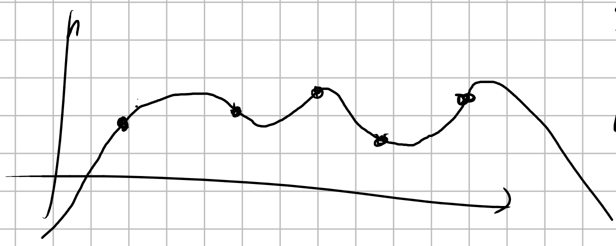
$P(\alpha) = 0 \Rightarrow \alpha^3 - 3\alpha + 1 = 0 \Rightarrow$

$$\begin{aligned} P(\alpha^2-2) &= (\alpha^2-2)^3 - 3(\alpha^2-2) + 1 \\ &= \alpha^6 - 6\alpha^4 + 12\alpha^2 - 8 - 3\alpha^2 + 6 + 1 \\ &= \alpha^6 - 6\alpha^4 + 9\alpha^2 - 1 \\ &= (3\alpha-1)^2 - 6\alpha\cdot(3\alpha-1) + 9\alpha^2 - 1 \\ &= 9\alpha^2 - 6\alpha + 1 - 18\alpha^2 + 6\alpha + 9\alpha^2 - 1 = 0. \end{aligned}$$

a é <u>única raiz</u> $\Rightarrow a \equiv a^2 - 2 \Rightarrow 0 \equiv (a+1)(a-2)$

- $P(2) = 3 \equiv 0 \pmod p \Rightarrow p = 3$
- $P(-1) = 3 \equiv 0 \pmod p \Rightarrow p = 3$

$(x, f(x))$



Polinômio
Interpolador
de
Lagrange

$\left(\frac{1+\sqrt{5}}{2}\right)^k \quad \left(\frac{1-\sqrt{5}}{2}\right)^k$

$F_k = \frac{\phi^k - \bar{\phi}^k}{\sqrt{5}}$

$F_n = \frac{\phi^n - \bar{\phi}^n}{\sqrt{5}}$?

$=$

$2^k \to (n+1)^k$

$P(k) = \#(\text{subconjuntos de } \{1, \dots, k\})$, $k \in \{0, \dots, n\}$

$= \binom{k}{0} + \binom{k}{1} + \binom{k}{2} + \dots + \binom{k}{k}$

$P(n+1) = \binom{n+1}{0} + \binom{n+1}{1} + \dots + \binom{n+1}{n} + \binom{n+1}{n+1} - 1$

$= 2^{n+1} - 1$

$P(x) = \binom{x}{0} + \binom{x}{1} + \dots + \binom{x}{n}$

$= \binom{k}{0} + \dots + \binom{k}{k} + \underbrace{\dots}_{0}$

$= 2^k$

$F_n = \frac{\phi^n - \bar{\phi}^n}{\sqrt{5}}$ $\quad \{ \phi^2 - \phi - 1 = 0. \}$

Palpite: • $P(2n+3) \neq F_{2n+3}$.
• $P(2n+3) = F_{2n+3}$

se P tem grau $\leq n$ $\Rightarrow$ ΔP tem grau $\leq n-1$

Seja
$\Delta P(x) = P(x+1) - P(x)$.

$n \to$ $P(n+2) = F_{n+2}$ $\quad P(n+3) = F_{n+3}$ $\quad P(n+4) = F_{n+4}$ $\quad \dots \quad P(2n+1) = F_{2n+1}$ $\quad P(2n+2) = F_{2n+2}$

$n-1 \to$ $\Delta P(n+2) = F_{n+1}$ $\quad \Delta P(n+3) = F_{n+2}$ $\quad \dots \quad \Delta P(2n+1) = F_{2n}$

$n-2$ $\quad \Delta^2 P(n+2) = F_n$ $\quad \Delta^2 P(n+3) = F_{n+1}$ $\quad \dots \quad \Delta^2 P(2n) = F_{2n-2}$

$\vdots$

$n-\ell \to$ $\Delta^\ell P(n+2) = F_{n+2-\ell}$ $\quad \dots \quad \Delta^\ell P(2n+2-\ell) = F_{(2n+2-\ell)-\ell}$

$\Delta^{n-1} P(n+2) = F_3$ $\quad \Delta^{n-1} P(n+3) = F_4$

$0 \to$ $\Delta^n P(n+2) = F_2 = 1.$ $\quad \Delta^n P(n+3) = F_2 = 1.$

$P(n+3) = F_{2n+2} + F_{2n} + F_{2n-2} + \dots + F_4 + (F_2 + F_1) - F_1$

$= F_{2n+3} - 1$

$\cos(3\theta) = 4\cos^3\theta - 3\cos\theta$

se $x = 2\cos\theta$

$2\cos(3\theta) = (2\cos\theta)^3 - 3(2\cos\theta) = -1$

$x^3 - 3x + 1 = 0$

$\cos(3\theta) = -1/2$

$2\cos\theta$

$\longrightarrow (2\cos\theta)^3 - 3(2\cos\theta) + 1 = 0$

$\rightarrow \theta = 40^\circ$

$\rightarrow \theta = 80^\circ$