Martingales

MIT Lecture

§1 Motivation Problem

Problem 1 23 candidates are running for a political office. There is an efficient betting market and p_i is the "market percent probability" that the ith candidate wins. Assume each p_i is an integer greater than 1 and that $\sum p_i = 100$.

The p_i evolve in time. Write $p_i(t)$ for the value at time t.

Assume that if p_i is a number $k \in \{1, 2, \ldots, 99\}$ at some given time, then the next integer value that p_i attains is $k+1$ with probability $1/2$ and $k-1$ with probability $1/2$.

The ith candidate makes an epic comeback if p_i gets all the way down 1 before getting to 100.

What is the probability that somebody will make an epic comeback?

§2 Martingale definition

Let S be a probability space.

Let $X_0, X_1, \ldots$ be a sequence of random quantities (a.k.a. random variables).

Definition 1 (The Martingale Property)

$$\mathbb{E}[X_{n+1} | \mathcal{F}_n] = X_n$$