P6, Álgebra Eleva 2020.

- $P(x)$ tem grau 990
- $P(K) = F_K$ para $K = 992, \dots, 1982$.

Prove que $P(1983) = F_{1983} - 1$.

$\Delta P(K) = F_{K+1} - F_K$, para $K = 992, \dots, 1981$

$= F_{K-1}$

$\Delta^2 P(K) = F_K - F_{K-1}$, para $K = 992, \dots, 1980$

$= F_{K-2}$

$\vdots$

$\Delta^i P(K) = F_{K+2-i} - F_{K+1-i}$, para $K = 992, \dots, 1982-i$

$= F_{K-i}$

$\vdots$

$\Delta^{990} P(K) = F_{K-990}$, para $K = 992, \dots, 992$

Mas $\Delta^{990} P(K)$ tem grau $\leq 0 \Rightarrow \Delta^{990} P(K) = F_2 = 1$

$1 = \Delta^{990} P(993) = \Delta^{989} P(994) - \Delta^{989} P(993)$

$= \Delta^{988} P(995) - \Delta^{988} P(994) - \Delta^{989} P(993)$

$= \dots$

$= \Delta^0 P(1983) - \Delta^0 P(1982) - \Delta^1 (1981) - \dots - \Delta^{989} (993)$

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$$P(1983) - 1 = F_{1982} + F_{1980} + F_{1978} + \cdots + F_4$$

$$= F_{1982} + \cdots + F_6 + F_4 + F_3 - F_3$$

$$= F_{1982} + \cdots + F_8 + F_6 + F_5 - F_3$$

$$\vdots$$

$$= F_{1983} - F_3$$

$$= F_{1983} - 2$$

Logo: $P(1983) = F_{1983} - 1$