

$$\begin{aligned}
 P(1983) &= \sum_{k=0}^{990} \left(\prod_{\substack{j=0 \\ j \neq k}}^{990} \frac{991-j}{k-j} \right) F_{k+992} \\
 &= \sum_{k=0}^{990} \frac{991!}{(991-k) \cdot k! (990-k)! (-1)^k} \left(\frac{1}{\sqrt{5}} \left(\varphi^{k+992} - \bar{\varphi}^{k+992} \right) \right) \\
 &= \frac{1}{\sqrt{5}} \sum_{k=0}^{990} \binom{991}{k} (-1)^k \varphi^{k+992} - \dots \\
 &= \frac{\varphi^{992}}{\sqrt{5}} \sum_{k=0}^{990} \binom{991}{k} (-\varphi)^k - \dots
 \end{aligned}$$

$\varphi + \bar{\varphi} = 1, \varphi \bar{\varphi} = -1$
 $1 - \varphi = 1 - \left(\frac{1+\sqrt{5}}{2} \right)$
 $= \frac{1}{2} - \frac{\sqrt{5}}{2} = \bar{\varphi}$

$$\begin{aligned}
 &= \frac{\varphi^{992}}{\sqrt{5}} \left[(1-\varphi)^{991} - (-\varphi)^{991} \right] - \dots \\
 &= \frac{\varphi^{992}}{\sqrt{5}} \left[\bar{\varphi}^{991} + \varphi^{991} \right] - \dots \\
 &= \frac{1}{\sqrt{5}} \left[(\varphi \bar{\varphi})^{991} \varphi + \varphi^{1983} \right] - \dots \\
 &= \frac{1}{\sqrt{5}} \left[-\varphi + \varphi^{1983} + \bar{\varphi} - \bar{\varphi}^{1983} \right] \\
 &= \frac{1}{\sqrt{5}} \left[-\sqrt{5} + \varphi^{1983} - \bar{\varphi}^{1983} \right] = F_{1983} - 1
 \end{aligned}$$