

**Problem 1** Find all integers  $m$  and  $n$  such that the fifth power of  $m$  minus the fifth power of  $n$  is equal to  $16mn$ .

**Problem 2** Find max number  $n$  of numbers of three digits such that :

1. Each has digit sum 9
2. No one contains digit 0
3. Each 2 have different unit digits
4. Each 2 have different decimal digits
5. Each 2 have different hundreds digits

**Problem 3** Let  $k > 1$  be a positive integer and  $n > 2018$  an odd positive integer. The non-zero rational numbers  $x_1, x_2, \dots, x_n$  are not all equal and:

$$x_1 + \frac{k}{x_2} = x_2 + \frac{k}{x_3} = x_3 + \frac{k}{x_4} = \dots = x_{n-1} + \frac{k}{x_n} = x_n + \frac{k}{x_1}$$

Find the minimum value of  $k$ , such that the above relations hold.

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**Problem 4** Let  $\triangle ABC$  and  $A', B', C'$  the symmetric of vertex over opposite sides. The intersection of the circumcircles of  $\triangle ABB'$  and  $\triangle ACC'$  is  $A_1$ .  $B_1$  and  $C_1$  are defined similarly. Prove that lines  $AA_1, BB_1$  and  $CC_1$  are concurrent.

**Problem 5** Problema math/jbmo/2018/5 não encontrado!

**Problem 6** Problema math/jbmo/2018/6 não encontrado!

**Problema 1** Determine all the sets of six consecutive positive integers such that the product of some two of them . added to the product of some other two of them is equal to the product of the remaining two numbers.

**Problema 2** Let  $x, y, z$  be positive integers such that  $x \neq y \neq z \neq x$  .Prove that

$$(x + y + z)(xy + yz + zx - 2) \geq 9xyz.$$

When does the equality hold?Proposed by Dorlir Ahmeti, Albania

**Problema 3** Let  $ABC$  be an acute triangle such that  $AB \neq AC$  ,with circumcircle  $\Gamma$  and circumcenter  $O$ . Let  $M$  be the midpoint of  $BC$  and  $D$  be a point on  $\Gamma$  such that  $AD \perp BC$ . let  $T$  be a point such that  $BDCT$  is a parallelogram and  $Q$  a point on the same side of  $BC$  as  $A$  such that  $\angle BQM = \angle BCA$  and  $\angle CQM = \angle CBA$ . Let the line  $AO$  intersect  $\Gamma$  at  $E$  ( $E \neq A$ ) and let the circumcircle of  $\triangle ETQ$  intersect  $\Gamma$  at point  $X \neq E$ . Prove that the point  $A, M$  and  $X$  are collinear .

JBMO 2017, Q3

**Problema 4** Consider a regular  $2n$ -gon  $P, A_1, A_2, \dots, A_{2n}$  in the plane ,where  $n$  is a positive integer . We say that a point  $S$  on one of the sides of  $P$  can be seen from a point  $E$  that is external to  $P$  , if the line segment  $SE$  contains no other points that lie on the sides of  $P$  except  $S$  .We color the sides of  $P$  in 3 different colors (ignore the vertices of  $P$ ,we

consider them colorless), such that every side is colored in exactly

one color, and each color is used at least once . Moreover ,from every

point in the plane external to  $P$  , points of most 2 different colors on  $P$  can be seen .Find the number of distinct such colorings of  $P$  (two colorings are considered distinct if at least one of sides is colored differently).Proposed by Viktor Simjanoski Macedonia

JBMO 2017, Q4

**Problema 5** Problema [math/jbmo/2017/5](https://math.jbmo/2017/5) não encontrado!

**Problema 6** Problema [math/jbmo/2017/6](https://math.jbmo/2017/6) não encontrado!

**Problema 1** Seja  $ABCD$  um trapézio ( $AB \parallel CD$ ,  $AB > CD$ ) circunscrito na circunferência  $\omega$ , isto é,  $\omega$  é tangente à  $AB$ ,  $BC$ ,  $CD$  e  $DA$ . O incírculo de  $ABC$  toca  $AB$  e  $AC$  nos pontos  $M$  e  $N$ , respectivamente.

Prove que o incentro  $ABCD$  cai na reta  $MN$ .

**Problema 2** Let  $a, b, c$  be positive real numbers. Prove that  $\frac{8}{(a+b)^2+4abc} + \frac{8}{(b+c)^2+4abc} + \frac{8}{(a+c)^2+4abc} + a^2 + b^2 + c^2 \geq \frac{8}{a+3} + \frac{8}{b+3} + \frac{8}{c+3}$ .

**Problema 3** Find all triplets of integers  $(a, b, c)$  such that the number

$$N = \frac{(a-b)(b-c)(c-a)}{2} + 2$$

is a power of 2016.

(A power of 2016 is an integer of form  $2016^n$ , where  $n$  is a non-negative integer.)

**Problema 4** A  $5 \times 5$

table is called regular if each of its cells contains one of four pairwise distinct real numbers, such that each of them occurs exactly once in every  $2 \times 2$

subtable. The sum of all numbers of a regular table is called the total sum of the table. With any four numbers, one constructs all possible regular tables, computes their total sums and counts the distinct outcomes. Determine the maximum possible count.

**Problema 5** Problema [math/jbmo/2016/5](#) não encontrado!

**Problema 6** Problema [math/jbmo/2016/6](#) não encontrado!

**Problema 1** Find all prime numbers  $a, b, c$  and positive integers  $k$  satisfying the equation

$$a^2 + b^2 + 16c^2 = 9k^2 + 1.$$

Proposed by Moldova

**Problema 2** Let  $a, b, c$  be positive real numbers such that  $a + b + c = 3$ . Find the minimum value of the expression

$$A = \frac{2 - a^3}{a} + \frac{2 - b^3}{b} + \frac{2 - c^3}{c}.$$

**Problema 3** Let  $ABC$  be an acute triangle. The lines  $l_1$  and  $l_2$  are perpendicular to  $AB$  at the points  $A$  and  $B$ , respectively. The perpendicular lines from the midpoint  $M$  of  $AB$  to the lines  $AC$  and  $BC$  intersect  $l_1$  and  $l_2$  at the points  $E$  and  $F$ , respectively. If  $D$  is the intersection point of the lines  $EF$  and  $MC$ , prove that

$$\angle ADB = \angle EMF.$$

**Problema 4** An L-shape is one of the following four pieces, each consisting of three unit squares: [asy]

```
size(300);
defaultpen(linewidth(0.8));
path P=(1,2)--(0,2)--origin--(1,0)--(1,2)--(2,2)--(2,1)--(0,1);
draw(P);
draw(shift((2.7,0))*rotate(90,(1,1))*P);
draw(shift((5.4,0))*rotate(180,(1,1))*P);
draw(shift((8.1,0))*rotate(270,(1,1))*P);
[/asy]
```

A  $5 \times 5$  board, consisting of 25 unit squares, a positive integer  $k \leq 25$  and an unlimited supply of L-shapes are given. Two players A and B, play the following game: starting with A they play alternatively mark a previously unmarked unit square until they marked a total of  $k$  unit squares.

We say that a placement of L-shapes on unmarked unit squares is called *good* if the L-shapes do not overlap and each of them covers exactly three unmarked unit squares of the board.

B wins if every *good* placement of L-shapes leaves uncovered at least three unmarked unit squares. Determine the minimum value of  $k$  for which B has a winning strategy.

**Problema 5** Problema math/jbmo/2015/5 não encontrado!

**Problema 6** Problema math/jbmo/2015/6 não encontrado!

**Problema 1** Find all triples of primes  $(p, q, r)$  satisfying  $3p^4 - 5q^4 - 4r^2 = 26$ .

**Problema 2** Consider an acute triangle  $ABC$  of area  $S$ . Let  $CD \perp AB$  ( $D \in AB$ ),  $DM \perp AC$  ( $M \in AC$ ) and  $DN \perp BC$  ( $N \in BC$ ). Denote by  $H_1$  and  $H_2$  the orthocentres of the triangles  $MNC$ , respectively  $MND$ . Find the area of the quadrilateral  $AH_1BH_2$  in terms of  $S$ .

**Problema 3** For positive real numbers  $a, b, c$  with  $abc = 1$  prove that  $(a + \frac{1}{b})^2 + (b + \frac{1}{c})^2 + (c + \frac{1}{a})^2 \geq 3(a+b+c+1)$

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**Problema 4** For a positive integer  $n$ , two players  $A$  and  $B$  play the following game: Given a pile of  $s$  stones, the players take turn alternatively with  $A$

going first. On each turn the player is allowed to take either one stone, or a prime number of stones, or a positive multiple of  $n$  stones. The winner is the one who takes the last stone. Assuming both  $A$  and  $B$  play perfectly, for how many values of  $s$  the player  $A$  cannot win?

**Problema 5** Problema [math/jbmo/2014/5](#) não encontrado!

**Problema 6** Problema [math/jbmo/2014/6](#) não encontrado!

**Problema 1** Find all ordered pairs  $(a, b)$  of positive integers for which the numbers  $\frac{a^3b-1}{a+1}$  and  $\frac{b^3a+1}{b-1}$  are both positive integers.

**Problema 2** Let  $ABC$  be an acute-angled triangle with  $AB \perp AC$  and let  $O$  be the centre of its circumcircle  $\omega$ . Let  $D$  be a point on the line segment  $BC$  such that  $\angle BAD = \angle CAO$ . Let  $E$  be the second point of intersection of  $\omega$  and the line  $AD$ . If  $M$ ,  $N$  and  $P$  are the midpoints of the line segments  $BE$ ,  $OD$  and  $AC$ , respectively, show that the points  $M$ ,  $N$  and  $P$  are collinear.

**Problema 3** Show that

$$\left(a + 2b + \frac{2}{a+1}\right) \left(b + 2a + \frac{2}{b+1}\right) \geq 16$$

for all positive real numbers  $a$  and  $b$  such that  $ab \geq 1$ .

**Problema 4** Let  $n$  be a positive integer. Two players, Alice and Bob, are playing the following game:

- Alice chooses  $n$  real numbers; not necessarily distinct.
- Alice writes all pairwise sums on a sheet of paper and gives it to Bob. (There are  $\frac{n(n-1)}{2}$  such sums; not necessarily distinct.)
- Bob wins if he finds correctly the initial  $n$  numbers chosen by Alice with only one guess.

Can Bob be sure to win for the following cases?

- a.  $n = 5$
- b.  $n = 6$
- c.  $n = 8$

Justify your answer(s).

[For example, when  $n = 4$ ,

Alice may choose the numbers 1, 5, 7, 9, which have the same pairwise sums as the numbers 2, 4, 6, 10, and hence Bob cannot be sure to win.]

**Problema 5** Problema math/jbmo/2013/5 não encontrado!

**Problema 6** Problema math/jbmo/2013/6 não encontrado!

**Problema 1** Let  $a, b, c$  be positive real numbers such that  $a + b + c = 1$ . Prove that

$$\frac{a}{b} + \frac{a}{c} + \frac{c}{b} + \frac{c}{a} + \frac{b}{c} + \frac{b}{a} + 6 \geq 2\sqrt{2} \left( \sqrt{\frac{1-a}{a}} + \sqrt{\frac{1-b}{b}} + \sqrt{\frac{1-c}{c}} \right).$$

When does equality hold?

**Problema 2** Let the circles  $k_1$  and  $k_2$  intersect at two points  $A$  and  $B$ , and let  $t$  be a common tangent of  $k_1$  and  $k_2$  that touches  $k_1$  and  $k_2$  at  $M$  and  $N$  respectively. If  $t \perp AM$  and  $MN = 2AM$ , evaluate the angle  $NMB$ .

**Problema 3** On a board there are  $n$  nails, each two connected by a rope. Each rope is colored in one of  $n$  given distinct colors. For each three distinct colors, there exist three nails connected with ropes of these three colors.

- a) Can  $n$  be 6 ?
- b) Can  $n$  be 7 ?

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**Problema 4** Find all positive integers  $x, y, z$  and  $t$  such that  $2^x 3^y + 5^z = 7^t$ .

**Problema 5** Problema math/jbmo/2012/5 não encontrado!

**Problema 6** Problema math/jbmo/2012/6 não encontrado!

**Problema 1** Let  $a, b, c$  be positive real numbers such that  $abc = 1$ . Prove that:  $\prod (a^5 + a^4 + a^3 + a^2 + a + 1) \geq 8(a^2 + a + 1)(b^2 + b + 1)(c^2 + c + 1)$

**Problema 2** Find all primes  $p$  such that there exist positive integers  $x, y$  that satisfy  $x(y^2 - p) + y(x^2 - p) = 5p$

**Problema 3** Let  $n \geq 3$  be a positive integer. Equilateral triangle  $ABC$  is divided into  $n^2$  smaller congruent equilateral triangles (with sides parallel to its sides). Let  $m$  be the number of rhombuses that contain two small equilateral triangles and  $d$  the number of rhombuses that contain eight small equilateral triangles. Find the difference  $m - d$  in terms of  $n$ .

**Problema 4** Let  $ABCD$  be a convex quadrilateral and points  $E$  and  $F$  on sides  $AB, CD$  such that

$$\frac{AB}{AE} = \frac{CD}{DF} = n$$

If  $S$  is the area of  $AEFD$  show that  $S \leq \frac{AB \cdot CD + n(n-1)AD^2 + n^2 DA \cdot BC}{2n^2}$

**Problema 5** Problema math/jbmo/2011/5 não encontrado!

**Problema 6** Problema math/jbmo/2011/6 não encontrado!



**Problema 1** The real numbers  $a, b, c, d$  satisfy simultaneously the equations

$$abc - d = 1, \quad bcd - a = 2, \quad cda - b = 3, \quad dab - c = -6.$$

Prove that  $a + b + c + d \neq 0$ .

**Problema 2** Find all integers  $n, n \geq 1$ , such that  $n \cdot 2^{n+1} + 1$  is a perfect square.

**Problema 3** Let  $AL$  and  $BK$  be angle bisectors in the non-isosceles triangle  $ABC$  ( $L$  lies on the side  $BC$ ,  $K$  lies on the side  $AC$ ). The perpendicular bisector of  $BK$  intersects the line  $AL$  at point  $M$ . Point  $N$  lies on the line  $BK$  such that  $LN$  is parallel to  $MK$ . Prove that  $LN = NA$ .

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**Problema 4** A  $9 \times 7$  rectangle is tiled with tiles of the two types: L-shaped tiles composed by three unit squares (can be rotated repeatedly with  $90^\circ$ ) and square tiles composed by four unit squares.

Let  $n \geq 0$  be the number of the  $2 \times 2$  tiles which can be used in such a tiling. Find all the values of  $n$ .

**Problema 5** [Problema math/jbmo/2010/5](#) não encontrado!

**Problema 6** [Problema math/jbmo/2010/6](#) não encontrado!

**Problema 1** Let  $ABCDE$  be a convex pentagon such that  $AB + CD = BC + DE$  and  $k$  a circle with center on side  $AE$  that touches the sides  $AB$ ,  $BC$ ,  $CD$  and  $DE$  at points  $P$ ,  $Q$ ,  $R$  and  $S$  (different from vertices of the pentagon) respectively. Prove that lines  $PS$  and  $AE$  are parallel.

**Problema 2** Solve in non-negative integers the equation  $2^a 3^b + 9 = c^2$

**Problema 3** Let  $x, y, z$  be real numbers such that  $0 \leq x, y, z \leq 1$  and  $xyz = (1-x)(1-y)(1-z)$ . Show that at least one of the numbers  $(1-x)y$ ,  $(1-y)z$ ,  $(1-z)x$  is greater than or equal to  $\frac{1}{4}$

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**Problema 4** Each one of 2009 distinct points in the plane is coloured in blue or red, so that on every blue-centered unit circle there are exactly two red points. Find the greatest possible number of blue points.

**Problema 5** Problema [math/jbmo/2009/5](#) não encontrado!

**Problema 6** Problema [math/jbmo/2009/6](#) não encontrado!

**Problema 1** Find all real numbers  $a, b, c, d$  such that

$$\begin{cases} a + b + c + d = 20, \\ ab + ac + ad + bc + bd + cd = 150. \end{cases}$$

**Problema 2** The vertices  $A$  and  $B$  of an equilateral triangle  $ABC$  lie on a circle  $k$  of radius 1, and the vertex  $C$  is in the interior of the circle  $k$ . A point  $D$ , different from  $B$ , lies on  $k$  so that  $AD = AB$ . The line  $DC$  intersects  $k$  for the second time at point  $E$ . Find the length of the line segment  $CE$ .

**Problema 3** Find all prime numbers  $p, q, r$ , such that  $\frac{p}{q} - \frac{4}{r+1} = 1$

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**Problema 4** A  $4 \times 4$  table is divided into 16 white unit square cells. Two cells are called neighbors if they share a common side. A move consists in choosing a cell and the colors of neighbors from white to black or from black to white. After exactly  $n$  moves all the 16 cells were black. Find all possible values of  $n$ .

**Problema 5** Problema [math/jbmo/2008/5](#) não encontrado!

**Problema 6** Problema [math/jbmo/2008/6](#) não encontrado!

**Problema 1** Let  $a$  be positive real number such that  $a^3 = 6(a+1)$ . Prove that the equation  $x^2 + ax + a^2 - 6 = 0$  has no real solution.

**Problema 2** Let  $ABCD$  be a convex quadrilateral with  $\angle DAC = \angle BDC = 36^\circ$ ,  $\angle CBD = 18^\circ$  and  $\angle BAC = 72^\circ$ . The diagonals intersect at point  $P$ . Determine the measure of  $\angle APD$ .

**Problema 3** Given are 50 points in the plane, no three of them belonging to a same line. Each of these points is colored using one of four given colors. Prove that there is a color and at least 130 scalene triangles with vertices of that color.

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**Problema 4** Prove that if  $p$  is a prime number, then  $7p + 3^p - 4$  is not a perfect square.

**Problema 5** Problema [math/jbmo/2007/5](#) não encontrado!

**Problema 6** Problema [math/jbmo/2007/6](#) não encontrado!

**Problema 1** If  $n \geq 4$  is a composite number, then  $2n$  divides  $(n-1)!$ .

**Problema 2** The triangle  $ABC$  is isosceles with  $AB = AC$ , and  $\angle BAC = 60^\circ$ . The points  $D$  and  $E$  are chosen on the side  $AC$  such that,  $EB = ED$ , and  $\angle ABD \equiv \angle CBE$ . Denote by  $O$  the intersection point between the internal bisectors of the angles  $\angle BDC$  and  $\angle ACB$ . Compute  $\angle COD$ .

**Problema 3** We call a number perfect if the sum of its positive integer divisors (including 1 and  $n$ ) equals  $2n$ . Determine all perfect numbers  $n$  for which  $n-1$  and  $n+1$  are prime numbers.

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**Problema 4** Consider a  $2n \times 2n$  board. From the  $i$ th line we remove the central  $2(i-1)$  unit squares. What is the maximal number of rectangles  $2 \times 1$  and  $1 \times 2$  that can be placed on the obtained figure without overlapping or getting outside the board?

**Problema 5** Problema [math/jbmo/2006/5](#) não encontrado!

**Problema 6** Problema [math/jbmo/2006/6](#) não encontrado!

**Problema 1** Find all positive integers  $x, y$  satisfying the equation

$$9(x^2 + y^2 + 1) + 2(3xy + 2) = 2005.$$

**Problema 2** Let  $ABC$  be an acute-angled triangle inscribed in a circle  $k$ . It is given that the tangent from  $A$  to the circle meets the line  $BC$  at point  $P$ . Let  $M$  be the midpoint of the line segment  $AP$  and  $R$  be the second intersection point of the circle  $k$  with the line  $BM$ . The line  $PR$  meets again the circle  $k$  at point  $S$  different from  $R$ .

Prove that the lines  $AP$  and  $CS$  are parallel.

**Problema 3** Prove that there exist

(a) 5 points in the plane so that among all the triangles with vertices among these points there are 8 right-angled ones;

(b) 64 points in the plane so that among all the triangles with vertices among these points there are at least 2005 right-angled ones.

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**Problema 4** Find all 3-digit positive integers  $\overline{abc}$  such that

$$\overline{abc} = abc(a + b + c),$$

where  $\overline{abc}$  is the decimal representation of the number.

**Problema 5** Problema [math/jbmo/2005/5](#) não encontrado!

**Problema 6** Problema [math/jbmo/2005/6](#) não encontrado!

**Problema 1** Prove that the inequality

$$\frac{x+y}{x^2-xy+y^2} \leq \frac{2\sqrt{2}}{\sqrt{x^2+y^2}}$$

holds for all real numbers  $x$  and  $y$ , not both equal to 0.

**Problema 2** Let  $ABC$  be an isosceles triangle with  $AC = BC$ , let  $M$  be the midpoint of its side  $AC$ , and let  $Z$  be the line through  $C$  perpendicular to  $AB$ . The circle through the points  $B$ ,  $C$ , and  $M$  intersects the line  $Z$  at the points  $C$  and  $Q$ . Find the radius of the circumcircle of the triangle  $ABC$  in terms of  $m = CQ$ .

**Problema 3** If the positive integers  $x$  and  $y$  are such that  $3x + 4y$  and  $4x + 3y$  are both perfect squares, prove that both  $x$  and  $y$  are both divisible with 7.

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**Problema 4** Consider a convex polygon having  $n$  vertices,  $n \geq 4$ .

We arbitrarily decompose the polygon into triangles having all the vertices among the vertices of the polygon, such that no two of the triangles have interior points in common. We paint in black the triangles that have two sides that are also sides of the polygon, in red if only one side of the triangle is also a side of the polygon and in white those triangles that have no sides that are sides of the polygon. Prove that there are two more black triangles than white ones.

**Problema 5** Problema [math/jbmo/2004/5](#) não encontrado!

**Problema 6** Problema [math/jbmo/2004/6](#) não encontrado!

**Problema 1** Let  $n$  be a positive integer. A number  $A$  consists of  $2n$  digits, each of which is 4; and a number  $B$  consists of  $n$  digits, each of which is 8. Prove that  $A + 2B + 4$  is a perfect square.

**Problema 2** Suppose there are  $n$  points in a plane no three of which are collinear with the property that if we label these points as  $A_1, A_2, \dots, A_n$  in any way whatsoever, the broken line  $A_1 A_2 \dots A_n$  does not intersect itself. Find the maximum value of  $n$ . Dinu Serbanescu, Romania

**Problema 3** Let  $D, E, F$  be the midpoints of the arcs  $BC, CA, AB$  on the circumcircle of a triangle  $ABC$  not containing the points  $A, B, C$ , respectively. Let the line  $DE$  meet  $BC$  and  $CA$  at  $G$  and  $H$ , and let  $M$  be the midpoint of the segment  $GH$ . Let the line  $FD$  meet  $BC$  and  $AB$  at  $K$  and  $J$ , and let  $N$  be the midpoint of the segment  $KJ$ .

- a) Find the angles of triangle  $DMN$ ;
- b) Prove that if  $P$  is the point of intersection of the lines  $AD$  and  $EF$ , then the circumcenter of triangle  $DMN$  lies on the circumcircle of triangle  $PMN$ .

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**Problema 4** Let  $x, y, z \geq -1$ . Prove that

$$\frac{1+x^2}{1+y+z^2} + \frac{1+y^2}{1+z+x^2} + \frac{1+z^2}{1+x+y^2} \geq 2.$$

Laurentiu Panaitopol

**Problema 5** Problema [math/jbmo/2003/5](#) não encontrado!

**Problema 6** Problema [math/jbmo/2003/6](#) não encontrado!



**Problema 1** The triangle  $ABC$  has  $CA = CB$ .  $P$  is a point on the circumcircle between  $A$  and  $B$  (and on the opposite side of the line  $AB$  to  $C$ ).  $D$  is the foot of the perpendicular from  $C$  to  $PB$ . Show that  $PA + PB = 2 \cdot PD$ .

**Problema 2** Two circles with centers  $O_1$  and  $O_2$  meet at two points  $A$  and  $B$  such that the centers of the circles are on opposite sides of the line  $AB$ . The lines  $BO_1$  and  $BO_2$  meet their respective circles again at  $B_1$  and  $B_2$ . Let  $M$  be the midpoint of  $B_1B_2$ . Let  $M_1, M_2$  be points on the circles of centers  $O_1$  and  $O_2$  respectively, such that  $\angle AO_1M_1 = \angle AO_2M_2$ , and  $B_1$  lies on the minor arc  $AM_1$  while  $B$  lies on the minor arc  $AM_2$ . Show that  $\angle MM_1B = \angle MM_2B$ . Cyprus

**Problema 3** Find all positive integers which have exactly 16 positive divisors  $1 = d_1lt;d_2lt;\dots lt;d_{16} = n$  such that the divisor  $d_k$ , where  $k = d_5$ , equals  $(d_2 + d_4)d_6$ .

**Problema 4** Prove that for all positive real numbers  $a, b, c$  the following inequality takes place

$$\frac{1}{b(a+b)} + \frac{1}{c(b+c)} + \frac{1}{a(c+a)} \geq \frac{27}{2(a+b+c)^2}.$$

Laurentiu Panaitopol, Romania

**Problema 5** Problema [math/jbmo/2002/5](#) não encontrado!

**Problema 6** Problema [math/jbmo/2002/6](#) não encontrado!

**Problema 1** Solve the equation  $a^3 + b^3 + c^3 = 2001$  in positive integers. Mircea Becheanu, Romania

**Problema 2** Let  $ABC$  be a triangle with  $\angle C = 90^\circ$  and  $CA \neq CB$ . Let  $CH$  be an altitude and  $CL$  be an interior angle bisector. Show that for  $X \neq C$  on the line  $CL$ , we have  $\angle XAC \neq \angle XBC$ . Also show that for  $Y \neq C$  on the line  $CH$  we have  $\angle YAC \neq \angle YBC$ . Bulgaria

**Problema 3** Let  $ABC$  be an equilateral triangle and  $D, E$  points on the sides  $[AB]$  and  $[AC]$  respectively. If  $DF, EF$  (with  $F \in AE, G \in AD$ ) are the interior angle bisectors of the angles of the triangle  $ADE$ , prove that the sum of the areas of the triangles  $DEF$  and  $DEG$  is at most equal with the area of the triangle  $ABC$ . When does the equality hold? Greece

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**Problema 4** Let  $N$  be a convex polygon with 1415 vertices and perimeter 2001. Prove that we can find 3 vertices of  $N$  which form a triangle of area smaller than 1.

**Problema 5** Problema [math/jbmo/2001/5](https://artofproblemsolving.com/wiki/index.php/JBMO_2001/5) não encontrado!

**Problema 6** Problema [math/jbmo/2001/6](https://artofproblemsolving.com/wiki/index.php/JBMO_2001/6) não encontrado!

**Problema 1** Let  $x$  and  $y$  be positive reals such that

$$x^3 + y^3 + (x + y)^3 + 30xy = 2000.$$

Show that  $x + y = 10$ .

**Problema 2** Find all positive integers  $n \geq 1$  such that  $n^2 + 3^n$  is the square of an integer. Bulgaria

**Problema 3** A half-circle of diameter  $EF$  is placed on the side  $BC$  of a triangle  $ABC$  and it is tangent to the sides  $AB$  and  $AC$  in the points  $Q$  and  $P$  respectively. Prove that the intersection point  $K$  between the lines  $EP$  and  $FQ$  lies on the altitude from  $A$  of the triangle  $ABC$ . Albania

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**Problema 4** At a tennis tournament there were  $2n$  boys and  $n$  girls participating. Every player played every other player. The boys won  $\frac{7}{5}$  times as many matches as the girls. It is known that there were no draws. Find  $n$ . Serbia

**Problema 5** Problema math/jbmo/2000/5 não encontrado!

**Problema 6** Problema math/jbmo/2000/6 não encontrado!

**Problem 1** Let  $a, b, c, x, y$  be five real numbers such that  $a^3 + ax + y = 0$ ,  $b^3 + bx + y = 0$  and  $c^3 + cx + y = 0$ . If  $a, b, c$  are all distinct numbers prove that their sum is zero. Cyprus

**Problem 2** For each nonnegative integer  $n$  we define  $A_n = 2^{3n} + 3^{6n+2} + 5^{6n+2}$ . Find the greatest common divisor of the numbers  $A_0, A_1, \dots, A_{1999}$ . Romania

**Problem 3** Let  $S$  be a square with the side length 20 and let  $M$  be the set of points formed with the vertices of  $S$  and another 1999 points lying inside  $S$ . Prove that there exists a triangle with vertices in  $M$  and with area at most equal with  $\frac{1}{10}$ . Yugoslavia

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**Problem 4** Let  $ABC$  be a triangle with  $AB = AC$ . Also, let  $D \in [BC]$  be a point such that  $BC \perp AD$ ;  $BD \perp AD$ ;  $DC \perp AD$ ;  $0$ , and let  $C_1, C_2$  be the circumcircles of the triangles  $ABD$  and  $ADC$  respectively. Let  $BB'$  and  $CC'$  be diameters in the two circles, and let  $M$  be the midpoint of  $B'C'$ . Prove that the area of the triangle  $MBC$  is constant (i.e. it does not depend on the choice of the point  $D$ ). Greece

**Problem 5** Problema math/jbmo/1999/5 não encontrado!

**Problem 6** Problema math/jbmo/1999/6 não encontrado!

**Problema 1** Prove that the number  $\underbrace{111 \dots 11}_{1997} \underbrace{22 \dots 22}_{1998} 5$  (which has 1997 of 1-s and 1998 of 2-s) is a perfect square.

**Problema 2** Let  $ABCDE$  be a convex pentagon such that  $AB = AE = CD = 1$ ,  $\angle ABC = \angle DEA = 90^\circ$  and  $BC + DE = 1$ . Compute the area of the pentagon.

**Problema 3** Find all pairs of positive integers  $(x, y)$  such that

$$x^y = y^{x-y}.$$

Albania

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**Problema 4** Do there exist 16 three digit numbers, using only three different digits in all, so that the all numbers give different residues when divided by 16?Bulgaria

**Problema 5** Problema [math/jbmo/1998/5](#) não encontrado!

**Problema 6** Problema [math/jbmo/1998/6](#) não encontrado!

**Problema 1** Show that given any 9 points inside a square of side 1 we can always find 3 which form a triangle with area less than  $\frac{1}{8}$ . Bulgaria

**Problema 2** Let  $\frac{x^2+y^2}{x^2-y^2} + \frac{x^2-y^2}{x^2+y^2} = k$ . Compute the following expression in terms of  $k$ :

$$E(x, y) = \frac{x^8 + y^8}{x^8 - y^8} - \frac{x^8 - y^8}{x^8 + y^8}.$$

Ciprus

**Problema 3** Let  $ABC$  be a triangle and let  $I$  be the incenter. Let  $N, M$  be the midpoints of the sides  $AB$  and  $CA$  respectively. The lines  $BI$  and  $CI$  meet  $MN$  at  $K$  and  $L$  respectively. Prove that  $AI + BI + CI \geq BC + KL$ . Greece

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**Problema 4** Determine the triangle with sides  $a, b, c$  and circumradius  $R$  for which  $R(b + c) = a\sqrt{bc}$ . Romania

**Problema 5** Let  $n_1, n_2, \dots, n_{1998}$  be positive integers such that

$$n_1^2 + n_2^2 + \dots + n_{1997}^2 = n_{1998}^2.$$

Show that at least two of the numbers are even.

**Problema 6** Problema math/jbmo/1997/6 não encontrado!

**Problema 1** Problema [math/jbmo/1996/1](#) não encontrado!

**Problema 2** Problema [math/jbmo/1996/2](#) não encontrado!

**Problema 3** Problema [math/jbmo/1996/3](#) não encontrado!

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**Problema 1** Problema [math/jbmo/1995/1](#) não encontrado!

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**Problema 6** Problema [math/jbmo/1994/6](#) não encontrado!

**Problema 1** Problema [math/jbmo/1993/1](#) não encontrado!

**Problema 2** Problema [math/jbmo/1993/2](#) não encontrado!

**Problema 3** Problema [math/jbmo/1993/3](#) não encontrado!

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**Problema 4** Problema [math/jbmo/1993/4](#) não encontrado!

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