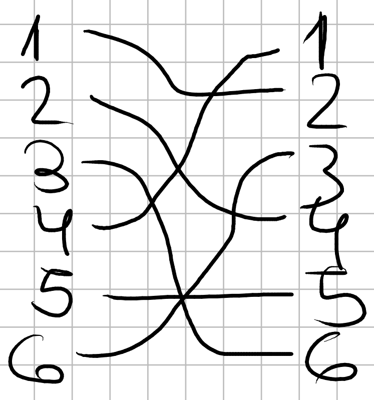
Exemplo.

D_6 $\quad$ D_6' $\qquad$ $D_6 + D_6'$



3
6
9
5
10
9

$D_6 \to 7/2$ $\quad$ $D_6' \to 7/2$ $\quad \longrightarrow \quad$ $D_6 + D_6' \to 7$

Problema 1.1:

$$\frac{1}{n} + \underbrace{\left(\frac{n-1}{n}\cdot\frac{1}{n-1}\right)}_{1/n} + \underbrace{\left(\qquad\right)}_{1/n} \ldots$$

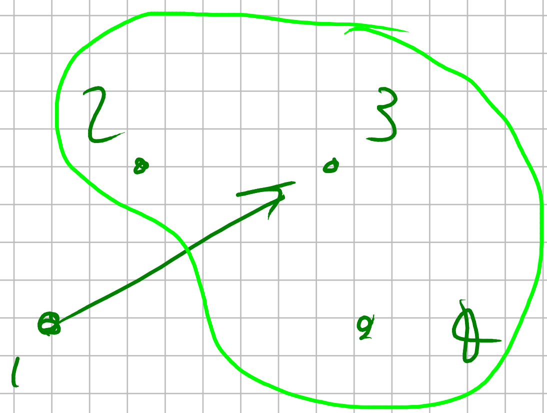
$$= 1$$

$$\mathbb{E}[S_n] = \frac{1}{n}\cdot\left(1+\mathbb{E}[S_{n-1}]\right) +$$

$$+ \frac{n-2}{n}\left(\cancel{\mathbb{E}[S_{n-1}]}\right)$$

$$+ \frac{1}{n}\left(\mathbb{E}[S_{n-1}]-1\right)$$

$n=4$



Problema 1.1.

Seja $S_i = \begin{cases} 1, & \text{se a pessoa } i \text{ pega sua etiqueta} \\ 0, & \text{caso contrário.} \end{cases}$

Mágico:

$$S = S_1 + S_2 + \cdots + S_n$$

$$\mathbb{E}[S] = \underbrace{\mathbb{E}[S_1]}_{1/n} + \underbrace{\mathbb{E}[S_2]}_{1/n} + \cdots + \underbrace{\mathbb{E}[S_n]}_{1/n}$$

$$= n \cdot 1/n = 1$$

$\star$ $\mathbb{E}[S_i] = 1 \cdot \mathbb{P}[S_i = 1]$

$$\mathbb{E}[X] = \sum_x x \cdot \mathbb{P}[X = x].$$

$= \mathbb{P}[\text{pessoa } i \text{ pega sua etiqueta}]$

Problema 1.2

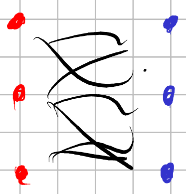
Em um berçário, 2006 bebês sentam em um círculo. De repente, cada bebê cutuca aleatóriamente o bebê imediatamente a sua direita ou o bebê imediatamente a sua esquerda. Qual é o valor esperado do $\underbrace{\text{número de bebês que não foram cutucados?}}_{S}$

$$S_i = \begin{cases} 1, & \text{se bebê } i \text{ não é cutucado} \\ 0, & \text{c.c.} \end{cases}$$

$$\mathbb{E}[S] = \mathbb{E}[S_1 + S_2 + \cdots + S_{2006}]$$
$$= \mathbb{E}[S_1] + \mathbb{E}[S_2] + \cdots + \mathbb{E}[S_{2006}]$$
$$= 2006/4 = 1003/2.$$

$$\star \quad \mathbb{E}[S_i] = 1 \cdot \mathbb{P}[S_i = 1] + 0 \cdot \mathbb{P}[S_i = 0]$$
$$= 1/4$$

Problema 1.4 (SJSU M179 Midterm)
Prove que qualquer subgrafo de $K_{n,n}$, o grafo bipartido completo com n vértices em cada partição, que possui pelo menos $n^2 - n + 1$ arestas possui um pareamento perfeito, i.e., n arestas disjuntas, i.e., possui um subgrafo em que todos os vértices tem grau 1.



$n^2 - n + 1$ arestas.

- Parir uma quantidade que:
  - → Represente "pareamento perfeito"
  - → Usar linearidade da esperança

Quantos pareamentos perfeitos tem $K_{n,n}$?

$1 \quad 1'$

$2 \quad 2'$

$\vdots \quad \vdots$

$n \quad n'$

$n!$

se tiver $1 \leftrightarrow 3'$?

$(n-1)!$

$$\mathbb{P}[1 - 3' \in \text{pareamento}] = 1/n$$

Começar com um pareamento perfeito P de $K_{n,n}$.

Será que P é pareamento perfeito de G?

$\iff$ P é subgrafo de G?

· Parir uma quantidade que

→ Medida p/ "ser subgrupo".

→ Usar lin. da $\mathbb{E}$.

$$(S = \sum S_i)$$

Seja S a quantidade de arestas que estão em P e em G.

$$S_a = \begin{cases} 1, & \text{se } a \in P \\ 0, & \text{c.c.} \end{cases}, \quad a \in G$$

$$S = \sum_{a \in G}' S_a$$

$$\mathbb{E}[S] = \sum_{a \in G}' \mathbb{E}[S_a] = \sum_{a \in G}' \mathbb{P}[a \in P]$$

$$= \sum_{a \in G}' 1/n \geq (n^2 - n + 1)\frac{1}{n}$$

$$\geq n - 1 + 1/n$$

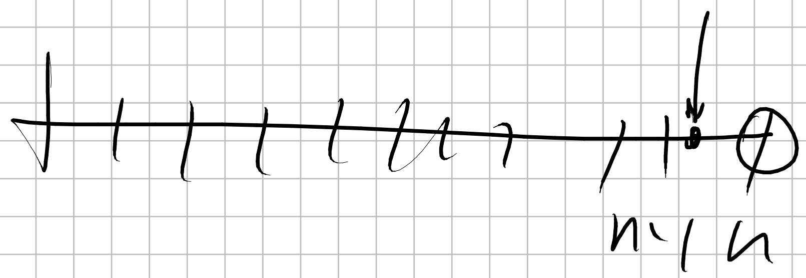
$$\mathbb{E}[S] = n - 1 + 1/n$$

Ou seja $\exists P$ t.q. $S = n \iff P \subseteq G \iff P$ p.p de G.

$$\mathbb{E}[S] = \sum_{x=0}^{n} x \cdot \mathbb{P}[S = x] = n - 1 + 1/n$$

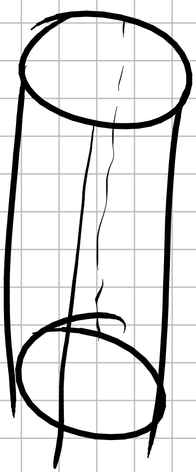
Se S sempre fosse $\leq n-1$, $\mathbb{E}[S] \leq n-1$. Abs!

Logo, $\exists P$, $S > n-1 \Rightarrow S = n$. ∎



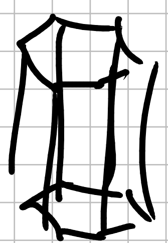
(Método Probabilístico)

Prisma (reto)

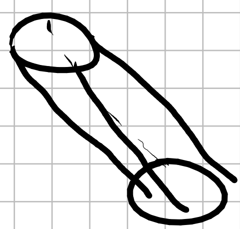


(reto)
Prisma circular
=
cilindro

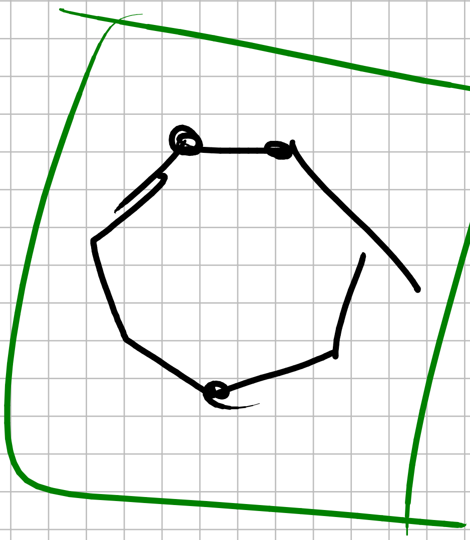
(reto)
Prisma retangular
=
paralelepípedo



(reto
Prisma hexagonal

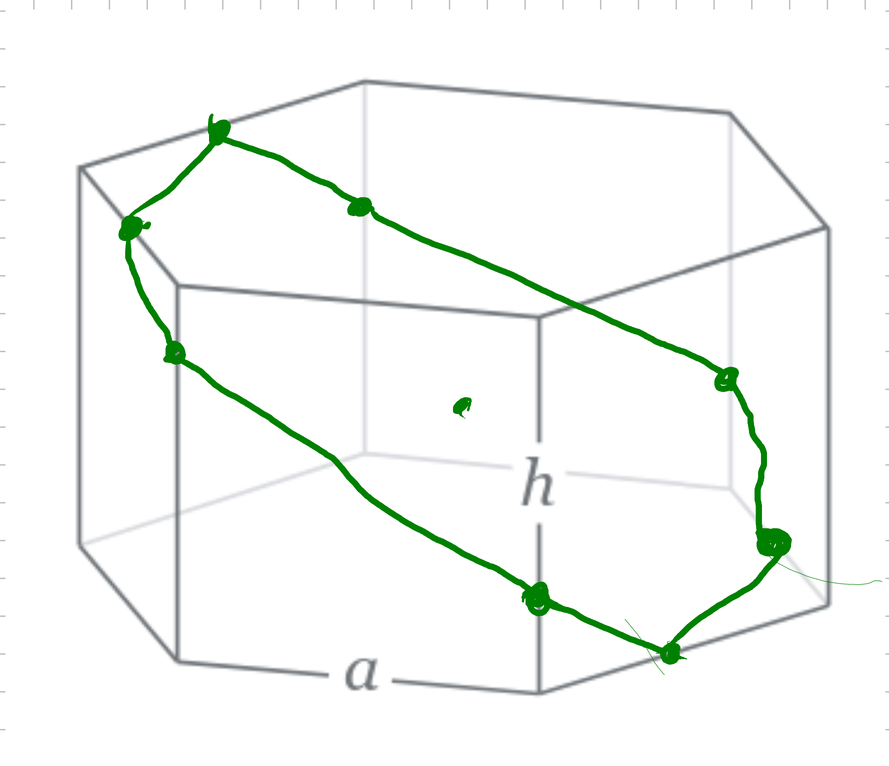


Prisma oblíquo
circular

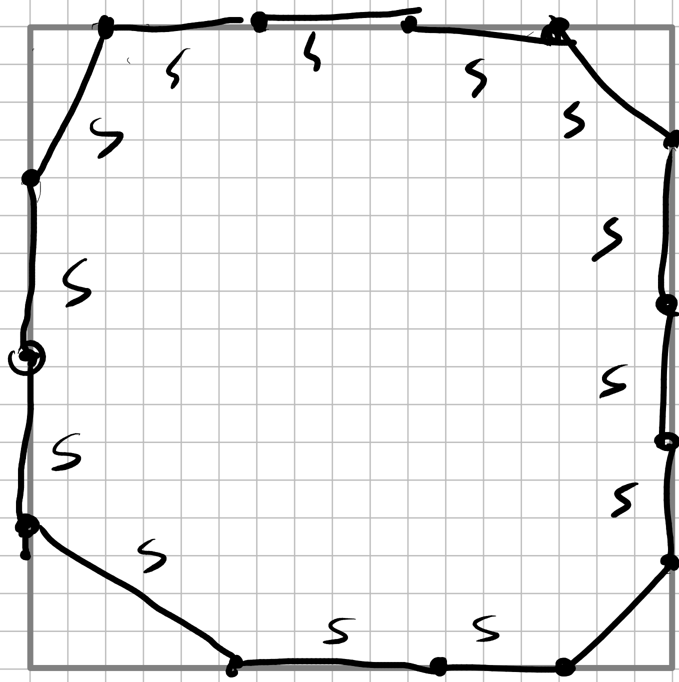


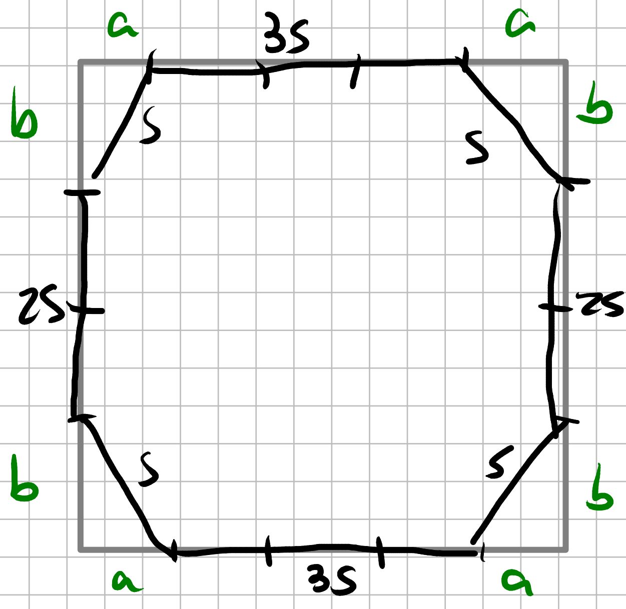
Plano

≤ 8



8 lados!





$$2a+3s=n \Rightarrow a=\frac{n-3s}{2}$$

$$2b+2s=n \Rightarrow b=\frac{n-2s}{2}$$

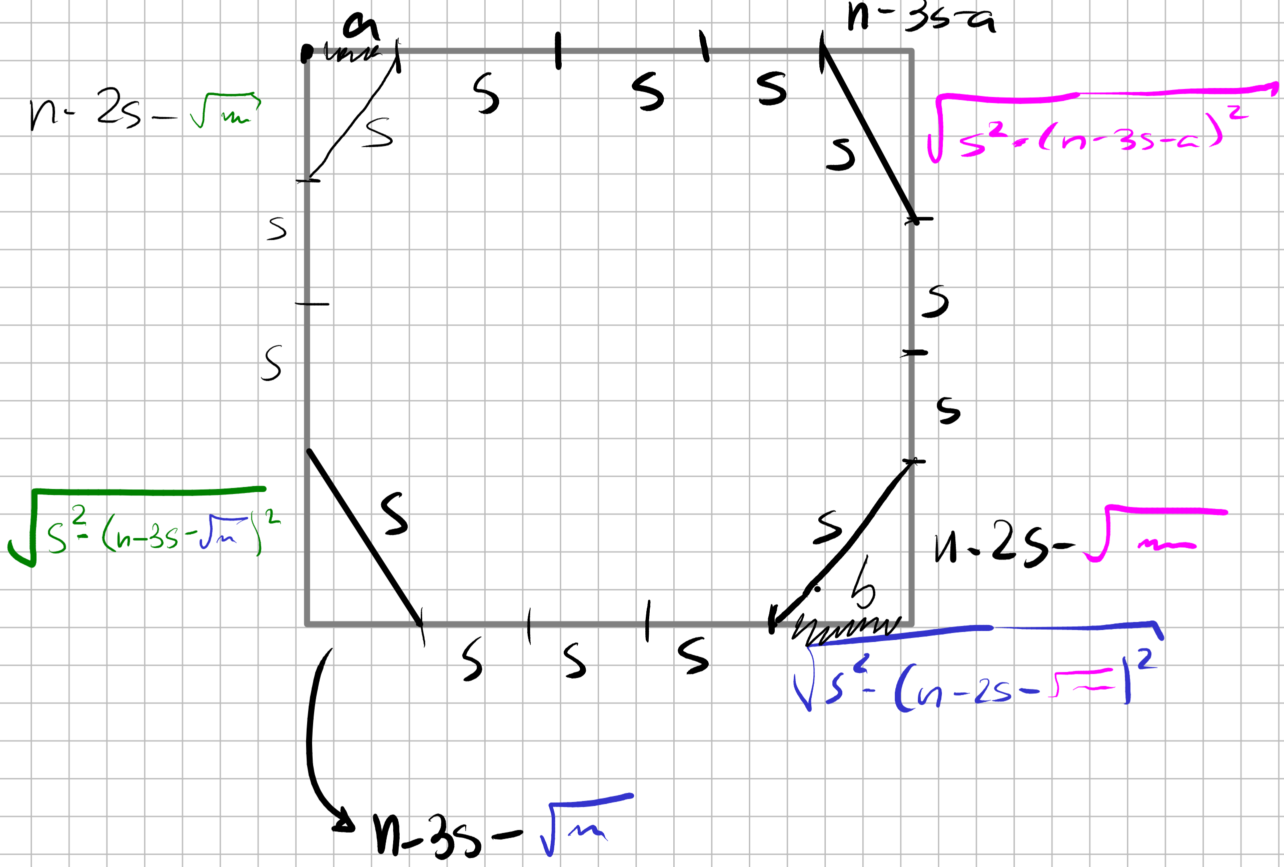
$$a^2+b^2=s^2$$

$$(n-3s)^2+(n-2s)^2=4s^2$$

$$2n^2-10ns+9s^2=0$$

$$s=\frac{10n \pm \sqrt{100n^2-4\cdot 2\cdot 9n^2}}{2\cdot 9}$$

$$=\left(\frac{10-\sqrt{28}}{18}\right)n$$



$$\begin{cases} b^2 = s^2 - \left(n - 2s - \sqrt{s^2 - (n-3s-a)^2}\right)^2 \\ a^2 = s^2 - \left(n - 2s - \sqrt{s^2 - (n-3s-b)^2}\right)^2 \end{cases}$$

$$b^2 - a^2 = \left(n-2s-\sqrt{s^2-(n-3s-b)^2}\right)^2 - \left(n-2s-\sqrt{s^2-(n-3s-a)^2}\right)^2$$

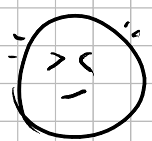
$$b^2 - a^2 = \left(\sqrt{s^2-(n-3s-a)^2} - \sqrt{s^2-(n-3s-b)^2}\right)\left(2n-4s-\left(\sqrt{s^2-(n-3s-a)^2}+\sqrt{s^2-(n-3s-b)^2}\right)\right)$$

$$b^2 - a^2 = \left(\cancel{s^2} - (n-3s-b)^2\right) - \left(\cancel{s^2} - (n-3s-a)^2\right) + 2(n-2s)\left(\sqrt{s^2-(n-3s-a)^2} - \sqrt{s^2-(n-3s-b)^2}\right)$$

$$b^2 - a^2 = (b-a)(2n-6s-a-b) + 2(n-2s)\left(\sqrt{s^2-(n-3s-a)^2} - \sqrt{s^2-(n-3s-b)^2}\right)$$

$$(b-a)(a+b+3s-n) = (n-2s)\left(\sqrt{s^2-(n-3s-a)^2} - \sqrt{s^2-(n-3s-b)^2}\right)$$

$$(b-a)^2(a+b+3s-n)^2 = (n-2s)^2\left(2s^2-(n-3s-a)^2-(n-3s-b)^2-2\sqrt{(n-2s-a)(n-2s-b)(4s-n+a)(4s-n+b)}\right)$$



Problema 3.1.

H T T H H T T H

1 2 3 4 5 6 7

H T T H T T H T

1 2 3 4 5 6 7

$$\#\left(\text{Sequência de } 4 \text{ c/ propriedade}\right) = 2 \cdot \#\left(\text{escolhes 2 numeros de mesma paridade } 1,\dots,7\right)$$

$$= 2 \cdot \left[\binom{4}{2} + \binom{3}{2}\right]$$

$$= 2 \cdot 9$$

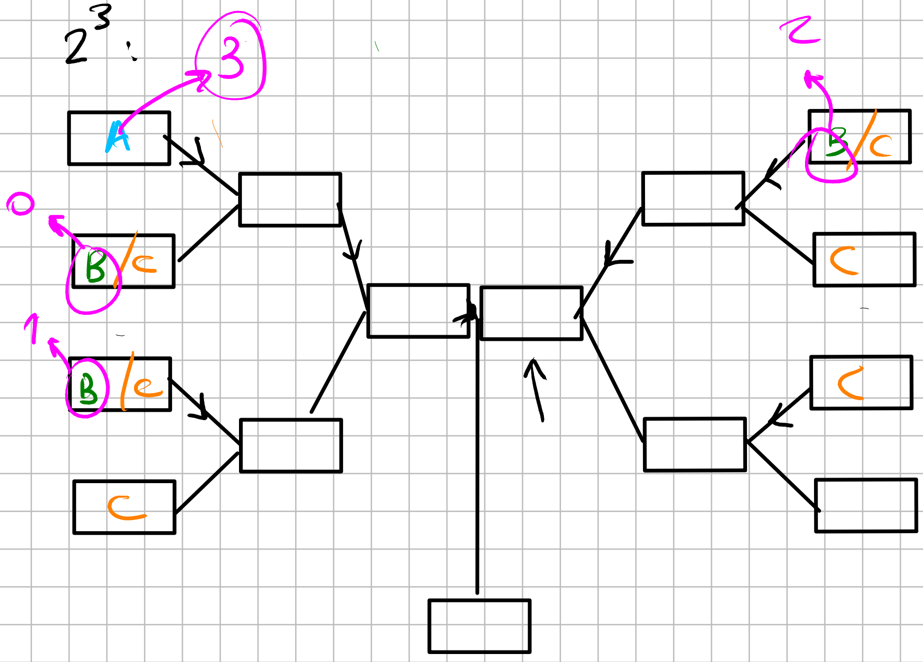
$\#$total $= 2^8$

$$\text{prob} = \frac{2 \cdot 9}{2^8} = \frac{9}{2^7} = \frac{9}{128}$$

~~R: 100~~ R: 1028

Problema 3.2.

2^3:



A: ganha do B em algum momento

A ganha do B na 1ª rodada: Bj. 0

" " 2ª rodada: Bj. 1.

" " 3ª rodada: Bj. 2.

⋮

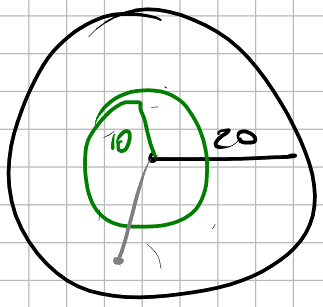
" " 9ª rodada Bj. 8.

A ganhado do C: Aj. 9.

$\oplus$

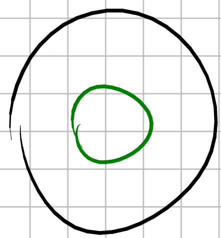
45/

Problema 3.3.



$\mathbb{P}[\text{dardo } \phi] = 1/4$

$\mathbb{E}$. n



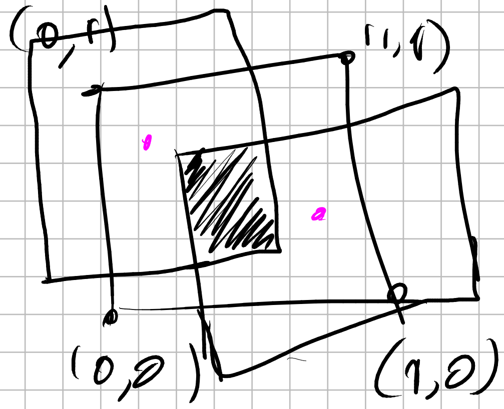
Quantos dardos a gente precisa jogar t.q n dardos caem em O.

$$\mathbb{E}[T_n] = \frac{1}{4}\left(1+\mathbb{E}[T_{n-1}]\right) + \frac{3}{4}\left(1+\mathbb{E}[T_n]\right)$$

$$\frac{1}{4}\mathbb{E}[T_n] = 1 + \frac{1}{4}\mathbb{E}[T_{n-1}]$$

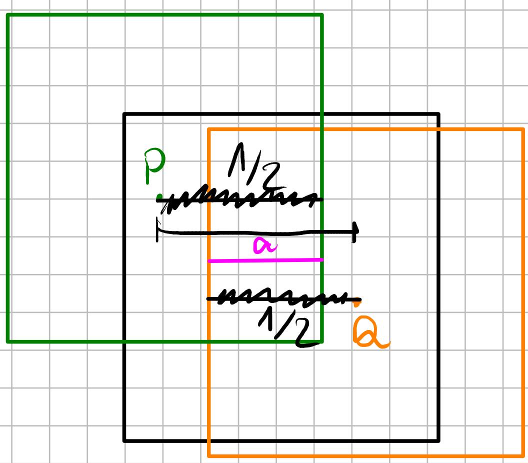
$$\mathbb{E}[T_n] = 4 + \mathbb{E}[T_{n-1}]$$
$$= 4n$$

$3n$ - que faltam.



$\mathbb{E}[S_u] = ?$

$P = (x, y)$; $Q = (x', y')$



$a = 1 - |x - x'|$

$b = 1 - |y - y'|$

$x, y, x', y' \in [0,1]$

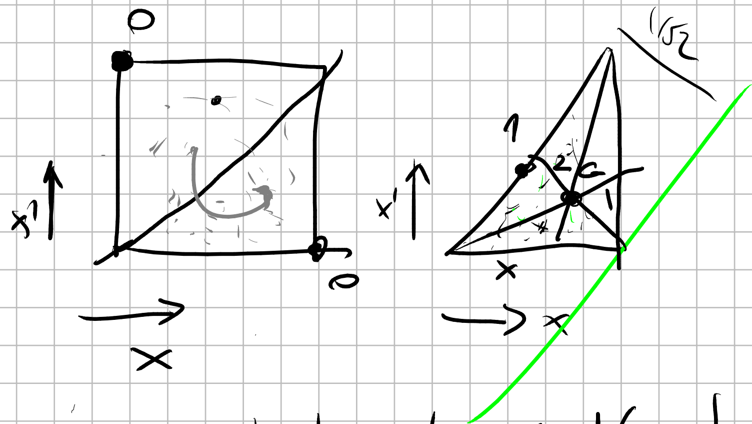
Quero

$$\mathbb{E}\left[(1-|x-x'|)\cdot(1-|y-y'|)\right] =$$

$$\mathbb{E}\left[(1-|x-x'|)\right]\cdot\mathbb{E}\left[(1-|y-y'|)\right] =$$

$$\mathbb{E}\left[(1-|x-x'|)\right]^2 = (1/3)^2$$

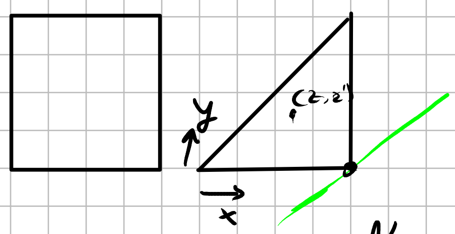
$$(x, x') \in [0,1]^2$$



$$1 - |x - x'| = \sqrt{2}\cdot d(\cdot, \;)$$

$$\mathbb{E}(1 - |x - x'|) = \sqrt{2}\cdot d(G, \;) = 1/3.$$

$\mathbb{E}\left[1-|z-z'|\right]$, $(z,z')\in[0,1]^2$



$1-x+y=0$

$d((z,z'),r)$:

$$\mathbb{E}\left[1-z+z'\right] = \sqrt{2}\cdot\frac{1-z+z'}{\sqrt{1^2+1^2}}$$

$r:\ Ax+By+C=0$

$$d((x_0,y_0),r) = \frac{Ax_0+By_0+C}{\sqrt{A^2+B^2}}$$

$$\mathbb{E}\left[d(A,B)\right]$$