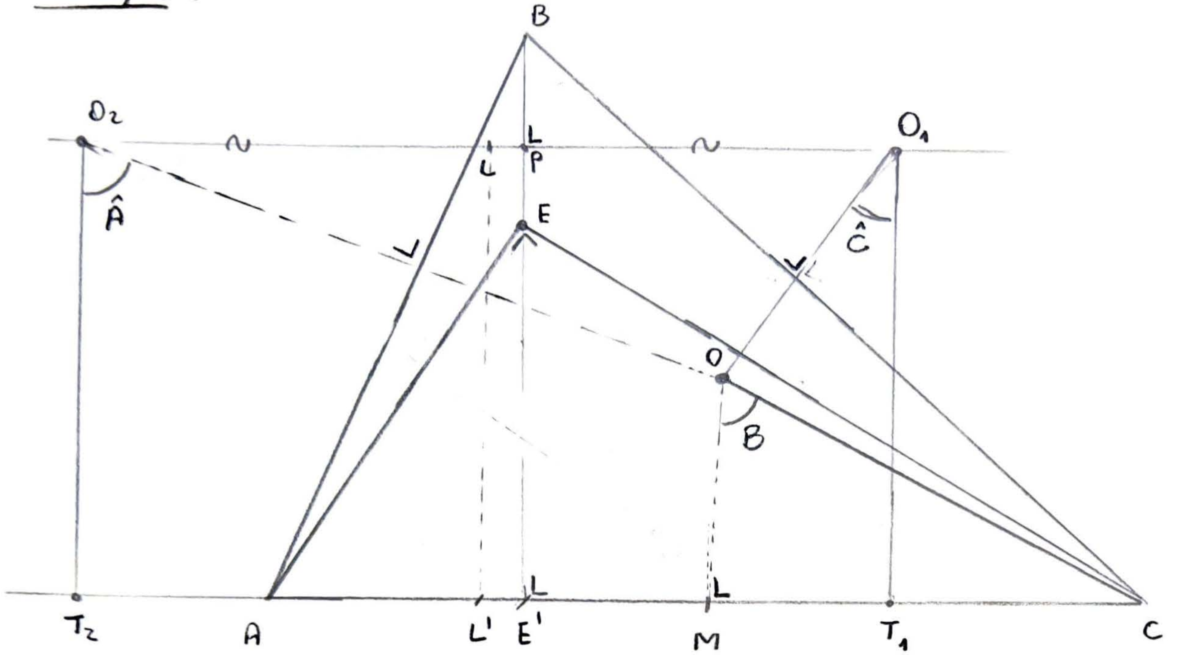
Triângulo ABC; E na altura tq. $A\hat{E}C = 90$; O_1 circ BEC; O_2 circ BEA, L ponto médio de O_1O_2, M ponto médio de AC. Prove L, M, E colineares.

Solução:



- $EE' = \sqrt{AL' \cdot L'C} = 2R\sqrt{\operatorname{sen}A\cos A\operatorname{sen}C\cos C}$

$BE' = 2R\operatorname{sen}A \cdot \operatorname{sen}C \Rightarrow BE = 2R\left(\operatorname{sen}A\operatorname{sen}C - \sqrt{\operatorname{sen}A\operatorname{sen}C\cos A\cos C}\right) \Rightarrow$

$\Rightarrow PE = R\left(\operatorname{sen}A\operatorname{sen}C - \sqrt{\operatorname{sen}A\operatorname{sen}C\cos A\cos C}\right) \Rightarrow PE' = R\left(\operatorname{sen}A\operatorname{sen}C + \sqrt{\operatorname{sen}A\cos A\operatorname{sen}C\cos C}\right)$.

- $OM = R \cdot \cos B \Rightarrow MT_1 = \operatorname{tg}\hat{C}\,(O_1T_1 - OM) = \operatorname{tg}\hat{C} \cdot R\left(\operatorname{sen}A\operatorname{sen}C - \cos B + \sqrt{\operatorname{sen}A\cos A\operatorname{sen}C\cos C}\right)$

$$MT_1 = \operatorname{tg}\hat{C} \cdot R \cdot \left(\cos A\cos C + \sqrt{\operatorname{sen}A\operatorname{sen}C\cos A\cos C}\right)$$

Analogamente, $MT_2 = \operatorname{tg}\hat{A} \cdot R\left(\cos A \cdot \cos C + \sqrt{\operatorname{sen}A\operatorname{sen}C\cos A\cos C}\right)$

$\Rightarrow ML' = \dfrac{MT_2 - MT_1}{2} = \dfrac{R}{2} \cdot \left(\cos A\cos C + \sqrt{\operatorname{sen}A\operatorname{sen}C\cos A\cos C}\right)(\operatorname{tg}A - \operatorname{tg}C)$

- $AE' = 2R\operatorname{sen}C\cos A$; $AM = R\operatorname{sen}B \Rightarrow E'M = R(\operatorname{sen}B - 2\operatorname{sen}C\cos A) = R(\operatorname{sen}A\cos C - \operatorname{sen}C\cos A)$

$$E'M = R\operatorname{sen}(A-C)$$

Mas, L, E e M são colineares $\Leftrightarrow \dfrac{LL'}{L'M} = \dfrac{EE'}{E'M} \Leftrightarrow \dfrac{\operatorname{sen}A\operatorname{sen}C + \sqrt{\quad}}{\frac{1}{2}\left(\cos A\cos C + \sqrt{\quad}\right)\left(\frac{\operatorname{sen}A}{\cos A} - \frac{\operatorname{sen}C}{\cos C}\right)} = \dfrac{2\sqrt{\quad}}{\operatorname{sen}(A-C)} \Leftrightarrow$

$\Leftrightarrow \dfrac{\operatorname{sen}A\operatorname{sen}C + \sqrt{\quad}}{\cos A\cos C + \sqrt{\quad}} = \dfrac{\sqrt{\quad}}{\cos A\cos C} \Leftrightarrow \dfrac{\operatorname{sen}A \cdot \operatorname{sen}C}{\sqrt{\quad}} = \dfrac{\sqrt{\quad}}{\cos A\cos C} \Leftrightarrow$

$\operatorname{sen}A\operatorname{sen}C\cos A\cos C = \sqrt{\quad}^{\,2}$, que é verdade $\square$