MATEMATICAMENTE

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# Banco de Problemas para a Tutoria

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**1.** Angel has a warehouse, which initially contains 100 piles of 100 pieces of rubbish each. Each morning, Angel performs exactly one of the following moves:

- He clears every piece of rubbish from a single pile.
- He clears one piece of rubbish from each pile.

However, every evening, a demon sneaks into the warehouse and performs exactly one of the following moves:

- He adds one piece of rubbish to each non-empty pile.
- He creates a new pile with one piece of rubbish.

What is the first morning when Angel can guarantee to have cleared all the rubbish from the warehouse?

**2.** Call a rational number short if it has finitely many digits in its decimal expansion. For a positive integer $m$ , we say that a positive integer $t$ is $m-$ tastic if there exists a number $c \in \{1, 2, 3, \ldots, 2017\}$ such that $\frac{10^t - 1}{c \cdot m}$ is short, and such that $\frac{10^k - 1}{c \cdot m}$ is not short for any $1 \leq k < t$ . Let $S(m)$ be the the set of $m-$ t astic numbers. Consider $S(m)$ for $m = 1, 2, \ldots$ . What is the maximum number of elements in $S(m)$ ?

**3.** Can we find $N$ such that all $m \times n$ rectangles with $m, n > N$ can be tiled with $4 \times 6$ and $5 \times 7$ rectangles?