Problema 1 Treinamento para IMO - Geometria - Rodada 1

(APMO 2017) ΔABC, $\hat{A}=60$, incentro I, ortocentro H. Prove: $\angle AHI = 3 \cdot A\hat{B}C$.

$\hat{B}=2\beta$. Quero: $A\hat{H}I = 3\beta$ • $B\hat{I}C = 90° + \hat{A}/2 = 120°$; $B\hat{H}C = 180° - \hat{A} = 120°$

Logo, # $BIHC$ cíclico $\therefore \angle IBC = \angle IHE = \beta$. Mas $\Delta ABD \sim \Delta AHE \Rightarrow$

$\angle AHE = 2\beta \Rightarrow \angle AHI = 3\beta$ □