Estudo em Grupo
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Problema 1 (HMMT 2020, Álgebra, 4 ☑). Para inteiros positivos n e k, seja $\mho(n,k)$ o número de fatores primos distintos de n maiores ou iguais a k. Por exemplo, $\mho(90,3) = 2$, dado que os únicos fatores primos de 90 que são pelo menos 3 são 3 e 5. Ache o inteiro mais próximo de

$$\sum_{n=1}^{\infty} \sum_{k=1}^{\infty} \frac{\mho(n,k)}{3^{n+k-7}}.$$

Problema 2 (Banco IMO 2009, C1). Consider 2009 cards, each having one gold side and one black side, lying on parallel on a long table. Initially all cards show their gold sides. Two player, standing by the same long side of the table, play a game with alternating moves. Each move consists of choosing a block of 50 consecutive cards, the leftmost of which is showing gold, and turning them all over, so those which showed gold now show black and vice versa. The last player who can make a legal move wins.
   (a) Does the game necessarily end?
   (b) Does there exist a winning strategy for the starting player?

Problema 3 (Berkeley Math Circle 2000). n squares in an infinite grid are coloured black; the rest are coloured white. When a square is the opposite colour from 2 or more of its 4 neighbours, its colour may be switched. Eventually, we get to having 2017 black squares, no two of which border along an edge, and all other squares white. Prove that $n \geq 2017$.

Problema 4. A maze consists of a finite grid of squares where the boundary and some internal edges are "walls" that cannot be crossed. For example:
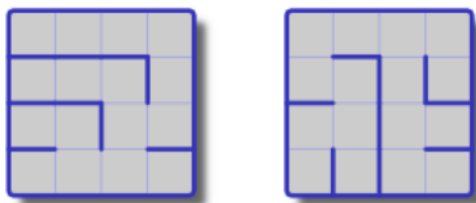
Two mazes are given, each with a robot in the top-left square. You may give a list of directions (up, down, left, or right) to the robots. Both robots will independently follow the same list of directions. For each direction, the robot will move one square in that direction if it can, or do nothing if there is a wall in the way. It will then proceed to the next direction, and repeat until it has gone through the whole list. Suppose that there is a list of directions that will get each robot individually from the top-left corner to the bottom-right corner of its maze. Prove there is also a list of directions that will get both robots to the bottom-right corner at the same time.

In the example mazes above, you could give the directions 'Right', 'Right', 'Down', 'Down', 'Down', 'Right', 'Down', 'Down', 'Left', 'Down', 'Right'.

Problema 5 (Colorado Math Olympiad 1997). On every square of a 2017×2017 board, either $(+1)$ or (-1) is written. For every row, we compute the product R_i of all numbers written in that row, and for every column, we compute the product C_i of all numbers written in that column. Is it possible to arrange the numbers in such a way that

$$\sum_{i=1}^{2017} (R_i + C_i) = 0?$$