Primeiro Teste de Seleção

LX Olímpíada Internacional de Matemática e XXXIV Olimpíada Iberoamericana

Problema 1 Determine all pairs (m, n) of positive integers for which there exists a positive integer s such that sm and sn have an equal number of divisors.

Problema 2 Let $n \geq 3$ be an integer. Prove that there exists a set S of $2n$ positive integers satisfying the following property: For every $m = 2, 3, \ldots, n$ the set S can be partitioned into two subsets with equal sum of elements, with one of the subsets of cardinality m.

Problema 3 Let $a_0, a_1, a_2, \ldots$ be a sequence of real numbers such that $a_0 = 0$, $a_1 = 1$, and for every $n \geq 2$ there exists $1 \leq k \leq n$ satisfying

$$a_n = \frac{a_{n-1} + \cdots + a_{n-k}}{k}.$$

Find the maximal value of $a_{2018} - a_{2017}$.

Problema 4 Let O be the circumcentre, and Ω be the circumcircle of an acute-angled triangle ABC. Let P be an arbitrary point on Ω, distict from A, B, C, and their antipodes in Ω. Denote the circumcentres od the triangles AOP, BOP, and COP by O_A, O_B, and O_C, respectively. The lines ℓ_A, ℓ_B, and ℓ_C are perpendicular to BC, CA, and AB pass through O_A, O_B, and O_C, respectively. Prove that the circumcircle of the triangle formed by ℓ_A, ℓ_B, and ℓ_C is tangent to the line OP.