Geometria 4

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Problema 1 **(IGO 2019, 1)** Os círculos ω_1 e ω_2 se intersectam nos pontos A e B. O ponto C está na reta tangente por A de ω_1 tal que $\angle ABC = 90°$. Uma reta ℓ passa por C e corta ω_2 nos pontos P e Q. As retas AP e AQ cortam ω_1 novamente em X e Z, respectivamente. Seja Y o pé da altitude de A na reta ℓ. Prove que os pontos X, Y e Z são colineares.

Problema 2 **(IGO 2018, 1)** Two circles ω_1, ω_2 intersect each other at points A, B. Let PQ be a common tangent line of these two circles with $P \in \omega_1$ and $Q \in \omega_2$. An arbitrary point X lies on ω_1. Line AX intersects ω_2 for the second time at Y . Point $Y' \neq Y$ lies on ω_2 such that $QY = QY'$. Line $Y'B$ intersects ω_1 for the second time at X'. Prove that $PX = PX'$.

Problema 3 **(IGO 2017, 1)** In triangle ABC, the incircle, with center I, touches the sides BC at point D. Line DI meets AC at X. The tangent line from X to the incircle (different from AC) intersects AB at Y. If YI and BC intersect at point Z, prove that $AB = BZ$.

Problema 4 **(IGO 2016, 1)** Let the circles ω and ω' intersect in A and B. Tangent to circleω at A intersectsω' in C and tangent to circle ω' at A intersects ω in D. Suppose that CD intersectsω and ω' in E and F, respectively (assume that E is between F and C). The perpendicular to AC from E intersects ω' in point P and perpendicular to AD from F intersectsω in point Q (The points A, P and Q lie on the same side of the line CD). Prove that the points A, P and Q are collinear.

Problema 5 **(IGO 2015, 1)** Let w_1 and w_2 be two circles such that $w_1 \cap w_2 = \{A, B\}$. Let X be a point on w_2 and Y be a point on w_1 such that $BY \perp BX$. Suppose that O is the center of w_1 and $X' = w_2 \cap OX$. If $K = w_2 \cap X'Y$, prove that X is the midpoint of the arc AK.

Problema 6 **(IGO 2014, 1)** ABC is a triangle with $\angle BAC = 90°$ and $\angle ACB = 30°$. Let M_1 be the midpoint of BC. Let W be a circle passing through A tangent in M_1 to BC. Let P be the circumcircle of ABC. W is intersecting AC in N and P in M. Prove that MN is perpendicular to BC.