$$\left(\tan\left(\frac{3\pi}{11}\right) + 4\operatorname{sen}\left(\frac{2\pi}{11}\right)\right)^2 = \left(\frac{\operatorname{sen}(3\pi/11) + 4\operatorname{sen}(2\pi/11)\cos(3\pi/11)}{\cos(3\pi/11)}\right)^2$$

$$= \frac{1-\cos^2(3\pi/11) + 4\operatorname{sen}(2\pi/11)\operatorname{sen}(6\pi/11) + 16\operatorname{sen}^2(2\pi/11)\cos^2(3\pi/11)}{\cos^2(3\pi/11)} =$$

$$= \frac{1 + 4\operatorname{sen}(2\pi/11)\operatorname{sen}(6\pi/11)}{\cos^2(3\pi/11)} - 1 + 16\operatorname{sen}^2(2\pi/11) =$$

$$= \frac{1+2\cos(4\pi/11) - 2\cos(8\pi/11)}{\frac{1}{2}\left(\cos(6\pi/11)+1\right)} - 1 - 8\cos(4\pi/11) + 8 \overset{?}{=} 11$$

Quero $1 + 2\cos(4\pi/11) - 2\cos(8\pi/11) - 4\cos(4\pi/11)\cos(6\pi/11) - 4\cos(4\pi/11) =$

$= 2\cos(6\pi/11) + 2.$

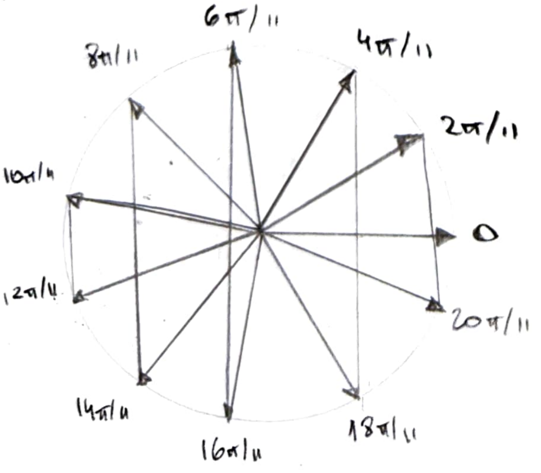
$\iff$

$$\cos(4\pi/11) + \cos(6\pi/11) + \cos(8\pi/11) + 2\cos(4\pi/11)\cos(6\pi/11) = -\frac{1}{2}$$

$\iff$

$$\cos(2\pi/11) + \cos(4\pi/11) + \cos(6\pi/11) + \cos(8\pi/11) + \cos(10\pi/11) = -1/2,$$

Que é verdade pois:




$$\Rightarrow \sum_{k=0}^{10} \cos(2k\pi/11) = 0$$

$$\Rightarrow \sum_{k=1}^{5} \cos(2k\pi/11) = -1/2$$