The 11^{th} Romanian Master of Mathematics

Sexta, 22 de fevereiro de 2019

Problema 1. Amy e Bob jogam um jogo. No começo, Amy escreve um inteiro positivo no quadro. Depois os jogadores jogam em turnos, Bob joga primeiro. Nos turnos de Bob, Bob troca o número n no quadro por um número da forma $n - a^2$, onde a é um inteiro positivo escolhido por Bob. Nos turnos de Amy, Amy troca o número n no quadro por um número da forma n^k, onde k é um inteiro positivo escolhido por Amy. Bob ganha se o número no quadro se tornar zero. Amy consegue prevenir a vitória de Bob?

Problema 2. Seja $ABCD$ um trapézio isósceles, com $AB||CD$. Seja E o ponto médio de AC. Sejam ω e Ω os circumcírculos dos triângulos ABE e CDE, respectivamente. Seja P a intersecção da tangente a ω em A com a tangente a Ω em D. Prove que PE é tangente a Ω.

Problema 3. Dado qualquer real positivo ϵ, prove que existem apenas finitos inteiros positivos v para os quais a seguinte propriedade é falsa:

Qualquer grafo com v vertices e pelo menos $(1+\epsilon)v$ arestas tem dois ciclos distintos de tamanhos iguais.

(Lembre-se de que a noção de um ciclo simples não permite a repetição de vértices em um ciclo.)

Sábado, 23 de fevereiro de 2019

Problema 4. Prove que para todo inteiro positivo n existe um polígono (não necessariamente convexo) sem três vértices colineares, o que admite exatamente n triangulações diferentes.

Problema 5. Determine todas as funções $f : \mathbb{R} \to \mathbb{R}$ que satisfazem

$$f(x + yf(x)) + f(xy) = f(x) + f(2019y),$$

para todos os números reais x e y.

Problema 6. Ache todos os pares de inteiros (c, d), ambos maiores que 1, com a seguinte propriedade: Para qualquer polinômio mônico Q de grau d com coeficientes inteiros e para qualquer primo $p > c(2c+1)$, existe um conjunto S com no máximo $\left(\frac{2c-1}{2c+1}\right)p$ inteiros, tal que

$$\bigcup_{s \in S} \{s, Q(s), Q(Q(s)), Q(Q(Q(s))), \dots\}$$

é um sistema completo de resíduos módulo p (i.e., intersecta com todas as classes de resíduos módulo p).

The 10^{th} Romanian Master of Mathematics

Fevereiro de 2018

Problema 1. Let $ABCD$ be a cyclic quadrilateral an let P be a point on the side AB. The diagonals AC meets the segments DP at Q. The line through P parallel to CD mmets the extension of the side CB beyond B at K. The line through Q parallel to BD meets the extension of the side CB beyond B at L. Prove that the circumcircles of the triangles BKP and CLQ are tangent.

Problema 2. Determine whether there exist non-constant polynomials $P(x)$ and $Q(x)$ with real coefficients satisfying

$$P(x)^{10} + P(x)^9 = Q(x)^{21} + Q(x)^{20}.$$

Problema 3. Ann and Bob play a game on the edges of an infinite square grid, playing in turns. Ann plays the first move. A move consists of orienting any edge that has not yet been given an orientation. Bob wins if at any point a cycle has been created. Does Bob have a winning strategy?

Fevereiro de 2018

Problema 4. Let a, b, c, d be positive integers such that $ad \neq bc$ and $\gcd(a, b, c, d) = 1$. Let S be the set of values attained by $\gcd(an + b, cn + d)$ as n runs through the positive integers. Show that S is the set of all positive divisors of some positive integer.

Problema 5. Let n be positive integer and fix $2n$ distinct points on a circle. Determine the number of ways to connect the points with n arrows (oriented line segments) such that all of the following conditions hold:

- each of the $2n$ points is a startpoint or endpoint of an arrow;
- no two arrows intersect; and
- there are no two arrows $\overrightarrow{AB}$ and $\overrightarrow{CD}$ such that A, B, C and D appear in clockwise order around the circle (not necessarily consecutively).

Problema 6. Fix a circle Γ, a line ℓ to tangent Γ, and another circle Ω disjoint from ℓ such that Γ and Ω lie on opposite sides of ℓ. The tangents to Γ from a variable point X on Ω meet ℓ at Y and Z. Prove that, as X varies over Ω, the circumcircle of XYZ is tangent to two fixed circles.

The 9^{TH} Romanian Master of Mathematics

Sexta, 24 de fevereiro de 2017

Problema 1. (a) Prove that every positive integer n can be written uniquely in the form

$$n = \sum_{j=1}^{2k+1} (-1)^{j-1} 2^{m_j},$$

where $k \geq 0$ and $0 \leq m_1 < m_2 \cdots < m_{2k+1}$ are integers. This number k is called *weight* of n.

(b) Find (in closed form) the difference between the number of positive integers at most 2^{2017} with even weight and the number of positive integers at most 2^{2017} with odd weight.

Problema 2. Determine all positive integers n satisfying the following condition: for every monic polynomial P of degree at most n with integer coefficients, there exists a positive integer $k \leq n$ and $k+1$ distinct integers $x_1, x_2, \ldots, x_{k+1}$ such that

$$P(x_1) + P(x_2) + \cdots + P(x_k) = P(x_{k+1}).$$

Nota. A polynomial is *monic* if the coefficient of the highest power is one.

Problema 3. Let n be an integer greater than 1 and let X be an n-element set. A non-empty collection of subsets $A_1, ..., A_k$ of X is tight if the union $A_1 \cup \cdots \cup A_k$ is a proper subset of X and no element of X lies in exactly one of the A_is. Find the largest cardinality of a collection of proper non-empty subsets of X, no non-empty subcollection of which is tight.

Nota. A subset A of X is proper if $A \neq X$. The sets in a collection are assumed to be distinct. The whole collection is assumed to be a subcollection.

Sábado, 25 de fevereiro de 2017

Problema 4. In the Cartesian plane, let G_1 and G_2 be the graphs of the quadratic functions $f_1(x) = p_1x^2 + q_1x + r_1$ and $f_2(x) = p_2x^2 + q_2x + r_2$, where $p_1 > 0 > p_2$. The graphs G_1 and G_2 cross at distinct points A and B. The four tangents to G_1 and G_2 at A and B form a convex quadrilateral which has an inscribed circle. Prove that the graphs G_1 and G_2 have the same axis of symmetry.

Problema 5. Fix an integer $n \geq 2$. An $n \times n$ sieve is an $n \times n$ array with n cells removed so that exactly one cell is removed from every row and every column. A stick is a $1 \times k$ or $k \times 1$ array for any positive integer k. For any sieve A, let $m(A)$ be the minimal number of sticks required to partition A. Find all possible values of $m(A)$, as A varies over all possible $n \times n$ sieves.

Problema 6. Let $ABCD$ be any convex quadrilateral and let P, Q, R, S be points on the segments AB, BC, CD, and DA, respectively. It is given that the segments PR and QS dissect $ABCD$ into four quadrilaterals, each of which has perpendicular diagonals. Show that the points P, Q, R, S are concyclic.

The 8^th^ Romanian Master of Mathematics

Fevereiro de 2016

Problema 1. Problem not found!

Problema 2. Problem not found!

Problema 3. Problem not found!

Fevereiro de 2016

Problema 4. Problem not found!

Problema 5. Problem not found!

Problema 6. Problem not found!

The 7$^{\text{th}}$ Romanian Master of Mathematics

Fevereiro de 2015

Problema 1. Problem not found!

Problema 2. Problem not found!

Problema 3. Problem not found!

Fevereiro de 2015

Problema 4. Problem not found!

Problema 5. Problem not found!

Problema 6. Problem not found!

The 6$^{\text{TH}}$ Romanian Master of Mathematics

Fevereiro de 2013

Problema 1. Problem not found!

Problema 2. Problem not found!

Problema 3. Problem not found!

Fevereiro de 2013

Problema 4. Problem not found!

Problema 5. Problem not found!

Problema 6. Problem not found!

The 5$^{\text{th}}$ Romanian Master of Mathematics

Fevereiro de 2012

Problema 1. Problem not found!

Problema 2. Problem not found!

Problema 3. Problem not found!

Fevereiro de 2012

Problema 4. Problem not found!

Problema 5. Problem not found!

Problema 6. Problem not found!

The 4^{th} Romanian Master of Mathematics

Fevereiro de 2011

Problema 1. Problem not found!

Problema 2. Problem not found!

Problema 3. Problem not found!

Fevereiro de 2011

Problema 4. Problem not found!

Problema 5. Problem not found!

Problema 6. Problem not found!

The 3^{th} Romanian Master of Mathematics

Fevereiro de 2010

Problema 1. Problem not found!

Problema 2. Problem not found!

Problema 3. Problem not found!

Fevereiro de 2010

Problema 4. Problem not found!

Problema 5. Problem not found!

Problema 6. Problem not found!

The 2$^{\text{ND}}$ Romanian Master of Mathematics

Fevereiro de 2009

Problema 1. Problem not found!

Problema 2. Problem not found!

Problema 3. Problem not found!

Fevereiro de 2009

Problema 4. Problem not found!

Problema 5. Problem not found!

Problema 6. Problem not found!

The 1st Romanian Master of Mathematics

Fevereiro de 2008

Problema 1. Problem not found!

Problema 2. Problem not found!

Problema 3. Problem not found!

Fevereiro de 2008

Problema 4. Problem not found!

Problema 5. Problem not found!

Problema 6. Problem not found!