Assignment

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Question 12.13.3.6 Explain why the experiment of tossing a coin three times is said to have binomial distribution

Solution: let X be the event of tossing coin and bernoulli distribution is

$$p_X(k) = \begin{cases} p & k = 0 \\ q = 1 - p & k = 1 \\ 0 & otherwise \end{cases} \tag{1}$$

$$\tag{2}$$

Then the Z transform of X is

$$M_X(z) = E[z^{-X}] = \sum_{k=-\infty}^{\infty} p_X(k) z^{-k} \tag{3}$$

$$= qz^0 + pz^{-1} \tag{4}$$

$$= q + pz^{-1} \tag{5}$$

Then for n trials, the Expectation is

$$M_X(z) = \left(q + pz^{-1}\right)^n \tag{6}$$

$$= \sum_{r=0}^{n} {}^nC_r \left(pz^{-1}\right)^r q^{n-r} \tag{7}$$

Mean of the given distribution is

$$E(X) = \frac{d(q + pz)^n}{dz}|_{z=1} \tag{8}$$

$$= np(q + pz)^{n-1}|_{z=1} \tag{9}$$

$$= np(q + p)^{n-1} \tag{10}$$

$$= np \tag{11}$$

which is the mean of a binomial distribution
$\therefore$ tossing 3 coins has binomial distribution