Assignment

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Consider the experiment of throwing a die. If a multiple of 3 comes up, throw the die again. If any other number comes up, toss a coin. Find the conditional probability of the event the coin shows a tail, given that at least one die shows a 3.

Solution: Let, the states S_0 and S_1 describe the outcomes of dice throws.
S_2 and S_3 describe the outcomes of coin toss.

$$S_0 = \Sigma(Y = k); k \in (3, 6) \tag{1}$$
$$S_1 = \Sigma(Y = k); k \in (1, 2, 4, 5) \tag{2}$$
$$S_2 = \text{Outcome of coin toss is heads} \tag{3}$$
$$S_3 = \text{Outcome of coin toss is tails} \tag{4}$$

Conditional Probability is that "The coin shows tails" given that "at least one die shows a 3". Since, a Markov chain does not depend on the past outcomes,

$$p_{S_3|S_0} = \Pr\left(X_n = S_3 | X_1 = S_0\right) \tag{5}$$

Transition Probability Matrix is given as,

$$\mathbf{P} = \begin{pmatrix} \frac{1}{3} & \frac{2}{3} & 0 & 0 \\ 0 & 0 & \frac{1}{2} & \frac{1}{2} \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \tag{6}$$

The given conditon is "3 occurs at least once", and we let occurrence of 3 as the initial state, Since Markov chain does not depend on past outcomes. So, $\Pr\left(X = S_0\right) = 1$ and 0 for all other states.
And State vector is ,

$$\mathbf{Q_0} = \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix} \tag{7}$$

The long term Probability that system will be in each state is called stationary state, and stationary state probability is given as,

$$\mathbf{Px} = \mathbf{x} \tag{8}$$

where $\mathbf{x}$ is steady state probability vector and $\mathbf{P}$ is the transition probability matrix.
So,after long time,

$$\mathbf{Q_1} = \mathbf{PQ_0} \tag{9}$$
$$\mathbf{Q_2} = \mathbf{PQ_1} \tag{10}$$
$$\vdots \tag{11}$$
$$\mathbf{Q_n} = \mathbf{PQ_{n-1}} \tag{12}$$

So substituting the state vectors we get,

$$\mathbf{Q_n} = \mathbf{P^nQ_0} \tag{13}$$
$$\tag{14}$$

applying limits to find the stationary probability vector,

$$\lim_{n\to\infty} \mathbf{Q_n} = \mathbf{P^nQ_0} \tag{15}$$

By substituting the values of Q_0 and P in the above equation, We get the steady state probability vector as,

$$\mathbf{x} = \begin{pmatrix} 0 \\ 0 \\ \frac{1}{2} \\ \frac{1}{2} \end{pmatrix} \tag{16}$$

So, the probability that "coin shows tails" given that "Die shows at least one 3" is,

$$p_{3|1} = \frac{1}{2} \tag{17}$$

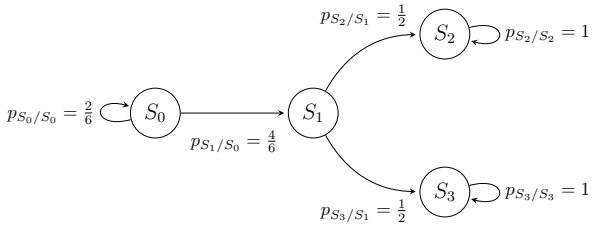

Fig. 1: State diagram generated using LatexTikZ