Assignment

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Question 9.3.3 Five cards are drawn successively with replacement from well shuffled deck of 52 cards, what is the probability that

1) all the five cards are spades?
2) only 3 cards are spades
3) None is a spade

Solution:

Binomial

Let X be a binomial random variable representing the number of spade cards among the five cards and with the parameters n and p as,

$$n = 5 \tag{1}$$

$$p = \frac{13}{52} \tag{2}$$

$$= 0.25 \tag{3}$$

PMF of the distribution is,

$$\Pr(X = k) = {}^{n}C_k p^k (1-p)^{n-k} \tag{4}$$

1)

$$k = 5 \tag{5}$$

$$\implies \Pr(X = 5) = {}^{5}C_5 (0.25)^5 (0.75)^0 \tag{6}$$

$$= 0.0009765625 \tag{7}$$

2)

$$k = 3 \tag{8}$$

$$\Pr(X = 3) = {}^{5}C_3 (0.25)^3 (0.75)^2 \tag{9}$$

$$= 0.087890625 \tag{10}$$

3)

$$k = 0 \tag{11}$$

$$\Pr(X = 0) = {}^{5}C_0 (0.25)^0 (0.75)^5 \tag{12}$$

$$= 0.2373046875 \tag{13}$$

Gaussian

$$X \sim Bin(n, p) \tag{14}$$

$$\sim Bin(5, 0.25) \tag{15}$$

Mean and varience of X are

$$\mu_X = np \tag{16}$$
$$= 1.25 \tag{17}$$
$$\sigma_X^2 = np\,(1-q) \tag{18}$$
$$= 0.9375 \tag{19}$$

Let,Z be a rondom variable with mean $\mu_Z = 0$ and $\sigma_Z = 1$, such that,

$$Z = \frac{X - \mu_X + 0.5}{\sigma_X} \tag{20}$$

0.5 is added for correction.
Z caoverges to normal distribution for large value of n

$$f\,(x) = \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}} \tag{21}$$

And the Q funtion is

$$Q\,(x) = \Pr\,(X > x) \tag{22}$$

1) all five cards are spades