

# Assignment 1

## Probability And Random Processes

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### 1 QUESTION 1.1.7

find the angles **A, B, C**, given that

$$\cos A \triangleq \frac{(\mathbf{B} - \mathbf{A})^\top (\mathbf{C} - \mathbf{A})}{\|\mathbf{B} - \mathbf{A}\| \|\mathbf{C} - \mathbf{A}\|} \quad (1)$$

### 2 SOLUTION

From the given values of **A, B, C**,

1) Finding the value of angle A

$$\mathbf{B} - \mathbf{A} = \begin{pmatrix} -5 \\ 7 \end{pmatrix} \quad (2)$$

and

$$\mathbf{C} - \mathbf{A} = \begin{pmatrix} -4 \\ -4 \end{pmatrix} \quad (3)$$

also calculating the values of norms

$$\|\mathbf{B} - \mathbf{A}\| = \sqrt{74} \quad (4)$$

$$\|\mathbf{C} - \mathbf{A}\| = \sqrt{32} \quad (5)$$

and by doing matrix multiplication we get,

$$\begin{aligned} (\mathbf{B} - \mathbf{A})^\top (\mathbf{C} - \mathbf{A}) &= \begin{pmatrix} -5 & 7 \end{pmatrix} \begin{pmatrix} -4 \\ -4 \end{pmatrix} \\ &= -8 \end{aligned} \quad (6)$$

so

$$\cos A = \frac{-8}{\sqrt{74} \sqrt{32}} \quad (7)$$

$$= \frac{-1}{\sqrt{37}} \quad (8)$$

$$\Rightarrow A = \cos^{-1} \frac{-1}{\sqrt{37}} \quad (9)$$

2) Finding the value of angle B

$$\mathbf{C} - \mathbf{B} = \begin{pmatrix} 1 \\ -11 \end{pmatrix} \quad (10)$$

and

$$\mathbf{A} - \mathbf{B} = \begin{pmatrix} 5 \\ -7 \end{pmatrix} \quad (11)$$

also calculating the values of norms

$$\|\mathbf{C} - \mathbf{B}\| = \sqrt{122} \quad (12)$$

$$\|\mathbf{A} - \mathbf{B}\| = \sqrt{74} \quad (13)$$

and by doing matrix multiplication we get,

$$\begin{aligned} (\mathbf{C} - \mathbf{B})^\top (\mathbf{A} - \mathbf{B}) &= \begin{pmatrix} 1 & -11 \end{pmatrix} \begin{pmatrix} 5 \\ -7 \end{pmatrix} \\ &= 82 \end{aligned} \quad (14)$$

so

$$\cos B = \frac{82}{\sqrt{74} \sqrt{122}} \quad (15)$$

$$= \frac{41}{\sqrt{2257}} \quad (16)$$

$$\Rightarrow B = \cos^{-1} \frac{41}{\sqrt{2257}} \quad (17)$$

3) Finding the value of angle C

$$\mathbf{A} - \mathbf{C} = \begin{pmatrix} 4 \\ 4 \end{pmatrix} \quad (18)$$

and

$$\mathbf{B} - \mathbf{C} = \begin{pmatrix} -1 \\ 11 \end{pmatrix} \quad (19)$$

also calculating the values of norms

$$\|\mathbf{A} - \mathbf{C}\| = \sqrt{32} \quad (20)$$

$$\|\mathbf{B} - \mathbf{C}\| = \sqrt{122} \quad (21)$$

and by doing matrix multiplication we get,

$$\begin{aligned} (\mathbf{A} - \mathbf{C})^\top (\mathbf{B} - \mathbf{C}) &= \begin{pmatrix} 4 & 4 \end{pmatrix} \begin{pmatrix} -1 \\ 11 \end{pmatrix} \\ &= 40 \end{aligned} \quad (22)$$

so

$$\cos C = \frac{40}{\sqrt{32} \sqrt{122}} \quad (23)$$

$$= \frac{5}{\sqrt{61}} \quad (24)$$

$$\Rightarrow C = \cos^{-1} \frac{5}{\sqrt{61}} \quad (25)$$