Assignment

Barath surya M — EE22BTECH11014

Question 9.3.3 Five cards are drawn successively with replacement from well shuffled deck of 52 cards, what is the probability that

1) all the five cards are spades?
2) only 3 cards are spades
3) None is a spade

Solution:
Binomial
Let X be a binomial random variable representing the number of spade cards among the five cards and with the parameters n and p as,

$$n = 5 \tag{1}$$
$$p = \frac{13}{52} \tag{2}$$
$$= 0.25 \tag{3}$$

PMF of the distribution is,

$$\Pr(X = k) = {}^nC_k p^k (1 - p)^{n-k} \tag{4}$$

1)

$$k = 5 \tag{5}$$
$$\implies \Pr(X = 5) = {}^5C_5 (0.25)^5 (0.75)^0 \tag{6}$$
$$= 0.0009765625 \tag{7}$$

2)

$$k = 3 \tag{8}$$
$$\implies \Pr(X = 3) = {}^5C_3 (0.25)^3 (0.75)^2 \tag{9}$$
$$= 0.087890625 \tag{10}$$

3)

$$k = 0 \tag{11}$$
$$\implies \Pr(X = 0) = {}^5C_0 (0.25)^0 (0.75)^5 \tag{12}$$
$$= 0.2373046875 \tag{13}$$

Gaussian

$$X \sim Bin(n, p) \tag{14}$$
$$\sim Bin(5, 0.25) \tag{15}$$

Mean and varience of X are

$$\mu_X = np \tag{16}$$
$$= 1.25 \tag{17}$$
$$\sigma_X^2 = np\left(1-q\right) \tag{18}$$
$$= 0.9375 \tag{19}$$

Let,Z be a rondom variable with mean $\mu_Z = 0$ and $\sigma_Z = 1$, such that,

$$Z = \frac{X-\mu_X}{\sigma_X} \tag{20}$$

Z caoverges to normal distribution for large value of n

$$f\left(x\right) = \frac{1}{\sqrt{2\pi}}e^{-\frac{x^2}{2}} \tag{21}$$

And the Q funtion is

$$Q\left(x\right) = \Pr\left(Z > x\right) \tag{22}$$

1)

$$X = 5 \tag{23}$$

$$\Pr\left(Z = \frac{X-\mu_X}{\sigma_X}\right) \approx \Pr\left(\frac{X+0.5-\mu_X}{\sigma_X} < Z < \frac{X-0.5-\mu_X}{\sigma_X}\right) \tag{24}$$
$$\approx \Pr\left(Z < \frac{X+0.5-\mu_X}{\sigma_X}\right) - \Pr\left(Z < \frac{X-0.5-\mu_X}{\sigma_X}\right) \tag{25}$$
$$\approx \Pr\left(Z > \frac{X-0.5-\mu_X}{\sigma_X}\right) - \Pr\left(Z > \frac{X+0.5-\mu_X}{\sigma_X}\right) \tag{26}$$
$$\approx Q\left(\frac{X-0.5-\mu_X}{\sigma_X}\right) - Q\left(\frac{X+0.5-\mu_X}{\sigma_X}\right) \tag{27}$$
$$\approx Q\left(3.356\right) - Q\left(4.389\right) \tag{28}$$
$$\approx 0.0003888 \tag{29}$$

2)

$$X = 3 \tag{30}$$

$$\Pr\left(Z = \frac{X-\mu_X}{\sigma_X}\right) \approx \Pr\left(\frac{X+0.5-\mu_X}{\sigma_X} < Z < \frac{X-0.5-\mu_X}{\sigma_X}\right) \tag{31}$$
$$\approx \Pr\left(Z < \frac{X+0.5-\mu_X}{\sigma_X}\right) - \Pr\left(Z < \frac{X-0.5-\mu_X}{\sigma_X}\right) \tag{32}$$
$$\approx \Pr\left(Z > \frac{X-0.5-\mu_X}{\sigma_X}\right) - \Pr\left(Z > \frac{X+0.5-\mu_X}{\sigma_X}\right) \tag{33}$$
$$\approx Q\left(\frac{X-0.5-\mu_X}{\sigma_X}\right) - Q\left(\frac{X+0.5-\mu_X}{\sigma_X}\right) \tag{34}$$
$$\approx Q\left(1.2909\right) - Q\left(2.3237\right) \tag{35}$$
$$\approx 0.08828 \tag{36}$$

3)

$$X = 0 \tag{37}$$

$$\Pr\left(Z=\frac{X-\mu_X}{\sigma_X}\right)\approx \Pr\left(\frac{X+0.5-\mu_X}{\sigma_X}<Z<\frac{X-0.5-\mu_X}{\sigma_X}\right) \tag{38}$$

$$\approx \Pr\left(Z<\frac{X+0.5-\mu_X}{\sigma_X}\right)-\Pr\left(Z<\frac{X-0.5-\mu_X}{\sigma_X}\right) \tag{39}$$

$$\approx \Pr\left(Z>\frac{X-0.5-\mu_X}{\sigma_X}\right)-\Pr\left(Z>\frac{X+0.5-\mu_X}{\sigma_X}\right) \tag{40}$$

$$\approx Q\left(\frac{X-0.5-\mu_X}{\sigma_X}\right)-Q\left(\frac{X+0.5-\mu_X}{\sigma_X}\right) \tag{41}$$

$$\approx Q(-1.8073)-Q(-0.7745) \tag{42}$$

$$\approx 0.1839 \tag{43}$$

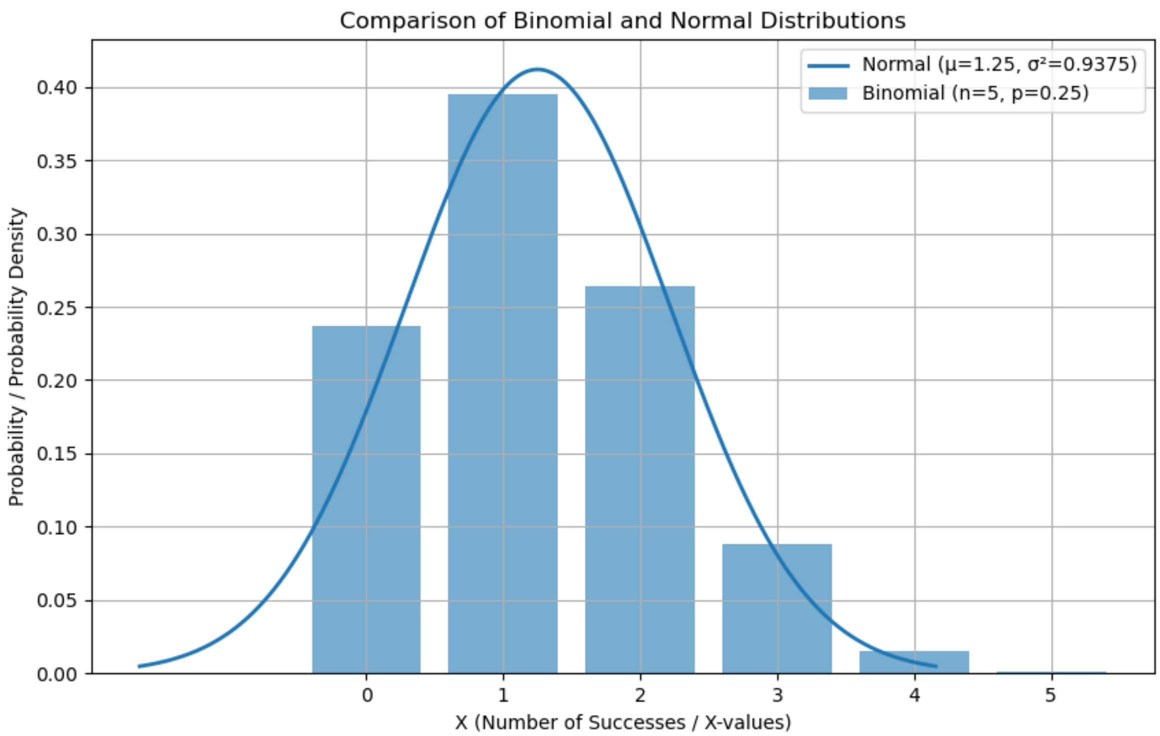

Fig. 1: Binomial and gaussian distribution