Assignment

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Question 9.3.3 Five cards are drawn successively with replacement from well shuffled deck of 52 cards, what is the probability that

1) all the five cards are spades?
2) only 3 cards are spades
3) None is a spade

Solution:
Binomial
Let X be a binomial random variable representing the number of spade cards among the five cards and with the parameters n and p as,

$$n = 5 \tag{1}$$
$$p = \frac{13}{52} \tag{2}$$
$$= 0.25 \tag{3}$$

PMF of the distribution is,

$$\Pr(X = k) = {}^{n}C_{k} p^{k} (1 - p)^{n-k} \tag{4}$$

1)

$$k = 5 \tag{5}$$
$$\implies \Pr(X = 5) = {}^{5}C_{5} (0.25)^{5} (0.75)^{0} \tag{6}$$
$$= 0.0009765625 \tag{7}$$

2)

$$k = 3 \tag{8}$$
$$\Pr(X = 3) = {}^{5}C_{3} (0.25)^{3} (0.75)^{2} \tag{9}$$
$$= 0.087890625 \tag{10}$$

3)

$$k = 0 \tag{11}$$
$$\Pr(X = 0) = {}^{5}C_{0} (0.25)^{0} (0.75)^{5} \tag{12}$$
$$= 0.2373046875 \tag{13}$$

Gaussian

$$X \sim Bin(n, p) \tag{14}$$
$$\sim Bin(5, 0.25) \tag{15}$$

Mean and varience of X are

$$\mu_X = np \tag{16}$$
$$= 1.25 \tag{17}$$
$$\sigma_X^2 = np(1-q) \tag{18}$$
$$= 0.9375 \tag{19}$$

Let,Z be a rondom variable with mean $\mu_Z = 0$ and $\sigma_Z = 1$, such that,

$$Z = \frac{X - \mu_X}{\sigma_X} \tag{20}$$

Z caoverges to normal distribution for large value of n

$$f(x) = \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}} \tag{21}$$

And the Q funtion is

$$Q(x) = \Pr(Z > x) \tag{22}$$

1)

$$X = 5 \tag{23}$$

$$\Pr\left(Z = \frac{X - \mu_X}{\sigma_X}\right) \approx \Pr\left(\frac{X + 0.5 - \mu_X}{\sigma_X} < Z < \frac{X - 0.5 - \mu_X}{\sigma_X}\right) \tag{24}$$
$$\approx \Pr\left(Z < \frac{X + 0.5 - \mu_X}{\sigma_X}\right) - \Pr\left(Z < \frac{X - 0.5 - \mu_X}{\sigma_X}\right) \tag{25}$$
$$\approx \Pr\left(Z > \frac{X - 0.5 - \mu_X}{\sigma_X}\right) - \Pr\left(Z > \frac{X + 0.5 - \mu_X}{\sigma_X}\right) \tag{26}$$
$$\approx Q\left(\frac{X - 0.5 - \mu_X}{\sigma_X}\right) - Q\left(\frac{X + 0.5 - \mu_X}{\sigma_X}\right) \tag{27}$$
$$\approx Q(3.356) - Q(4.389) \tag{28}$$
$$\approx 0.0003888 \tag{29}$$

2)

$$X = 3 \tag{30}$$

$$\Pr\left(Z = \frac{X - \mu_X}{\sigma_X}\right) \approx \Pr\left(\frac{X + 0.5 - \mu_X}{\sigma_X} < Z < \frac{X - 0.5 - \mu_X}{\sigma_X}\right) \tag{31}$$
$$\approx \Pr\left(Z < \frac{X + 0.5 - \mu_X}{\sigma_X}\right) - \Pr\left(Z < \frac{X - 0.5 - \mu_X}{\sigma_X}\right) \tag{32}$$
$$\approx \Pr\left(Z > \frac{X - 0.5 - \mu_X}{\sigma_X}\right) - \Pr\left(Z > \frac{X + 0.5 - \mu_X}{\sigma_X}\right) \tag{33}$$
$$\approx Q\left(\frac{X - 0.5 - \mu_X}{\sigma_X}\right) - Q\left(\frac{X + 0.5 - \mu_X}{\sigma_X}\right) \tag{34}$$
$$\approx Q(1.2909) - Q(2.3237) \tag{35}$$
$$\approx 0.08828 \tag{36}$$

3)

$$X = 0 \tag{37}$$

$$\Pr\left(Z = \frac{X - \mu_X}{\sigma_X}\right) \approx \Pr\left(\frac{X + 0.5 - \mu_X}{\sigma_X} < Z < \frac{X - 0.5 - \mu_X}{\sigma_X}\right) \tag{38}$$

$$\approx \Pr\left(Z < \frac{X + 0.5 - \mu_X}{\sigma_X}\right) - \Pr\left(Z < \frac{X - 0.5 - \mu_X}{\sigma_X}\right) \tag{39}$$

$$\approx \Pr\left(Z > \frac{X - 0.5 - \mu_X}{\sigma_X}\right) - \Pr\left(Z > \frac{X + 0.5 - \mu_X}{\sigma_X}\right) \tag{40}$$

$$\approx Q\left(\frac{X - 0.5 - \mu_X}{\sigma_X}\right) - Q\left(\frac{X + 0.5 - \mu_X}{\sigma_X}\right) \tag{41}$$

$$\approx Q(-1.8073) - Q(-0.7745) \tag{42}$$

$$\approx 0.1839 \tag{43}$$

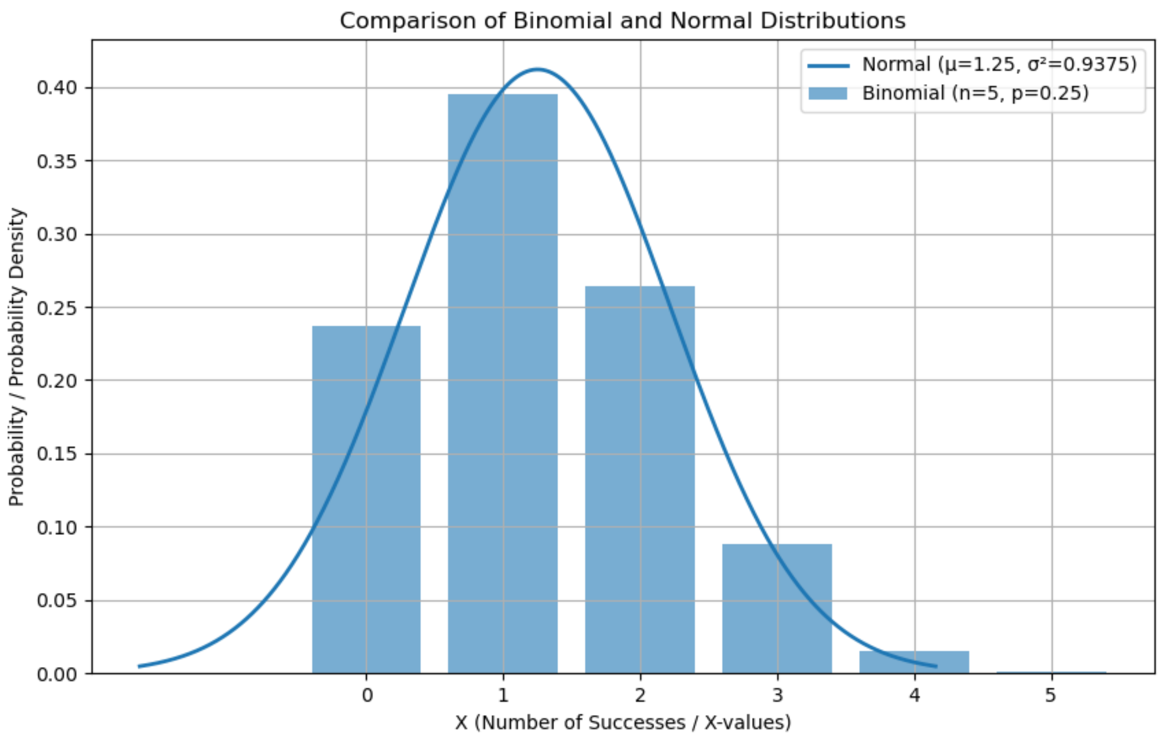

Fig. 1: Binomial and gaussian distribution