

Assignment

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Question 12.13.3.6 Explain why the experiment of tossing a coin three times is said to have binomial distribution

Solution: let X be the event of tossing coin and bernoulli distribution is

$$p_X(k) = \begin{cases} p & k = 0 \\ q = 1 - p & k = 1 \\ 0 & \text{otherwise} \end{cases} \quad (1)$$

(2)

Then the Z transform of X is

$$M_X(z) = E[z^{-X}] = \sum_{k=-\infty}^{\infty} p_X(k) z^{-k} \quad (3)$$

$$= qz^0 + pz^{-1} \quad (4)$$

$$= q + pz^{-1} \quad (5)$$

Then for n trials, the Expectation is

$$M_X(z) = (q + pz^{-1})^n \quad (6)$$

Mean of the given distribution is

$$E(X) = \frac{d(q + pz)^n}{dz} \Big|_{z=1} \quad (7)$$

$$= np(q + pz)^{n-1} \Big|_{z=1} \quad (8)$$

$$= np(q + p)^{n-1} \quad (9)$$

$$= np \quad (10)$$

which is the mean of a binomial distribution

\therefore Tossing 3 coins also has a binomial distribution