Assignment

Barath surya M — EE22BTECH11014

Question 12.13.3.6 Explain why the experiment of tossing a coin three times is said to have binomial distribution

Solution: Let X be discrete random variable which takes values $X \in \{0, 1, 2, 3 \dots\}$ and pmf is

$$p_i = \Pr(X = i) \tag{1}$$

Then Expectation is

$$E\left[z^{-X}\right] = \sum_{i=0}^{\infty} p_i z^{-i} \tag{2}$$

let X be the event of tossing coin and bernoulli distribution is

$$X = \begin{cases} 1 & \text{toss is heads ; probability } p \\ 0 & \text{toss is tails ; probability } q = 1 - p \end{cases} \tag{3}$$

$$E\left[z^{-X}\right] = qz^0 + pz^{-1} \tag{4}$$

$$= q + pz^{-1} \tag{5}$$

Then for n trials, the Expectation is

$$E\left[z^{-X}\right]^n = E\left[p_i^n\right] \tag{6}$$

$$= \left(q + pz^{-1}\right)^n \tag{7}$$

$$= \sum_{r=0}^{n} {}^nC_r \left(pz^{-1}\right)^r q^{n-r} \tag{8}$$

Now by getting pmf by comparing coefficients

$$\Pr(X = i) = {}^nC_i p^i (1 - p)^{n-i} \tag{9}$$

which is in binomial distribution

$\therefore$ tossing 3 coins has binomial distribution