

Assignment

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Question 9.3.3 There are 5 % defective items in a large bulk of items. What is the probability that a sample of 10 items will include not more than one defective item ?

Solution:

Binomial

$$p = \frac{5}{100} \quad (1)$$

$$= 0.05 \quad (2)$$

$$n = 10 \quad (3)$$

Let X be a Binomial random variable with parameters p and n

$$\Pr(X = k) = {}^nC_k p^k (1 - p)^{n-k} \quad (4)$$

$$= {}^{10}C_k (0.05)^k (0.95)^{10-k} \quad (5)$$

CDF of X

$$F_X(n) = \Pr(X \leq n) \quad (6)$$

$$= \sum_{k=0}^n \Pr(X = k) \quad (7)$$

$$= \sum_{k=0}^n {}^{10}C_k (0.05)^k (0.95)^{10-k} \quad (8)$$

Since, according to question n here equals,

$$\Rightarrow F_X(1) = \sum_{k=0}^1 {}^{10}C_k (0.05)^k (0.95)^{10-k} \quad (9)$$

$$= 0.9138 \quad (10)$$

Gaussian

$$X \sim \text{Bin}(n, p) \quad (11)$$

$$\sim \text{Bin}(10, 0.05) \quad (12)$$

Mean and Variance of X are

$$\mu_X = np \quad (13)$$

$$= 0.5 \quad (14)$$

$$\sigma_X^2 = np(1 - p) \quad (15)$$

$$= 0.475 \quad (16)$$

Let, Z be a random variable with mean $\mu_Z = 0$ and $\sigma_Z = 1$ such that,

$$Z = \frac{X - \mu_X}{\sigma_X} \quad (17)$$

$$\approx 0.7254 \quad (18)$$

Z converges to normal distribution for large value of n

$$f(x) = \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}} \quad (19)$$

And the Q function is defined as,

$$Q(x) = \Pr(X > x) \quad (20)$$

We need

$$\Pr(Z < x) = 1 - \Pr(Z > x) \quad (21)$$

$$= 1 - Q(x) \quad (22)$$

Upon computation for $Z = 0.7254$

$$\Pr(Z < 0.7254) = 1 - 0.2340 \quad (23)$$

$$= 0.7659 \quad (24)$$

Therefore the Gaussian approximation for the given question is 0.766