

Assignment

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Question 12.13.3.90

Suppose a random variable X follows binomial distribution with parameters n and p , where $0 < p < 1$. If $\Pr(X = r)/\Pr(X = n - r)$ is independent of n and r , then p equals,

- 1) $\frac{1}{2}$
- 2) $\frac{1}{3}$
- 3) $\frac{1}{5}$
- 4) $\frac{1}{7}$

Solution: In a binomial distribution, The PMF of random variable X ,

$$\Pr(X = r) = {}^nC_r p^r (1 - p)^{n-r} \quad (1)$$

Therefore,

$$\frac{\Pr(X = r)}{\Pr(X = n - r)} = \frac{{}^nC_r p^r (1 - p)^{n-r}}{{}^nC_{n-r} p^{n-r} (1 - p)^r} \quad (2)$$

$$= \frac{p^r (1 - p)^{n-r}}{p^{n-r} (1 - p)^r} \quad (3)$$

since

$${}^nC_r = {}^nC_{n-r} \quad (4)$$

$$\Rightarrow \frac{\Pr(X = r)}{\Pr(X = n - r)} = \left(\frac{1 - p}{p} \right)^{n-2r} \quad (5)$$

Since it is independent of n and r ,

$$\frac{1 - p}{p} = 1 \quad (6)$$

$$\Rightarrow p = \frac{1}{2} \quad (7)$$