Assignment

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Question 9.3.3 There are 5 % defective items in a large bulk of items. What is the probability that a sample of 10 items will include not more than one defective item ?

Solution:

Binomial

$$p = \frac{5}{100} \tag{1}$$
$$= 0.05 \tag{2}$$
$$n = 10 \tag{3}$$

Let X be a Binomial random variable with parameters p and n

$$\Pr(X = k) = {}^{n}C_k p^k (1-p)^{n-k} \tag{4}$$
$$= {}^{10}C_k (0.05)^k (0.95)^{10-k} \tag{5}$$

CDF of X

$$F_X(n) = \Pr(X \leq n) \tag{6}$$
$$= \sum_{k=0}^{n} \Pr(X = k) \tag{7}$$
$$= \sum_{k=0}^{n} {}^{10}C_k (0.05)^k (0.95)^{10-k} \tag{8}$$

Since, according to question n here equals,

$$\implies F_X(1) = \sum_{k=0}^{1} {}^{10}C_k (0.05)^k (0.95)^{10-k} \tag{9}$$
$$= 0.9138 \tag{10}$$

Gaussian

$$X \sim Bin(n, p) \tag{11}$$
$$\sim Bin(10, 0.05) \tag{12}$$

Mean and Varience of X are

$$\mu_X = np \tag{13}$$
$$= 0.5 \tag{14}$$
$$\sigma_X^2 = np(1-p) \tag{15}$$
$$= 0.475 \tag{16}$$

Let, Z be a random variable with mean $\mu_Z = 0$ and $\sigma_Z = 1$ such that,

$$Z = \frac{X - \mu_X + 0.5}{\sigma_X} \tag{17}$$
$$\approx 1.4509 \tag{18}$$

0.5 is added for correction.
Z converges to normal distribution for large value of n

$$f(x) = \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}} \tag{19}$$

And the Q funtion is defined as,

$$Q(x) = \Pr(X > x) \tag{20}$$

We need

$$\Pr(Z < x) = 1 - \Pr(Z > x) \tag{21}$$
$$= 1 - Q(x) \tag{22}$$

Upon computation for $Z = 1.4509$

$$\Pr(Z < 1.4509) = 1 - 0.0733 \tag{23}$$
$$= 0.9266 \tag{24}$$

Therefore the Gaussian approximation for the given question is 0.9266