Assignment

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Question 1.3.3

Find the equation of altitudes BE_1 and CF_1 to the sides AC and AB

Solution:

1) Finding the foot of perpendicular drawn from **B**

$$\mathbf{E_1} = \mathbf{C} + (\mathbf{A} - \mathbf{C}) \frac{(\mathbf{B} - \mathbf{C})^\top (\mathbf{A} - \mathbf{C})}{\|\mathbf{C} - \mathbf{A}\|^2} \tag{1}$$

$$= \begin{pmatrix} -3 \\ -5 \end{pmatrix} + \begin{pmatrix} 4 \\ 4 \end{pmatrix} \frac{\begin{pmatrix} -1 & 11 \end{pmatrix} \begin{pmatrix} 4 \\ 4 \end{pmatrix}}{\left\| \begin{pmatrix} 4 \\ 4 \end{pmatrix} \right\|^2} \tag{2}$$

$$= \begin{pmatrix} -3 \\ -5 \end{pmatrix} + \begin{pmatrix} 4 \\ 4 \end{pmatrix} \begin{pmatrix} \frac{40}{32} \end{pmatrix} \tag{3}$$

$$= \begin{pmatrix} -3 \\ -5 \end{pmatrix} + \begin{pmatrix} 5 \\ 5 \end{pmatrix} \tag{4}$$

$$= \begin{pmatrix} 2 \\ 0 \end{pmatrix} \tag{5}$$

So the equation of the line in parametric form is

$$x = \mathbf{a} + k\mathbf{b} \tag{6}$$

substituting the given values gives us,

$$BE_1 = \mathbf{B} + k(\mathbf{E_1} - \mathbf{B}) \tag{7}$$

$$= \begin{pmatrix} -4 \\ 6 \end{pmatrix} + k \begin{pmatrix} 6 \\ -6 \end{pmatrix} \tag{8}$$

2) Finding the foot of perpendicular drawn from **C**

$$\mathbf{F_1} = \mathbf{A} + (\mathbf{B} - \mathbf{A}) \frac{(\mathbf{B} - \mathbf{A})^\top (\mathbf{C} - \mathbf{A})}{\|\mathbf{B} - \mathbf{A}\|^2} \tag{9}$$

$$= \begin{pmatrix} 1 \\ -1 \end{pmatrix} + \begin{pmatrix} -5 \\ 7 \end{pmatrix} \frac{\begin{pmatrix} -5 & 7 \end{pmatrix} \begin{pmatrix} -4 \\ -4 \end{pmatrix}}{\left\| \begin{pmatrix} -5 \\ 7 \end{pmatrix} \right\|^2} \tag{10}$$

$$= \begin{pmatrix} 1 \\ -1 \end{pmatrix} + \begin{pmatrix} -5 \\ 7 \end{pmatrix} \begin{pmatrix} \frac{-8}{\sqrt{74}} \end{pmatrix} \tag{11}$$

$$= \begin{pmatrix} 1.5405 \\ -1.7567 \end{pmatrix} \tag{12}$$

So the equation of the line in parametric form is

$$x = \mathbf{a} + k\mathbf{b} \tag{13}$$

substituting the given values gives us,

$$CF_1 = \mathbf{C} + k(\mathbf{F_1} - \mathbf{C}) \tag{14}$$

$$= \begin{pmatrix} -3 \\ -5 \end{pmatrix} + k \begin{pmatrix} 4.5405 \\ 2.4324 \end{pmatrix} \tag{15}$$

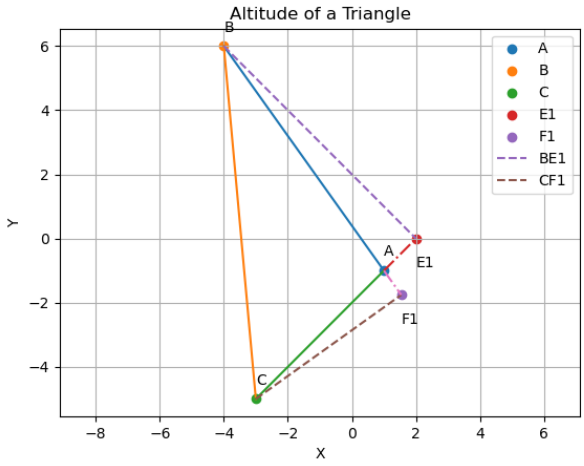

Fig. 2. Altitude BE1 and CF1 generated by python