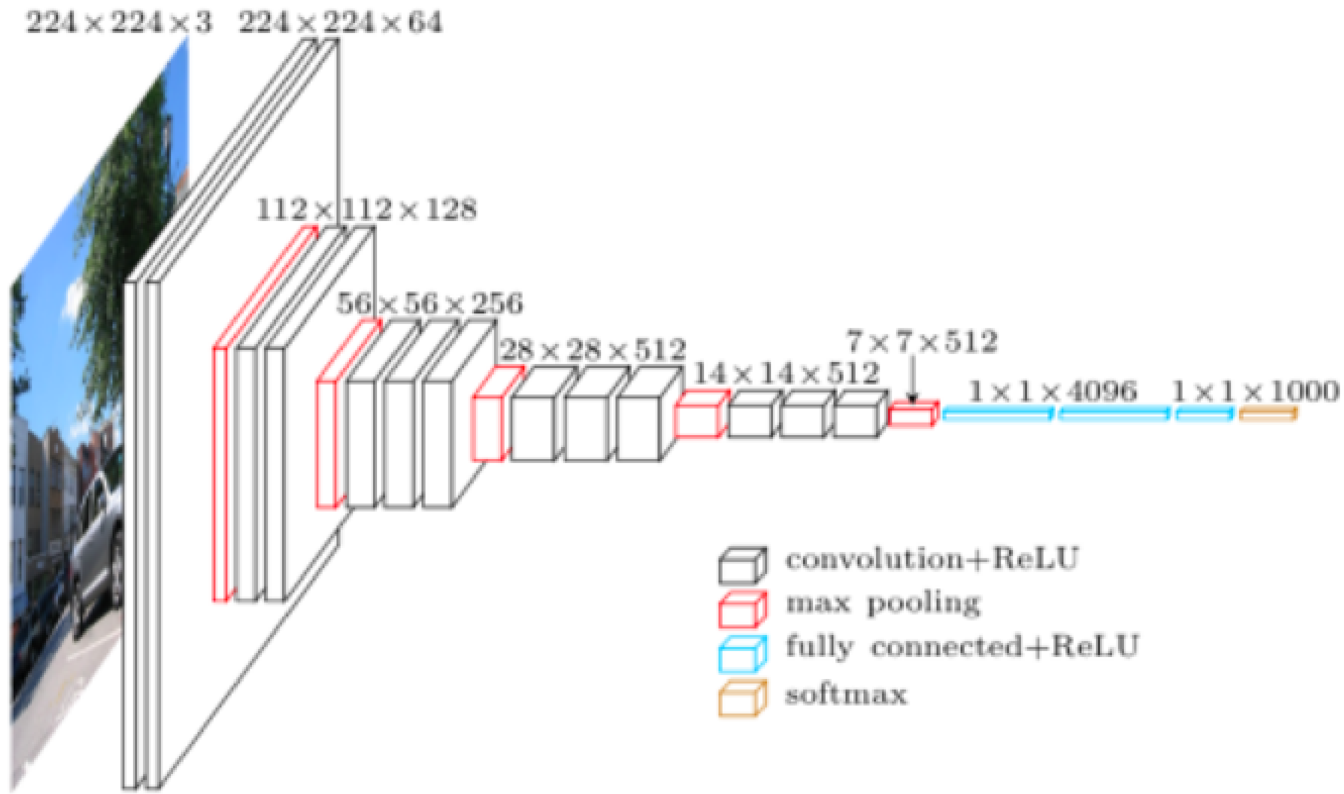
TTIC 31230, Fundamentals of Deep Learning

David McAllester, Winter 2019

Convolutional Neural Networks (CNNs)

What is a CNN?

VGG, Zisserman, 2014



Review: Einstein Notation for Linear Threshold Layer

$$y = \sigma\left(Wx - B\right)$$

is an abbreviation for

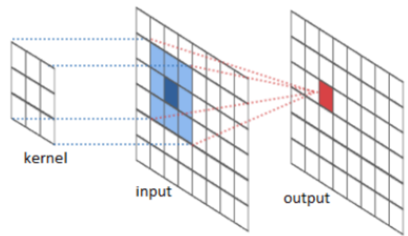
$$y[b,j] = \sigma\left(\left(\sum_i W[j,i]\,x[b,i]\right) - B[j]\right)$$

Think of this as a separate assignment statement for each (b, j).

Each $y[b,j]$ is the output of a "linear threshold unit".

Einstein notation makes all indeces and summations explicit.

A Convolution Layer



$$W[\Delta x, \Delta y, i, j] \qquad L_{\mathrm{IN}}[b, x, y, i] \qquad L_{\mathrm{out}}[b, x, y, j]$$

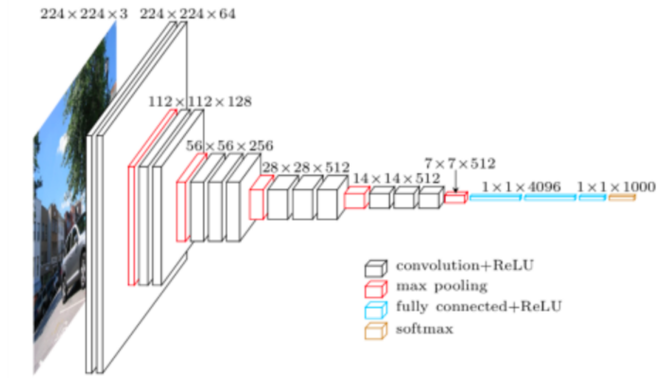
River Trail Documentation

$\texttt{Procedure CONV}(W[\Delta x, \Delta y, i, j], B[j], L_{\mathrm{in}}[b, x, y, i])$

Return $L_{\mathrm{out}}[b, x, y, j]$

$$= \sigma\left(\left(\sum_{\Delta x, \Delta y, i} W[\Delta x, \Delta y, i, j]\; L_{\mathrm{in}}[b, x + \Delta x, y + \Delta y, i]\right) - B[j]\right)$$

A Convolution Layer

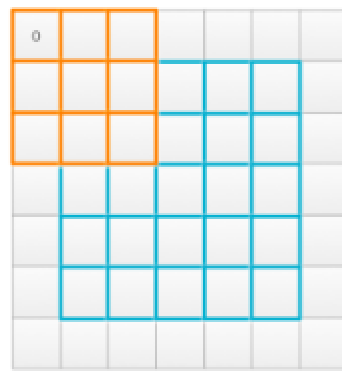


Each box is a tensor $L[b, x, y, i]$

$$
\begin{aligned}
&L_{\mathrm{out}}[b, x, y, j] \\
&= \sigma\left(\left(\sum_{\Delta x, \Delta y, i} W[\Delta x, \Delta y, i, j]\; L_{\mathrm{in}}[b, x + \Delta x, y + \Delta y, i]\right) - B[j]\right)
\end{aligned}
$$

Each $L_{\mathrm{out}}[b, x, y, j]$ is the output of a single linear threshold unit.

Padding



Jonathan Hui

If we pad the input with zeros then the input and output can have the same spatial dimensions.

Zero Padding in NumPy

In NumPy we can add a zero padding of width p to an image as follows:

```
padded = np.zeros(W + 2*p,  H + 2*p)

padded[p:W+p, p:H+p] = x
```

Padding

$\texttt{Procedure CONV}(\Phi,\ L_{\text{in}}[b,x,y,i],\ \text{padding } p)$

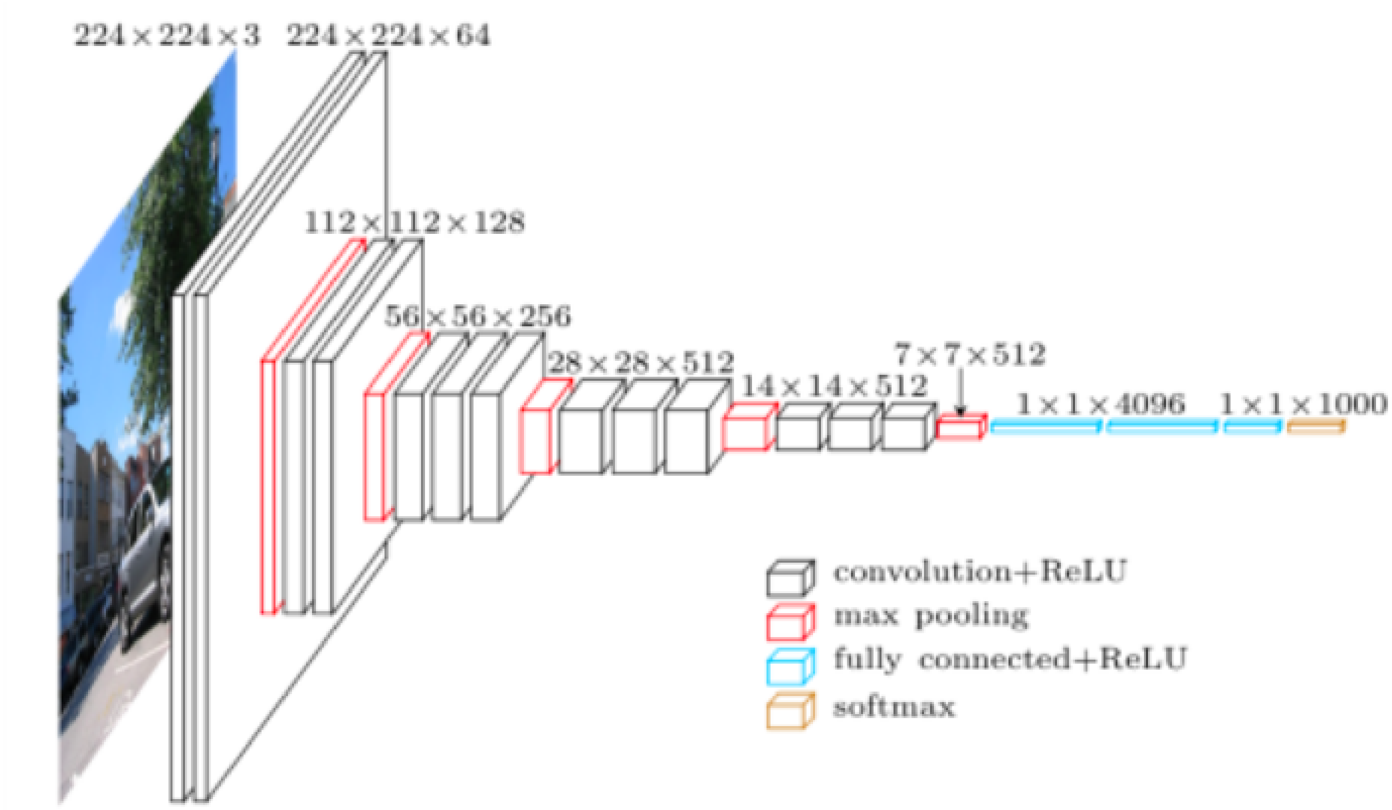
$L'_{\text{in}} = \text{Padd}(L_{\text{in}},\ p)$

$L_{\text{out}}[b,x,y,j] =$

$$\sigma\left(\left(\sum_{\Delta x,\Delta y,i}\ W[\Delta x,\Delta y,i,j]\ L'_{\text{in}}[b,x+\Delta x,y+\Delta y,i]\right) - B[j]\right)$$

Return $L_{\text{out}}[b,x,y,j]$

Reducing Spatial Dimention



Strides

We can move the filter by a "stride" s for each spatial step.

$$L_{\text{out}}[b, x, y, j] =$$

$$\sigma\left(\left(\sum_{\Delta x, \Delta y, i} W[\Delta x, \Delta y, i, j] L_{\text{in}}[b, s*x + \Delta x, s*y + \Delta y, i]\right) - B[j]\right)$$

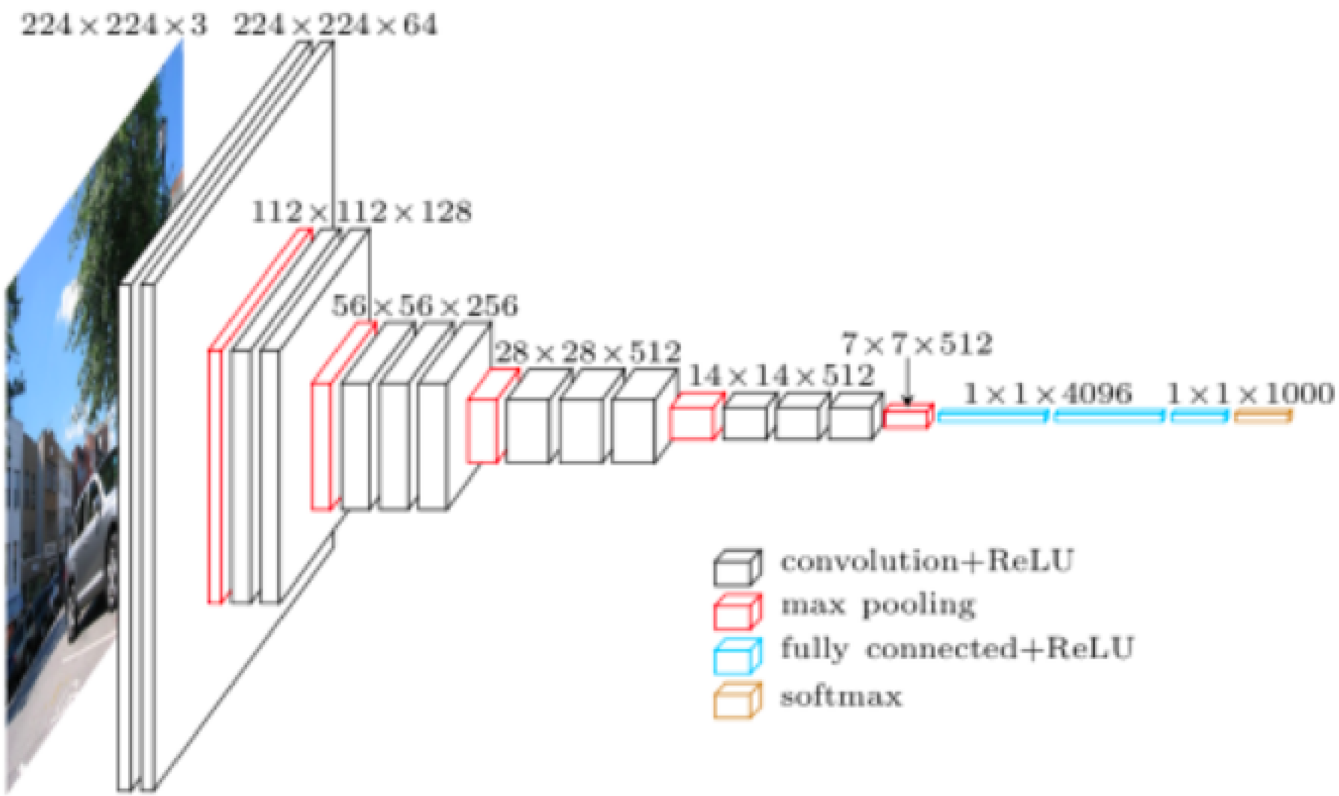
For strides greater than 1 the spatial dimention is reduced.

Max Pooling

$$L_{\mathrm{out}}[b, x, y, i] = \max_{\Delta x, \Delta y} L_{\mathrm{in}}[b, s * x + \Delta x,\ s * y + \Delta y,\ i]$$

This is typically done with a stride greater than one so that the image dimension is reduced.

Fully Connected (FC) Layers



Fully Connected (FC) Layers

We reshape $L_{\text{in}}[b, x, y, i]$ to $L_{\text{in}}[b, i']$ and then

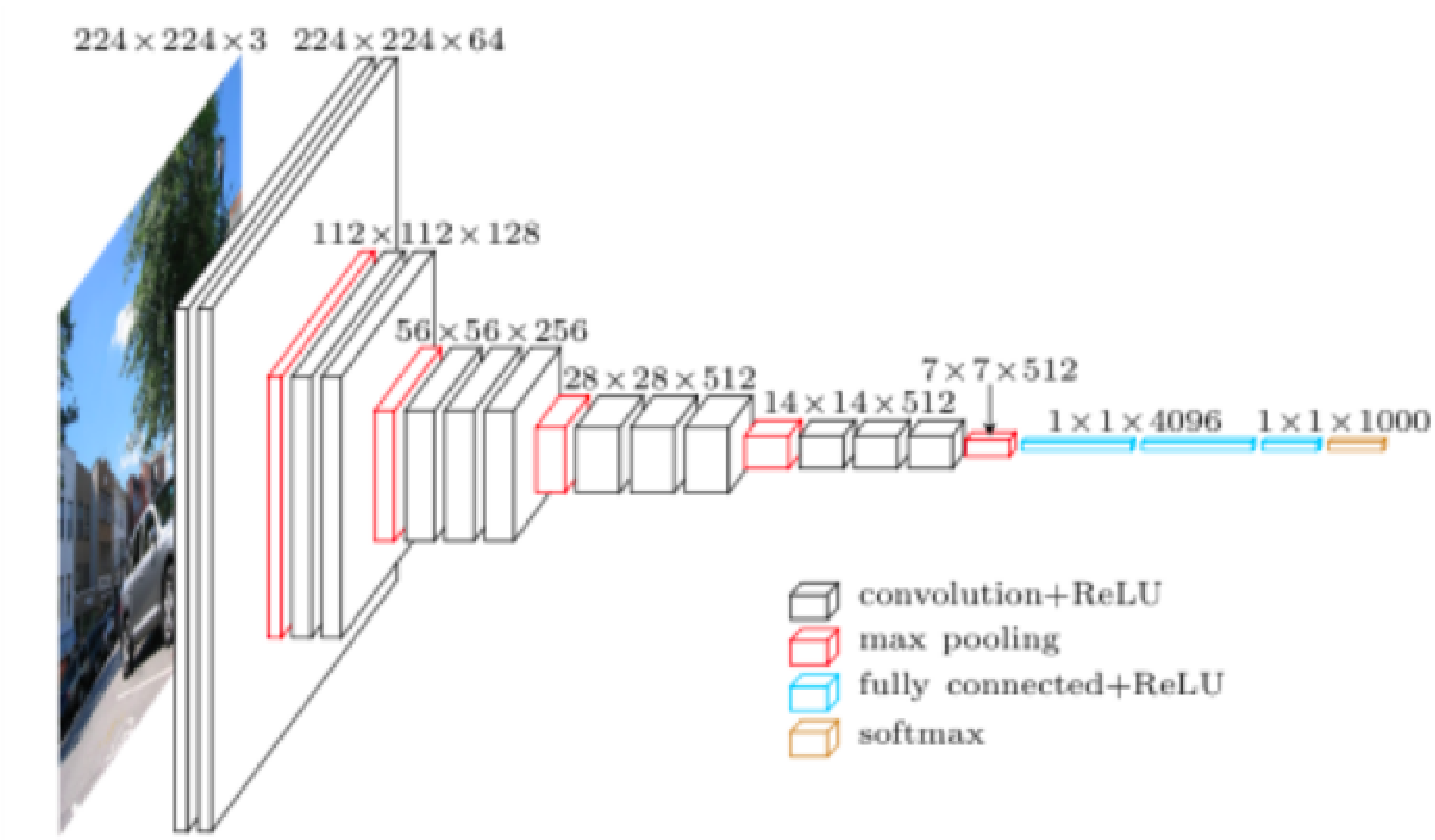
$$L_{\text{out}}[b, j] = \sigma\left(\left(\sum_{i'} W[j, i']\; L_{\text{in}}[b, i']\right) - B[j]\right)$$

Alexnet

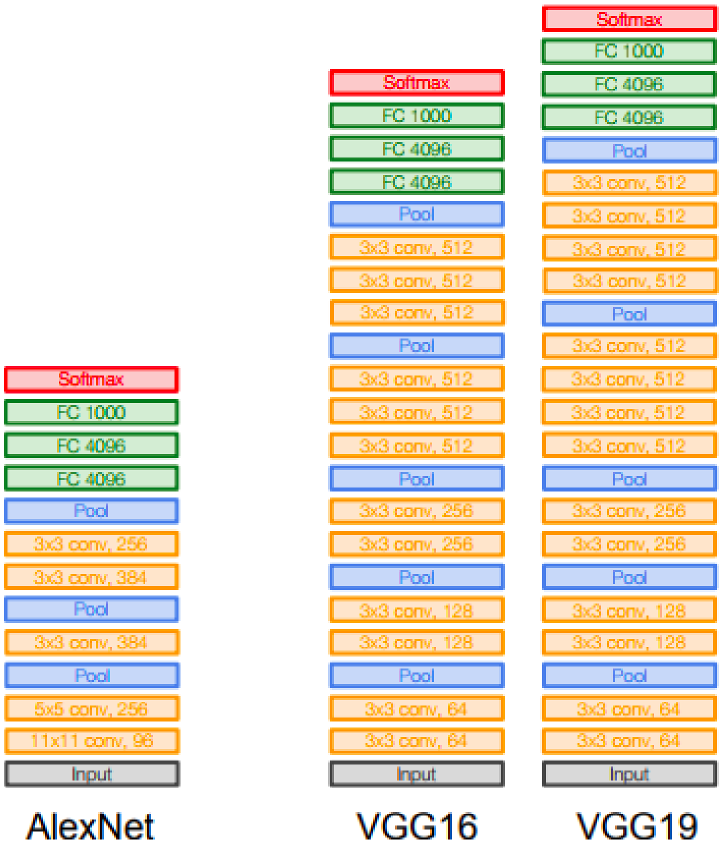
Given Input$[227, 227, 3]$

$$
\begin{aligned}
L_1[55 \times 55 \times 96] &= \mathrm{ReLU}(\mathrm{CONV}(\mathrm{Input}, \Phi_1, \text{width } 11, \text{pad } 0, \text{stride } 4)) \\
L_2[27 \times 27 \times 96] &= \mathrm{MaxPool}(L_1, \text{width } 3, \text{stride } 2)) \\
L_3[27 \times 27 \times 256] &= \mathrm{ReLU}(\mathrm{CONV}(L_2, \Phi_3, \text{width } 5, \text{pad } 2, \text{stride } 1)) \\
L_4[13 \times 13 \times 256] &= \mathrm{MaxPool}(L_3, \text{width } 3, \text{stride } 2)) \\
L_5[13 \times 13 \times 384] &= \mathrm{ReLU}(\mathrm{CONV}(L_4, \Phi_5, \text{width } 3, \text{pad } 1, \text{stride } 1)) \\
L_6[13 \times 13 \times 384] &= \mathrm{ReLU}(\mathrm{CONV}(L_5, \Phi_6, \text{width } 3, \text{pad } 1, \text{stride } 1)) \\
L_7[13 \times 13 \times 256] &= \mathrm{ReLU}(\mathrm{CONV}(L_6, \Phi_7, \text{width } 3, \text{pad } 1, \text{stride } 1)) \\
L_8[6 \times 6 \times 256] &= \mathrm{MaxPool}(L_7, \text{width } 3, \text{stride } 2)) \\
L_9[4096] &= \mathrm{ReLU}(\mathrm{FC}(L_8, \Phi_9)) \\
L_{10}[4096] &= \mathrm{ReLU}(\mathrm{FC}(L_9, \Phi_{10})) \\
s[1000] &= \mathrm{ReLU}(\mathrm{FC}(L_{10}, \Phi_s)) \ \ \text{class scores}
\end{aligned}
$$

VGG, Zisserman, 2014

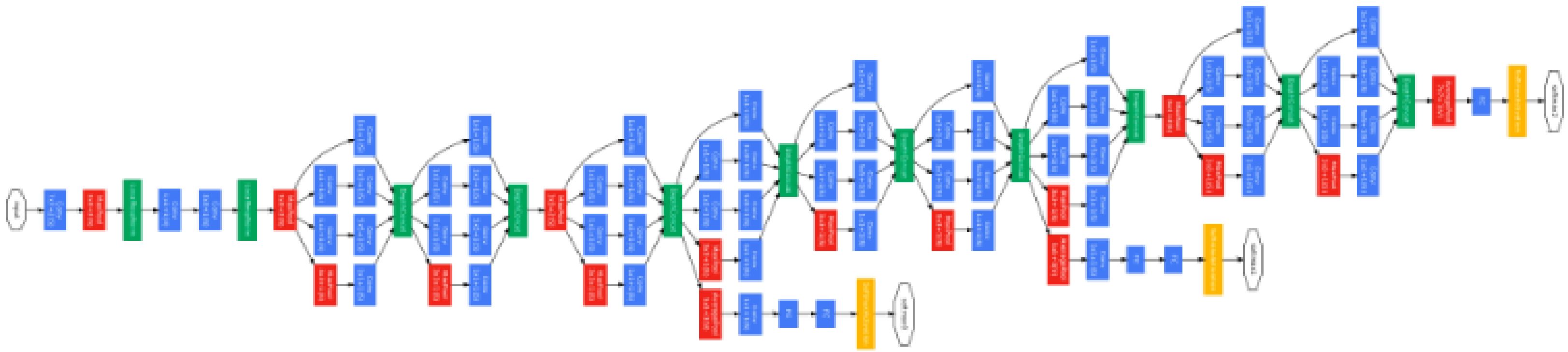


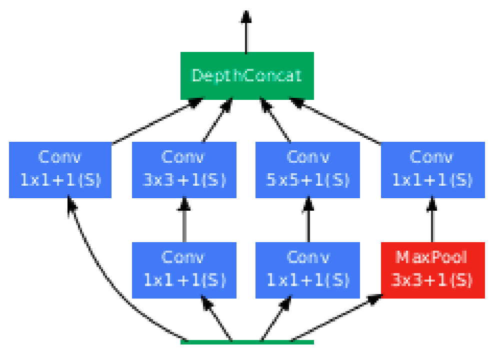
VGG



Stanford CS231

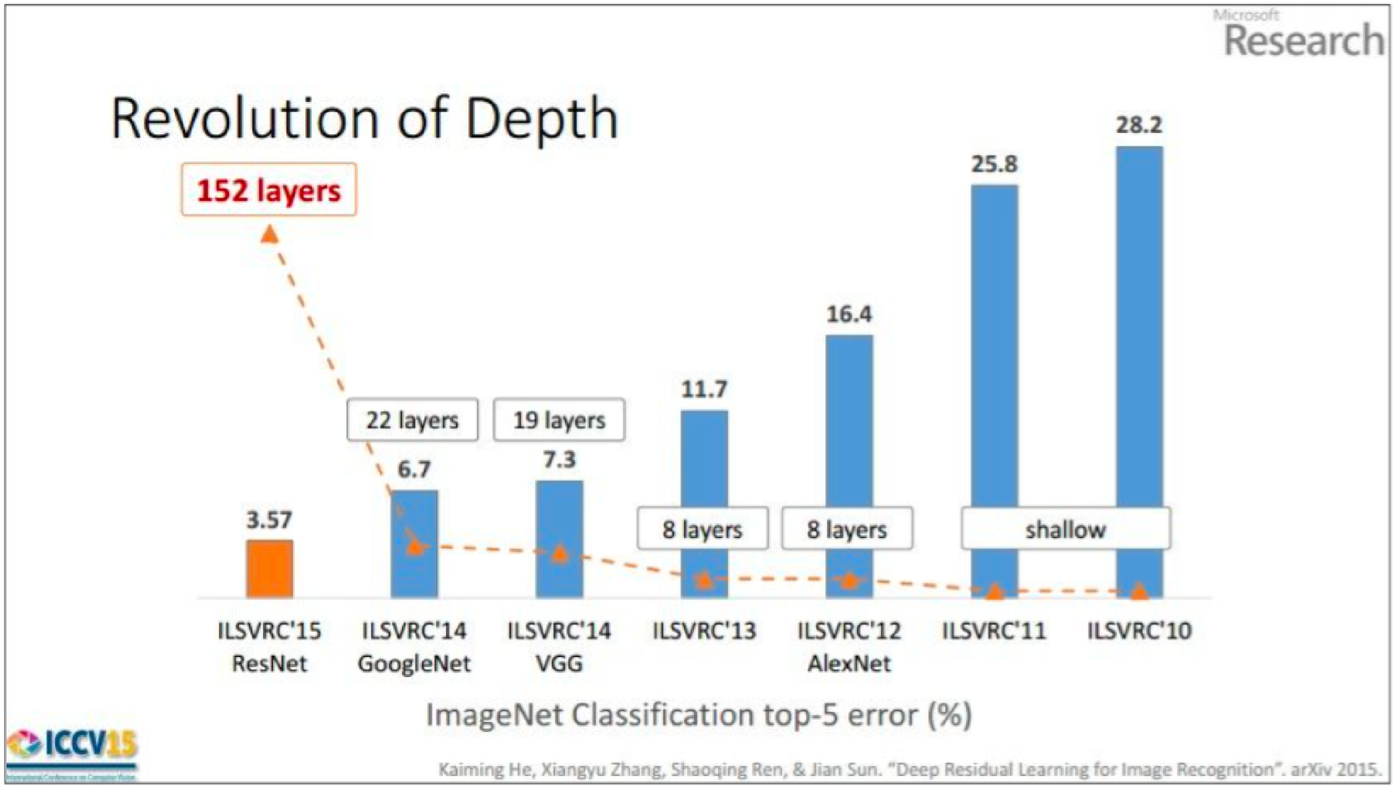
Inception, Google, 2014





Imagenet Classification

1000 kinds of objects.



(slide from Kaiming He's recent presentation)

2016 error rate is 3.0%   2017 error rate is 2.25%

Review of The Swap Rule

$$\tilde{y}[b,j] = \sum_i W[j,i]\, x[b,i]$$

$$y[b,j] = \sigma(\tilde{y}[b,j] - B[j])$$

$$x.\mathrm{grad}[b,i] \mathrel{+}= \sum_j \tilde{y}.\mathrm{grad}[b,j] W[j,i]$$

$$W.\mathrm{grad}[j,i] \mathrel{+}= \sum_b \tilde{y}.\mathrm{grad}[b,j] x[b,i]$$

Alternative Swap Rule

Intialize all computed tensors to zero and write the program using only **+=**.

$$\text{for } b, i, j \quad \tilde{y}[b, j] \mathrel{+}= W[j, i]\; x[b, i]$$

$$\text{for } b, i, j \quad x.\text{grad}[b, i] \mathrel{+}= \tilde{y}.\text{grad}[b, j] W[j, i]$$

$$\text{for } b, i, j \quad W.\text{grad}[j, i] \mathrel{+}= \tilde{y}.\text{grad}[b, j] x[b, i]$$

one swaps the output with one of the inputs inside the body of the loop.

Alternative Swap Rule for Convolution

$$\text{for } b, x, y, i, j, \Delta x, \Delta y$$

$$\tilde{L}_{\text{out}}[b, x, y, j] \mathrel{\textbf{+=}} W[\Delta x, \Delta y, i, j]\ L_{\text{in}}[b, x + \Delta x, y + \Delta y, i]$$

$$\text{for } b, x, y, i, j, \Delta x, \Delta y$$

$$W.\text{grad}[\Delta x, \Delta y, i, j] \mathrel{\textbf{+=}} \tilde{L}_{\text{out}}.\text{grad}[b, x, y, j]\ L_{\text{in}}[b, x{+}\Delta x, y{+}\Delta y, i]$$

$$\text{for } b, x, y, i, j, \Delta x, \Delta y$$

$$L_{\text{in}}.\text{grad}[b, x + \Delta x, y + \Delta y, i, j] \mathrel{\textbf{+=}} \tilde{L}_{\text{out}}.\text{grad}[b, x, y, j]\ W[\Delta x, \Delta y, i, j]$$

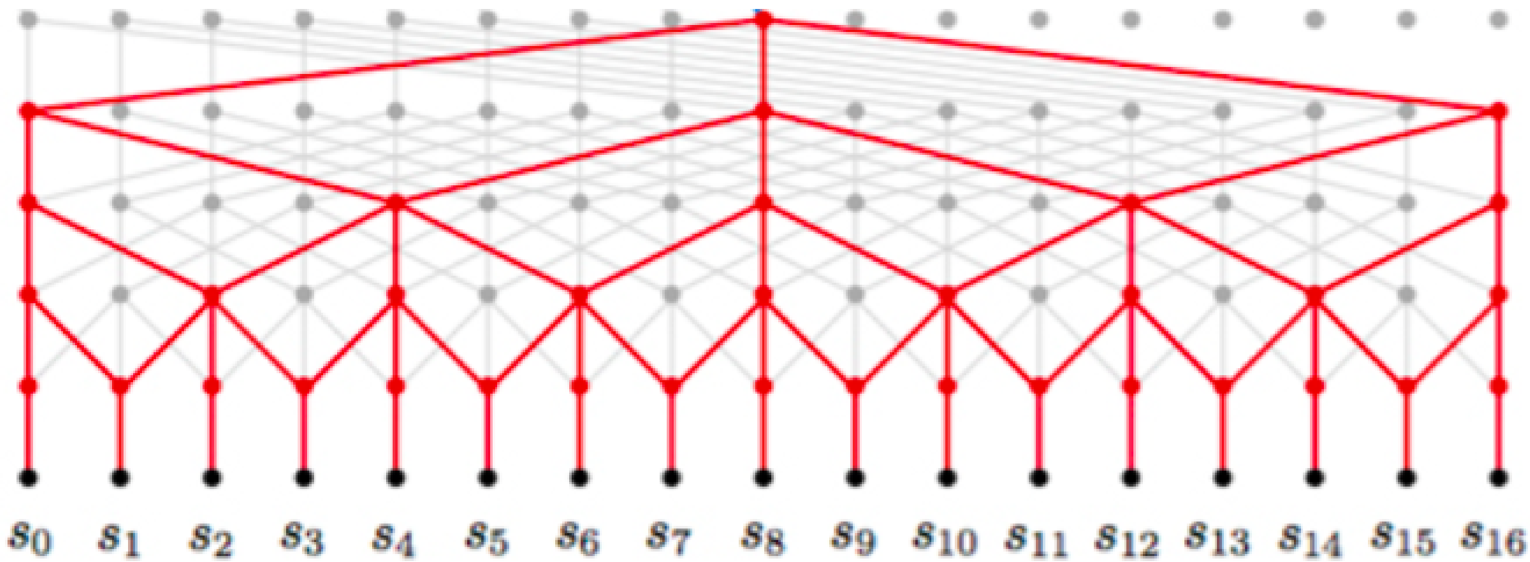
Image to Column (Im2C)

Reduce convolution to matrix multiplication — more space but faster.

$$L_{\text{in}}[b,x,y,\Delta x,\Delta y,i] = L_{\text{in}}[b,x+\Delta x,y+\Delta y,i]$$

$$\begin{aligned}
&\tilde{L}_{\text{out}}[b,x,y,j] \\
&= \left(\sum_{\Delta x,\Delta y,i} W[\Delta x,\Delta y,i,j] * L_{\text{in}}[b,x+\Delta x,\ y+\Delta y,\ i]\right) + B[j] \\
&= \left(\sum_{\Delta x,\Delta y,i} L_{\text{in}}[b,x,y,\Delta x,\Delta y,i] * W[\Delta x,\Delta y,i,j]\right) + B[j] \\
&= \left(\sum_{(\Delta x,\Delta y,i)} L_{\text{in}}[(b,x,y),(\Delta x,\Delta y,i)] * W[(\Delta x,\Delta y,i),j]\right) + B[j]
\end{aligned}$$

Fully Convolutional Networks



Dilation

We can "dilate" the filter by introducing an image step size d for each step in the filter coordinates.

$$\tilde{L}_{\mathrm{out}}[b, x, y, j] = W[\Delta x, \Delta y, i, j] L_{\mathrm{in}}[b, x + d * \Delta x, y + d * \Delta y, i] + B[j]$$

This is used for "fully convolutional" CNNs.

Summary

- Convolution
- Padding
- Stides
- Max Pooling
- Fully Connected Layers
- Dilation

Modern Trends

Modern Convolutions use 3X3 filters. This is faster and has fewer parameters. Expressive power is preserved by increasing depth with many stride 1 layers.

Max pooling and dilation seem to have disappeared.

Resnet and resnet-like architectures are now dominant (next lecture).

END