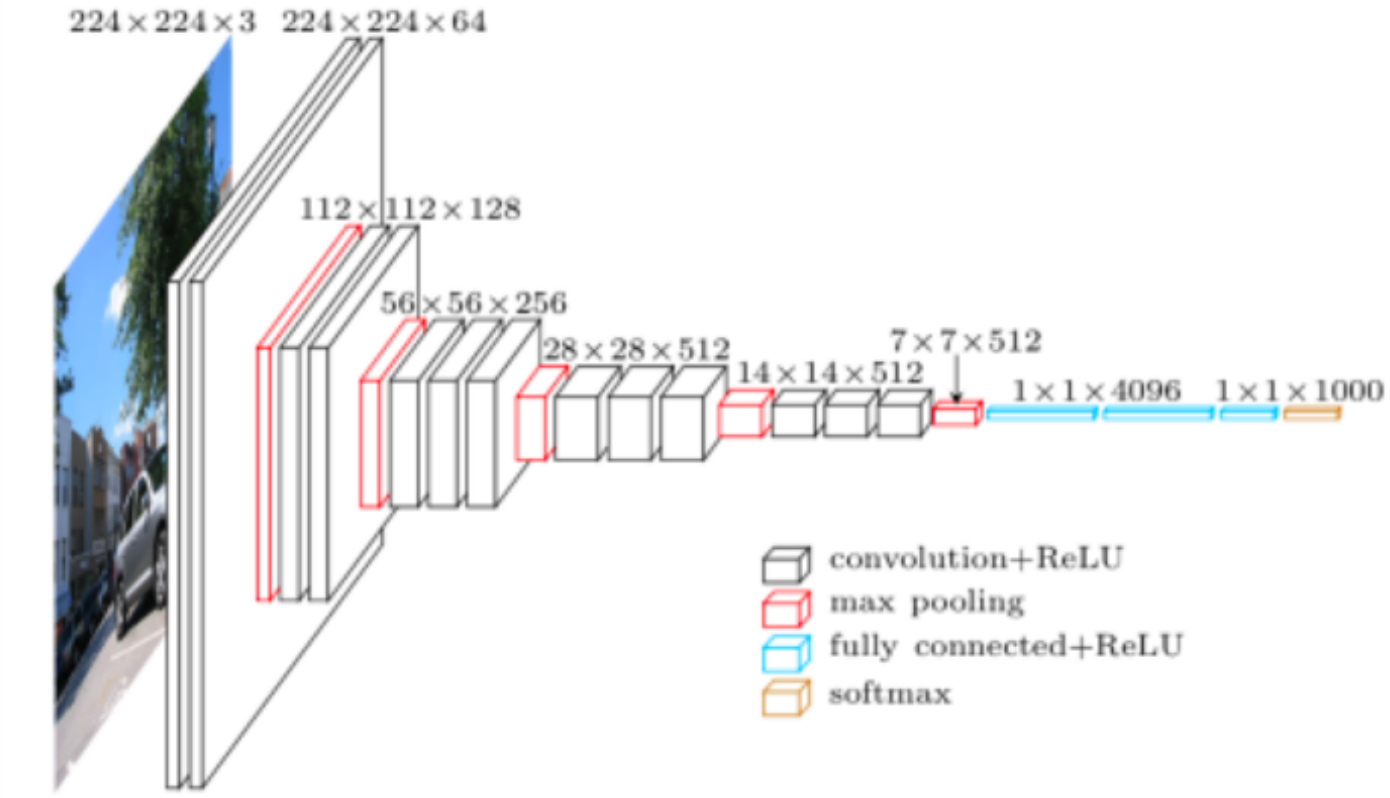
TTIC 31230, Fundamentals of Deep Learning

David McAllester, Winter 2019

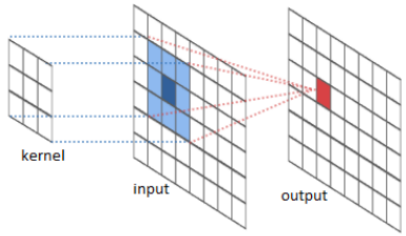
Convolutional Neural Networks (CNNs)

What is a CNN?

VGG, Zisserman, 2014



A Convolution Layer



$W[\Delta x, \Delta y, i, j]$ $L_{\text{IN}}[b, x, y, i]$ $L_{\text{out}}[b, x, y, j]$

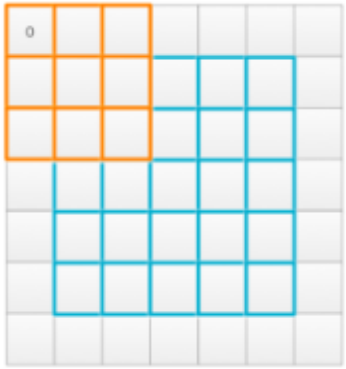
River Trail Documentation

$$\texttt{Procedure CONV}(W[\Delta x, \Delta y, i, y,], B[j], L_{\text{in}}[b, x, , y, i])$$

$$\text{Return} \;\; L_{\text{out}}[b, x, y, j]$$

$$= \left(\sum_{\Delta x, \Delta y, i} W[\Delta x, \Delta y, i, j] \; L_{\text{in}}[b, x + \Delta x, y + \Delta y, i] \right) + B[j]$$

Padding



Jonathan Hui

If we pad the input with zeros then the input and output can have the same spatial dimensions.

Zero Padding in NumPy

In NumPy we can add a zero padding of width p to an image as follows:

```
padded = np.zeros(W + 2*p,  H + 2*p)

padded[p:W+p, p:H+p] = x
```

Padding

`Procedure CONV`$(\Phi,\ L_{\text{in}}[b, x, , y, i],\ \text{padding}\ p)$

$$L'_{\text{in}} = \text{Padd}(L_{\text{in}},\ p)$$

$$L_{\text{out}}[b, x, y, j] = \left(\sum_{\Delta x, \Delta y, i} W[\Delta x, \Delta y, i, j]\ L'_{\text{in}}[b, x + \Delta x, y + \Delta y, i] \right) + B[j]$$

Return $L_{\text{out}}[b, x, y, j]$

Strides

We can move the filter by a "stride" s for each spatial step.

$$L_{\text{out}}[b, x, y, j] = W[\Delta x, \Delta y, i, j] L_{\text{in}}[b, s * x + \Delta x, s * y + \Delta y, i] + B[j]$$

Max Pooling

$$L_{\text{out}}[b, x, y, c] = \max_{\Delta x, \Delta y} L_{\text{in}}[b, s * x + \Delta x,\ s * y + \Delta y,\ c]$$

This is typically done with a stride greater than one so that the image dimension is reduced.

Fully Connected (FC) Layers

We reshape $L_{\text{in}}[b, x, y, c]$ to $L_{\text{in}}[b, (x, y, c)]$ and then

$$L_{\text{out}}[b, j] = \left(\sum_i W[i, j] \; L_{\text{in}}[b, i] \right) + B[j]$$

Basics

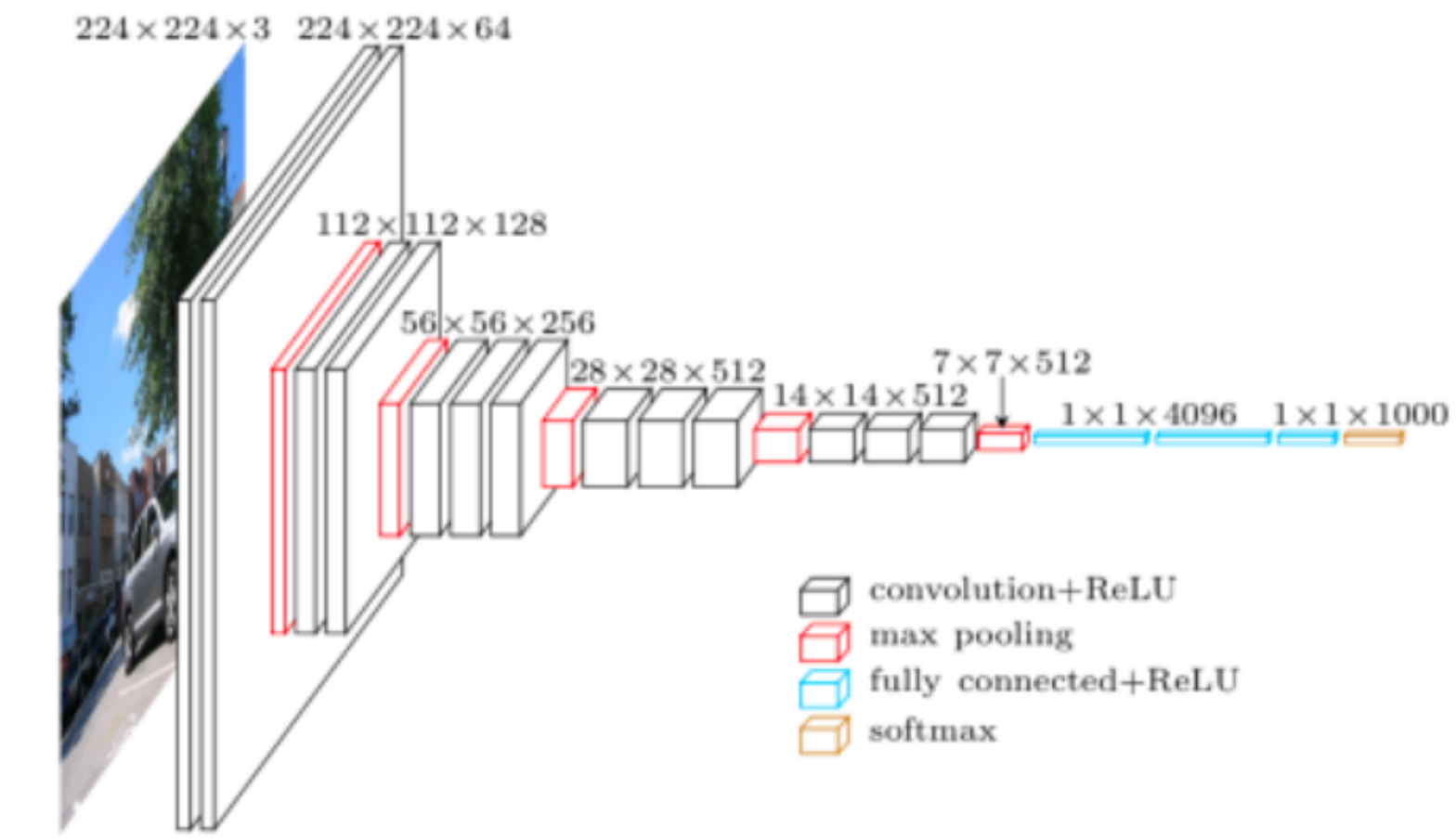
- Convolution
- Padding
- Stides
- Max Pooling
- Fully Connected Layers

Alexnet

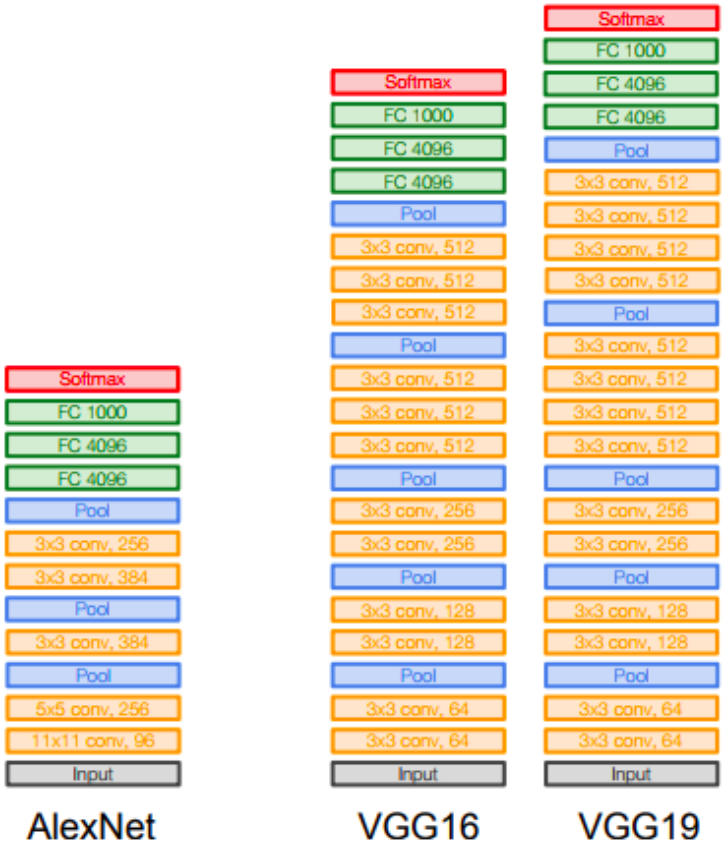
Given Input$[227, 227, 3]$

$$
\begin{aligned}
L_1[55 \times 55 \times 96] &= \text{ReLU}(\text{CONV}(\text{Input}, \Phi_1, \text{width } 11, \text{pad } 0, \text{stride } 4)) \\
L_2[27 \times 27 \times 96] &= \text{MaxPool}(L_1, \text{width } 3, \text{stride } 2)) \\
L_3[27 \times 27 \times 256] &= \text{ReLU}(\text{CONV}(L_2, \Phi_3, \text{width } 5, \text{pad } 2, \text{stride } 1)) \\
L_4[13 \times 13 \times 256] &= \text{MaxPool}(L_3, \text{width } 3, \text{stride } 2)) \\
L_5[13 \times 13 \times 384] &= \text{ReLU}(\text{CONV}(L_4, \Phi_5, \text{width } 3, \text{pad } 1, \text{stride } 1)) \\
L_6[13 \times 13 \times 384] &= \text{ReLU}(\text{CONV}(L_5, \Phi_6, \text{width } 3, \text{pad } 1, \text{stride } 1)) \\
L_7[13 \times 13 \times 256] &= \text{ReLU}(\text{CONV}(L_6, \Phi_7, \text{width } 3, \text{pad } 1, \text{stride } 1)) \\
L_8[6 \times 6 \times 256] &= \text{MaxPool}(L_7, \text{width } 3, \text{stride } 2)) \\
L_9[4096] &= \text{ReLU}(\text{FC}(L_8, \Phi_9)) \\
L_{10}[4096] &= \text{ReLU}(\text{FC}(L_9, \Phi_{10})) \\
s[1000] &= \text{ReLU}(\text{FC}(L_{10}, \Phi_s)) \;\; \text{class scores}
\end{aligned}
$$

VGG, Zisserman, 2014

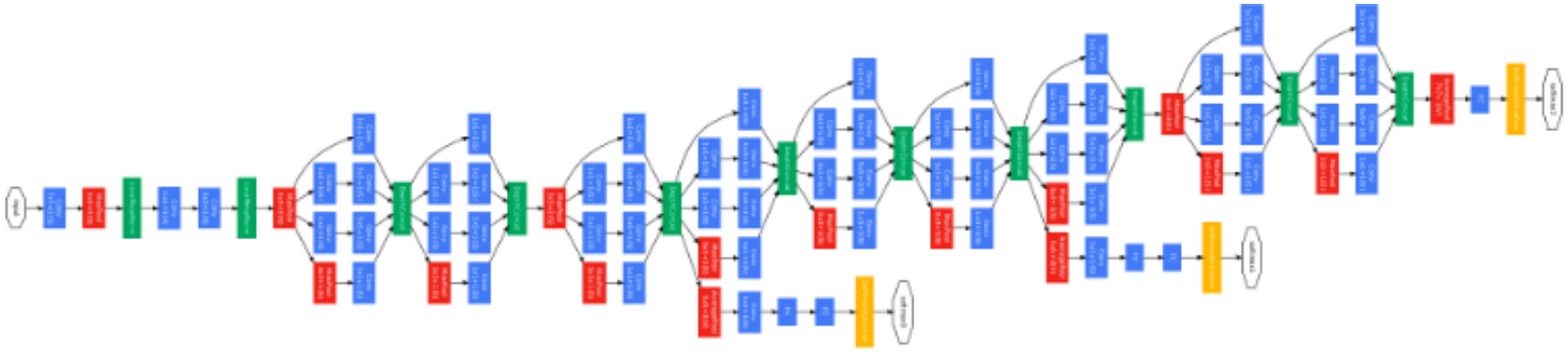


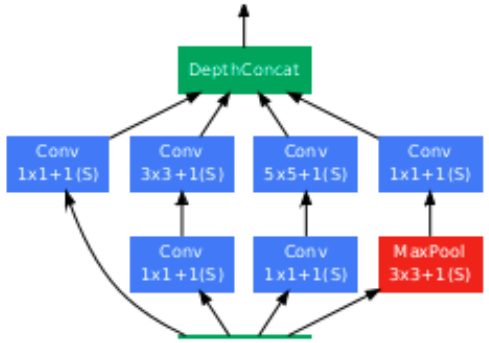
VGG



Stanford CS231

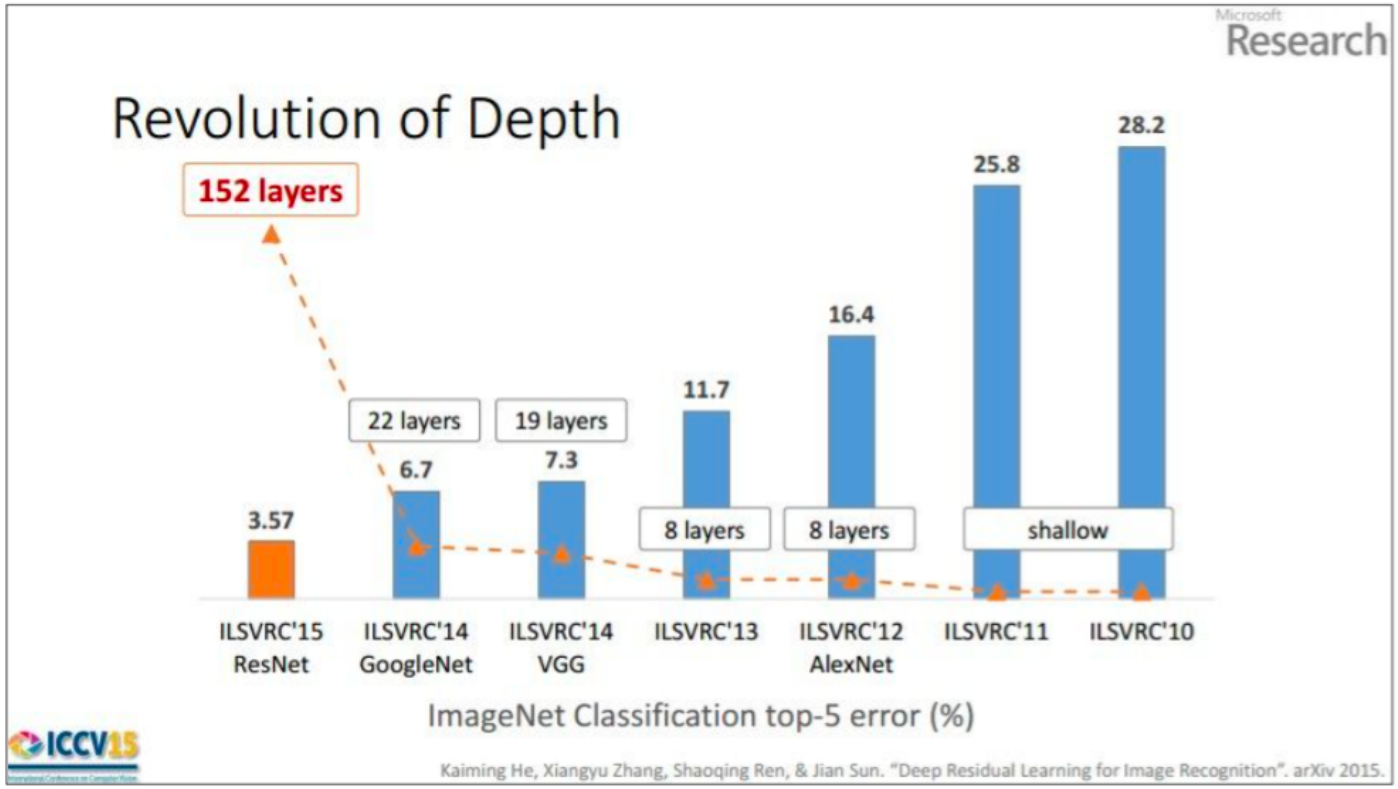
Inception, Google, 2014





Imagenet Classification

1000 kinds of objects.



(slide from Kaiming He's recent presentation)

2016 error rate is 3.0%

2017 error rate is 2.25%

Use Swap Rule to get Backward Method

$$L_{\text{out}}[b, x, y, j] \texttt{ += } W[\Delta x, \Delta y, i, j]\ L_{\text{in}}[b, x + \Delta x, y + \Delta y, i]$$

$$W.\text{grad}[\Delta x, \Delta y, i, j] \texttt{ += } \frac{1}{B}\ L_{\text{out}}.\text{grad}[b, x, y, j]\ L_{\text{in}}[b, x + \Delta x, y + \Delta_y, i]$$

$$L_{\text{in}}.\text{grad}[b, x{+}\Delta x, y{+}\Delta y, i, j] \texttt{ += } L_{\text{out}}.\text{grad}[b, x, y, j]\ W[\Delta x, \Delta y, i, j]$$

Image to Column (Im2C)

Reduce convolution to matrix multiplication — more space but faster.

$$L_{\text{in}}[b, x, y, \Delta x, \Delta y, i] = L_{\text{in}}[b, x + \Delta x, y + \Delta y, i]$$

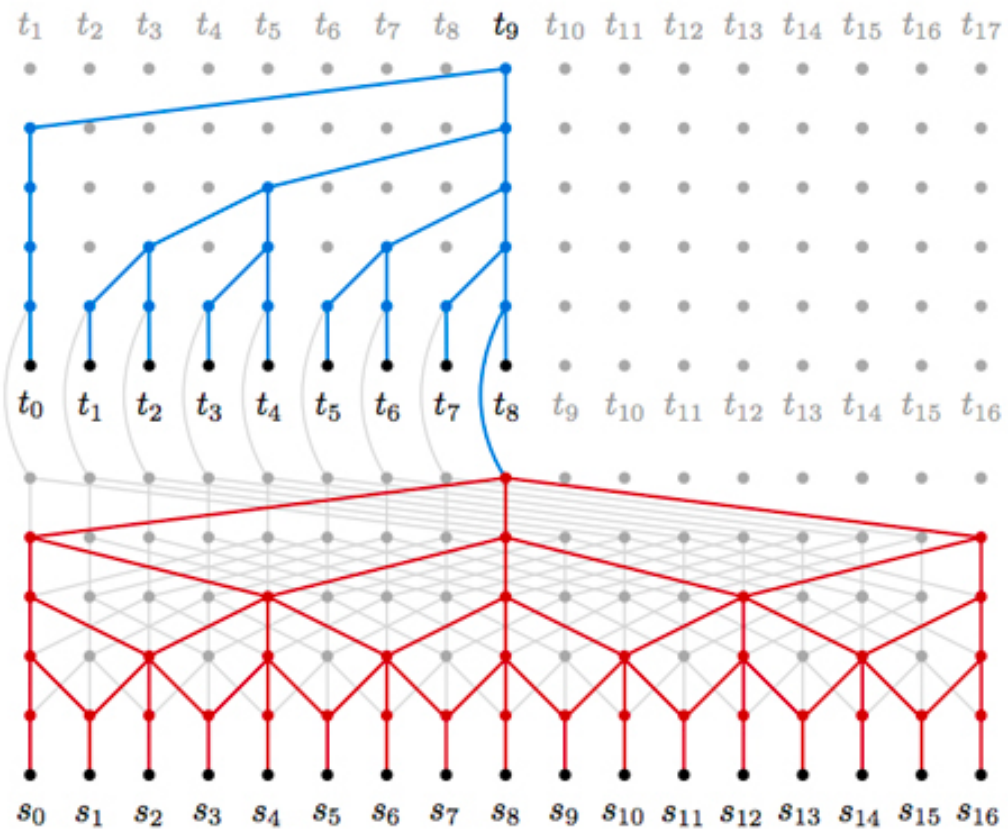
$$L_{\text{out}}[b, x, y, j]$$

$$= \left(\sum_{\Delta x, \Delta y, i} W[\Delta x, \Delta y, i, j] * L_{\text{in}}[b, x + \Delta x,\ y + \Delta y,\ i] \right) + B[j]$$

$$= \left(\sum_{\Delta x, \Delta y, i} L_{\text{in}}[b, x, y, \Delta x, \Delta y, i] * W[\Delta x, \Delta y, i, j] \right) + B[j]$$

$$= \left(\sum_{(\Delta x, \Delta y, i)} L_{\text{in}}[(b, x, j), (\Delta x, \Delta y, i)] * W[(\Delta x, \Delta y, i), j] \right) + B[j]$$

Fully Convolutional Networks



Dilation

We can "dilate" the filter by introducing an image step size d for each step in the filter coordinates.

$$L_{\text{out}}[b, x, y, j] = W[\Delta x, \Delta y, i, j] L_{\text{in}}[b, x + d * \Delta x, y + d * \Delta y, i] + B[j]$$

This is used for "fully convolutional" CNNs.

END