Notes on "Estimates of the Trade and Welfare Effects of NAFTA"(Caliendo et al. 2015, Economic Review)

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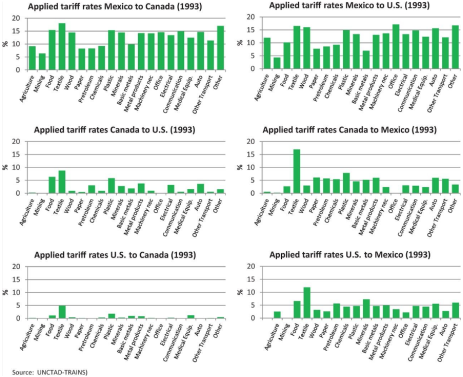
Introduction

- This paper builds a model based on the E-K model (Eaton and Kortum, 2002)to quantify the trade and welfare effects from tariff changes.
- The model has sectoral linkages, multiple sectors, intermediate goods trade, a production economy, non-tradable sectors.
- The theory in this paper: production is at constant returns to scale and markets are perfectly competitive.
- In this paper, Caliendo and Parro propose a new method to estimate sectoral trade elasticities consistent with any trade model that delivers a multiplicative gravity equation.

- Tariff rates vary substantially across sectors.
- Figure A1: Effective applied tariff rates before NAFTA



In 1993, sectoral tariff rates applied by Mexico, Canada and the US to NAFTA members were on average 12.5%, 4.2% and 2.7%. By 2005, they dropped to almost zero between NAFTA members but tariffs that Mexico, Canada and the US applied to the RoW were on average 7.1%, 2.2% and 1.7%, respectively.

- By 1993, most goods traded across NAFTA members were intermediate goods. Respectively 68%, 61.5% and 64.6% of Mexico's, Canada's and the US imports from non-NAFTA countries were intermediate goods, and 82.1%, 72.3% and 72.8 % of Mexico's, Canada's and the US imports from NAFTA countries were intermediate goods.
- I-O tables reflect that tradable and non-tradable sectors are interconnected. In the IO Tables, the mean share of own-sector inputs is around 15-20%; more than 70% of intermediate consumption is addressed to other sectors; average share of tradables in the production of non-tradables is 23% for the US and 32% for Mexico, and average shares of non-tradables in the production of tradables are 34% for the US and 26% for Mexico.

The model

- There are N countries and J sectors
- Denote countries by i and n and sectors by j and k
- Consider that sectors are of two types, either tradable or non-tradable
- There is only one factor of production, labor. And labor is mobile across sectors and not mobile across countries.
- Households supply labor L_n at a wage w_n and receive transfers on a lump-sum basis.
- I_n is households' income

- In each country, there are a measure of L_n representative households that maximize utility by consuming final goods C_n^j .
- The preferences of the households are given by

$$u(C_n) = \prod_{j=1}^{J} C_n^{j\,\alpha_n^j} \qquad (1)$$

where, $\sum_{j=1}^{J} \alpha_n^j = 1$

- Two types of inputs, labour $l_n^j(\omega^j)$ and composite intermediate goods $m_n^{k,j}(\omega^j)$ (also referred to as materials) from all sectors, are used for the production of each intermediate goods $\omega^j \in [0,1]$.
- Denote by $z_n^j(\omega^j)$ the efficiency of producing intermediate good in country n. Producers of intermediate goods across countries differ in the efficiency of production.
- The production technology of a good ω^j is

$$q_n^j(\omega^j) = z_n^j(\omega^j)[l_n^j(\omega^j)]^{\gamma_n^j} \prod_{k=1}^{J} [m_n^{k,j}(\omega^j)]^{\gamma_n^{k,j}}$$

where, $\gamma_n^{k,j} \geq 0$ is the share of materials from sector k used in the production of intermediate goods ω^j, with $\sum_{k=1}^{J} \gamma_n^{k,j} = 1 - \gamma_n^j$; γ_n^j is the share of value added.

Input-Output Linkages

- The producers of intermediate goods minimize the unit cost to get one unit of production.
- Let the unit cost is $w_n l_n^j + \sum_{k=1}^J P_n^k m_n^{k,j}$, where, P_n^k is the price of composite intermediate goods from sector k. Note that here I simplify $l_n^j(\omega^j),\ m_n^{k,j}(\omega^j)\ and\ z_n^j(\omega^j)\ to\ l_n^j,\ m_n^{k,j}\ and z_n^j$ for the calculations.
- Now the problem is :

$$\min_{l_n^j, \left\{m_n^{k,j}\right\}_{k=1}^J} \quad w_n l_n^j + \sum_{k=1}^J P_n^k m_n^{k,j}$$

subjet to

$$z_n^j [l_n^j]^{\gamma_n^j} \prod_{k=1}^J [m_n^{k,j}]^{\gamma_n^{k,j}} = 1$$

- The Lagrange function is

$$L = w_n l_n^j + \sum_{k=1}^{J} P_n^k m_n^{k,j} + \lambda[1 - z_n^j [l_n^j]^{\gamma_n^j} \prod_{k=1}^{J} [m_n^{k,j}]^{\gamma_n^{k,j}}]$$

- The first order conditions:

$$\frac{\partial L}{\partial l_n^j} = w_n - \lambda z_n^j \gamma_n^j [l_n^j]^{\gamma_n^j - 1} \prod_{k=1}^{J} [m_n^{k,j}]^{\gamma_n^{k,j}} = 0 \qquad (a)$$

$$\frac{\partial L}{\partial m_n^{k,j}} = P_n^k - \lambda z_n^j [l_n^j]^{\gamma_n^j} \frac{\prod_{k=1}^{J} (m_n^{k,j})^{\gamma_n^{k,j}}}{(m_n^{k,j})^{\gamma_n^{k,j}}} \gamma_n^{k,j} (m_n^{k,j})^{\gamma_n^{k,j} - 1} = 0 \qquad (b)$$

$$\frac{\partial L}{\partial \lambda} = 1 - z_n^j [l_n^j]^{\gamma_n^j} \prod_{k=1}^{J} [m_n^{k,j}]^{\gamma_n^{k,j}} = 0 \qquad (c)$$

- (a) divided by (b) , and rearrange to obtain

$$m_n^{k,j} = \frac{w_n l_n^j \gamma_n^{k,j}}{P_n^k \gamma_n^j} \qquad (d)$$

- Substitute (d) to (c), can get

$$1 = z_n^j [l_n^j]^{\gamma_n^j} \prod_{k=1}^{J} [\frac{w_n l_n^j \gamma_n^{k,j}}{P_n^k \gamma_n^j}]^{\gamma_n^{k,j}}$$

$$= z_n^j [l_n^j]^{\gamma_n^j} [\frac{w_n l_n^j}{\gamma_n^j}]^{\sum_{k=1}^{J} \gamma_n^{k,j}} \prod_{k=1}^{J} [\frac{\gamma_n^{k,j}}{P_n^k}]^{\gamma_n^{k,j}} \qquad (e)$$

- Let $B = z_n^j \prod_{k=1}^{J} [\frac{\gamma_n^{k,j}}{P_n^k}]^{\gamma_n^{k,j}}$, then rearrange (e) to get

$$l_n^{j*} = \frac{(\gamma_n^j)^{1-\gamma_n^j}}{Bw_n^{1-\gamma_n^j}} \qquad (f)$$

note that $\sum_{k=1}^{J} \gamma_n^{k,j} = 1 - \gamma_n^j$.

- Now substitute (f) to (d), get

$$m_n^{k,j*} = \frac{w_n^{\gamma_n^j}}{(\gamma_n^j)^{\gamma_n^j}} \frac{\gamma_n^{k,j}}{P_n^k} \frac{1}{B} \qquad (g)$$

- Now using (f) and (g), the unit cost is

$$w_n l_n^{j*} + \sum_{k=1}^{J} P_n^k m_n^{k,j*}$$

- Take the whole items of B back to get:

$$w_n^{\gamma_n^j} \frac{(\gamma_n^j)^{1-\gamma_n^j}}{z_n^j \prod_{k=1}^J [\frac{\gamma_n^{k,j}}{P_n^k}]^{\gamma_n^{k,j}}} + \sum_{k=1}^{J} \frac{w_n^{\gamma_n^j}}{(\gamma_n^j)^{\gamma_n^j}} \frac{\gamma_n^{k,j}}{z_n^j \prod_{k=1}^J [\frac{\gamma_n^{k,j}}{P_n^k}]^{\gamma_n^{k,j}}}$$

- Rearrange to get

$$w_n^{\gamma_n^j} \prod_{k=1}^{J} (P_n^k)^{\gamma_n^{k,j}} (z_n^j)^{-1} \prod_{k=1}^{J} (\gamma_n^{k,j})^{-\gamma_n^{k,j}} (\gamma_n^j)^{-\gamma_n^j} \qquad (h)$$

Input-Output Linkages

- Let $\Upsilon_n^j = \prod_{k=1}^{J} (\gamma_n^{k,j})^{-\gamma_n^{k,j}} (\gamma_n^j)^{-\gamma_n^j}$, then the unit cost is $c_n^j (z_n^j)^{-1}$.
- The cost of an input bundle is given by

$$c_n^j = \Upsilon_n^j w_n^{\gamma_n^j} \prod_{k=1}^{J} P_n^{k\,\gamma_n^{k,j}} \qquad (2)$$

- The cost of the input bundle depends on wages and on the price of all the composite intermediate goods in the economy, tradable and non-tradable.
- A change in policy that affects the price in any single sector will affect indirectly all the sectors in the economy via the input bundle.

- Producers of composite intermediate goods in sector j and country n, supply Q_n^j at minimum cost by purchasing intermediate goods ω^j, $\omega^j \in [0, 1]$, from the lowest cost suppliers across countries.
- The production of composite intermediate goods at minimun cost by buying tradable intermediate goods ω^j, is

$$Q_n^j = \left[\int r_n^j(\omega^j)^{1-\frac{1}{\sigma^j}} \mathrm{d}\omega^j \right]^{\frac{\sigma^j}{\sigma^j - 1}}$$

where, $\sigma^j > 0$ is the elasticity of substitution across intermediate goods within sector j, and $r_n^j(\omega^j)$ is the demand of intermediate goods ω^j from the lowest cost supplier. And here the production is CES.

- To solve the problem of the composite intermediate good producer, we can get the demand for good ω^j, $r_n^j(\omega^j)$.
- The problem is to maximize the profit:

$$\max_{r_n^j(\omega^j)} \pi(r_n^j(\omega^j)) = Q_n^j P_n^j - \int r_n^j(\omega^j) p_n^j(\omega^j) d\omega$$

where, P_n^j is unit price of the composite intermediate good, and $p_n^j(\omega^j)$ denotes the lowest price of intermediate good ω^j across all locations n.

- $$\max_{r_n^j(\omega^j)} \pi(r_n^j(\omega^j)) = Q_n^j P_n^j - \int r_n^j(\omega^j) p_n^j(\omega^j) d\omega$$

First order condition:

$$\frac{\mathrm{d}\pi}{\mathrm{d}r_n^j(\omega^j)} = P_n^j[\frac{\sigma^j}{\sigma^j - 1}][\int r_n^j(\omega^j)^{1-\frac{1}{\delta^j}} \mathrm{d}\omega^j]^{(\frac{\delta^j}{\delta^j-1}-1)} \times$$

$$[1 - \frac{1}{\sigma^j}] r_n^j(\omega^j)^{1-\frac{1}{\sigma^j}-1} - p_n^j(\omega^j) = 0 \qquad (\star)$$

Notes: when taking the derivation of $[\int r_n^j(\omega^j)^{1-\frac{1}{\sigma^j}} \mathrm{d}\omega^j]$, it can be viewd as : $\sum r_n^j(\omega^j)^{1-\frac{1}{\sigma^j}} \mathrm{d}\omega^j$. Then take the derivation of one of the items, $r_n^j(\omega^j)^{1-\frac{1}{\sigma^j}} \mathrm{d}\omega^j$. The same thing for $\int r_n^j(\omega^j) p_n^j(\omega^j) d\omega$ and also here hold $P_n^j, p_n^j(\omega^j)$ as constant.

- For equation ($\star$), cancle out $[\frac{\sigma^j}{\sigma^j-1}]$ and rearrange it to get:

$$P_n^j[\int r_n^j(\omega^j)^{1-\frac{1}{\delta^j}}\mathrm{d}\omega^j]^{(\frac{1}{\delta^j-1})} r_n^j(w^j)^{-\frac{1}{\sigma^j}} - p_n^j(\omega^j) = 0 \qquad (\star\star)$$

- Rearrange equation($\star\star$) to get :

$$r_n^j(\omega^j)^{-\frac{1}{\sigma^j}} = \frac{p_n^j(\omega^j)}{P_n^j[\int r_n^j(\omega^j)^{1-\frac{1}{\delta^j}}\mathrm{d}\omega^j]^{(\frac{1}{\delta^j-1})}} \qquad (\star\star\star)$$

- Then equation $(\star\star\star)$ becomes:

$$r_n^j(\omega^j) = \frac{p_n^j(\omega^j)^{-\sigma^j}}{P_n^j[\int r_n^j(\omega^j)^{1-\frac{1}{\delta^j}} \mathrm{d}\omega^j]^{(\frac{-\sigma^j}{\delta^j-1})}}$$

$$= \frac{p_n^j(\omega^j)^{-\sigma^j}}{{P_n^j}^{-\sigma^j}}[\int r_n^j(\omega^j)^{1-\frac{1}{\delta^j}} \mathrm{d}\omega^j]^{(\frac{\sigma^j}{\delta^j-1})}$$

- Then can get:

$$r_n^j(\omega^j) = [\frac{p_n^j(\omega^j)}{P_n^j}]^{-\sigma^j} Q_n^j$$

  which is the demand for good ω^j.
- There is free entry in the production of final goods with competition implying zero profit.

- Trade costs, k^j_{ni}, include iceberg trade costs, d^j_{ni} (Samuelson,1954) and an ad-valorem flat-rate tariffs, $\tilde{\tau}^j_{ni}$

$$k^j_{ni} = \tilde{\tau}^j_{ni} d^j_{ni} \qquad (3)$$

where, $\tilde{\tau}^j_{ni} = 1 + \tau^j_{ni}$
$d^j_{ni} \geq 1$
$d^j_{nn} = 1$
$k^j_{nh} k^j_{hi} \geq k^j_{ni}$
$k^j_{nn} = 1$

- For non-tradable goods setcor, $k^j_{ni} = \infty$.

Price

- A unit of a tradable intermediate good ω^j produced in country i is available in country n at unit prices $c_i^j k_n^j / z_i^j(\omega^j)$.
- The price of intermediate good ω^j in country n is given by the minimum of the unit costs across locations, adjusted by the transport costs k_{ni}^j:

$$p_n^j(\omega^j) = \min_i \left\{ \frac{c_i^j k_{ni}^j}{z_i^j(\omega^j)} \right\}$$

.

- In non-tradable sectors, $p_n^j(\omega^j) = \frac{c_n^j}{z_n^j(\omega^j)}$.

Price

- To get the unit price of the composite intermediate good, P_n^j, we let $Q_n^j = 1$ and then we can have $r_n^j(\omega^j) = [\frac{p_n^j(\omega^j)}{P_n^j}]^{-\sigma^j}$.
- Now $P_n^j = \int r_n^j(\omega^j) p_n^j(\omega^j) \mathrm{d}\omega^j$

$$
\begin{aligned}
&= \int [\frac{p_n^j(\omega^j)}{P_n^j}]^{-\sigma^j} p_n^j(\omega^j) \mathrm{d}\omega^j \\
&= \int p_n^j(\omega^j)^{1-\sigma^j} {P_n^j}^{\sigma^j} \mathrm{d}\omega^j \\
&= {P_n^j}^{\sigma^j} \int p_n^j(\omega^j)^{1-\sigma^j} \mathrm{d}\omega^j .
\end{aligned}
$$

- Rearrange it can get

$$
P_n^j = [\int p_n^j(\omega^j)^{1-\sigma^j} \mathrm{d}\omega^j]^{\frac{1}{1-\sigma^j}} \qquad (A).
$$

- The price of the composite intermediate good is given by

$$P_n^j = A^j \left[\sum_{i=1}^{N} \lambda_i^j (c_i^j k_{ni}^j)^{-\theta^j} \right]^{-\frac{1}{\theta^j}} \qquad (4)$$

- **Proof of Equation(4)** :
- Follow Eaton and Kortum (2002) in solving for the distribution of prices.
- Assume that the distributions of productivities are independent across goods, sectors and countries, and that $\sigma^j \leq 1 + \theta^j$.

- Denote $z_n^j(\omega^j)$ the efficiency of producing ω^j in country n for sector j
- Assume that $z_n^j(\omega^j)$ is the realization of a Fréchet distribution with with a location parameter that varies by country and sector, λ_n^j and shape parameter that varies by sector, θ^j:

$$F_n^j(z) = e^{-\lambda_n^j z^{-\theta^j}}$$

- By calculating the unit price of the composite intermediate good, P_n^j, from equation (A), we can rewrite it as:

$$(P_n^j)^{1-\sigma^j} = [\int p_n^j(\omega^j)^{1-\sigma^j} d\omega^j] \qquad (A^\star).$$

- Proof of Equation (4) Continued
- The cost of purchasing an intermediate good ω^j from country i is $p^j_{ni}(z^j_i) = c^j_i k^j_{ni}/z^j_i$, and it also has a Fréchet distribution. So the price distributiuon of the goods exported from country i to country n is

$$F_{p^j_{ni}}(p) = Pr[p^j_{ni} \leq p] = Pr[\frac{c^j_i k^j_{ni}}{z^j_i} \leq p] = Pr[\frac{c^j_i k^j_{ni}}{p} \leq z^j_i]$$

$$= 1 - e^{-T^j_{ni} p^{\theta^j}} \qquad (A1)$$

where $T^j_{ni} = \lambda^j_i (c^j_i k^j_{ni})^{-\theta^j}$.

Distribution of Prices

- Proof of Equation (4) Continued
- The lowest price of an intermediate good ω^j in country n, $p_n^j(\omega^j)$ also has a Fréchet distribution. So the price distribution in country n is

$$Pr[p_n^j \leq p] = Pr[\min_i \left\{ p_{ni}^j(z_i^j) \right\} \leq p]$$
$$= 1 - \prod_{i=1}^{N} Pr[p_{ni}^j \geq p]$$
$$= 1 - \prod_{i=1}^{N} Pr[\frac{c_i^j k_{ni}^j}{z_i^j} \geq p]$$
$$= 1 - \prod_{i=1}^{N} Pr[z_i^j \leq \frac{c_i^j k_{ni}^j}{p}] = 1 - e^{-\Phi_n^j p^{\theta^j}} \qquad (A2)$$

where,$\Phi_n^j = \sum_{i=1}^{N} T_{ni}^j = \sum_{i=1}^{N} \lambda_i^j (c_i^j k_{ni}^j)^{-\theta^j}$; Φ_n^j does not depend on i because we can integrating out the regional dimension.

- Proof of Equation (4) Continued
- Let $F_{p_n^j}(p)$ denote the CDF in equation (A2), then the associated pdf, denoted $f(p)$, is $\Phi_n^j \theta^j p^{\theta^j - 1} e^{-\Phi_n^j p^{\theta^j}}$, and the equation $(A^\star)$ can be written as:

$$(P_n^j)^{1-\sigma_n^j} = \int p^{1-\sigma_n^j} dF_{p_n^j}(p) = \int p^{1-\sigma_n^j} f(p) dp$$

$$= \int p^{1-\sigma_n^j} \Phi_n^j \theta^j p^{\theta^j - 1} e^{-\Phi_n^j p^{\theta^j}} dp \qquad (A3)$$

Note: using the definition of Lebesgue integration to understand the first " = " sign in equation (A3).

- Proof of Equation (4) Continued
- For the purpose of getting the distribution of price in country n, it will be convenient to work with the random variable $p_n^j(\omega^j)^{\theta^j}$, let $y = p_n^j(\omega^j)^{\theta^j}$.
- Now we can re-write equation (A3) as:

$$(P_n^j)^{1-\sigma_n^j} = \int (p^{\theta^j})^{\frac{1-\sigma_n^j}{\theta^j}} \Phi_n^j \theta^j p^{\theta^j - 1} e^{-\Phi_n^j p^{\theta^j}} dp$$

$$= \int y^{\frac{1-\sigma_n^j}{\theta^j}} \Phi_n^j e^{-\Phi_n^j y} dy. \qquad (A4)$$

- Proof of Equation (4) Continued
- Now consider the change of variables $u = \Phi_n^j y$. Then $y = (\Phi_n^j)^{-1} u$, $y^{\frac{1-\sigma_n^j}{\theta^j}} = (\Phi_n^j)^{-\frac{1-\sigma_n^j}{\theta^j}} u^{\frac{1-\sigma_n^j}{\theta^j}}$, and $du = \Phi_n^j dy$. So we can get:

$$(P_n^j)^{1-\sigma^j} = (\Phi_n^j)^{-(1-\sigma^j)/\theta^j} \int u^{(1-\sigma^j)/\theta^j} e^{-u} \mathrm{d}u$$

- or

$$P_n^j = \Gamma(\xi^j)^{1/(1-\sigma^j)} (\Phi_n^j)^{-1/\theta^j}$$

Where, $\Gamma(\xi^j)$ is a Gamma function, i.e. $\Gamma(a) \equiv \int_0^\infty x^{a-1} e^{-x} \mathrm{d}x$. Here, $\Gamma(\xi^j)$ is evaluated at $\xi^j = 1 + (1 - \sigma^j)/\theta^j$. And with a CES utility, the elasticity of substitution $\sigma < 1 + \theta$.

- Proof of Equation (4) Continued
- Let $A^j = \Gamma(\xi^j)^{1/(1-\sigma^j)}$, the price of composite sectoral goods in tradeable sector j can then also be expressed as

$$P_n^j = A^j \Big[\sum_{i=1}^{N} \lambda_i^j (c_i^j k_{ni}^j)^{-\theta^j}\Big]^{-\frac{1}{\theta^j}}$$

  Equation (4) is proved.
- In a non-tradable goods sector, the price index is given by $P_n^j = A^j {\lambda_n^j}^{-1/\theta^j} c_n^j$ because $k_{in}^j = \infty$.

- With Cobb-Douglas preferences(equation 1), the consumption price index is given by

$$P_n = \prod_{j=1}^{J} \left(P_n^j/\alpha_n^j\right)^{\alpha_n^j} \qquad (5)$$

- **Proof of Equation**(**5**) :
- Households maximize utility by consuming final goods C_n^j. Denote by I_n households' income.
  Consumers purchase final goods at price P_n^j.
- The problem is : $\max u$
  subject to $\sum_{j=1}^{J} C_n^j P_n^j = I_n$.

- Take logs of the utility function from both sides, we can write it as :

$$lnu = \sum_{j=1}^{J} \alpha_n^j lnC_n^j, \ where \ \sum_{j=1}^{J} \alpha_n^j = 1$$

- Now the problem is

$$\max_{C_n^j} lnu \quad subject\ to \quad \sum_{j=1}^{J} C_n^j P_n^j = I_n$$

- The lagrangian function is

$$L = \sum_{j=1}^{J} \alpha_n^j lnC_n^j + \lambda \left[I_n - \sum_{j=1}^{J} C_n^j P_n^j \right]$$

- First order conditions:

$$\frac{\partial L}{\partial C_n^j} = \frac{\alpha_n^j}{C_n^j} - \lambda P_n^j = 0 \qquad (5.1)$$

so: $C_n^j = \frac{\alpha_n^j}{\lambda P_n^j} \qquad (5.2)$

$$\frac{\partial L}{\partial \lambda} = I_n - \sum_{j=1}^{J} C_n^j P_n^j = 0$$

so $I_n = \sum_{j=1}^{J} C_n^j P_n^j \qquad (5.3)$

- Rearrange (5.1) can get $\frac{\alpha_n^j}{C_n^j} = \lambda P_n^j$;
- Multiply both sides by C_n^j to get $\alpha_n^j = \lambda P_n^j C_n^j$;
- Take sum for all sector j to get $\lambda \sum_{j=1}^J C_n^j P_n^j = \sum_{j=1}^J \alpha_n^j$;
- Using (5.3) can get $\lambda I_n = 1$;
- So we can get $\lambda = \frac{1}{I_n}$.
- Substitute into (5.2) can get:

$$C_n^j = \frac{\alpha_n^j I_n}{P_n^j} \qquad (5.4)$$

- Put (5.4) back to the utility function to get :

$$u = \prod_{j=1}^J \left(\frac{\alpha_n^j I_n}{P_n^j}\right)^{\alpha_n^j} = \left[I_n^{\sum_{j=1}^J \alpha_n^j}\right] \left[\prod_{j=1}^J \left(\frac{\alpha_n^j}{P_n^j}\right)^{\alpha_n^j}\right] \qquad (5.5)$$

- The consumption price index P_n is the income needed to buy one unit of utility:

$$
\begin{aligned}
P_n &= I_n/u \\
&= \frac{I_n}{\left[I_n^{\sum_{j=1}^{J}\alpha_n^j}\right]\left[\prod_{j=1}^{J}\left(\frac{\alpha_n^j}{P_n^j}\right)^{\alpha_n^j}\right]} \\
&= \prod_{j=1}^{J}\left(\frac{P_n^j}{\alpha_n^j}\right)^{\alpha_n^j}
\end{aligned}
$$

.

- Using the properties of the Fréchet distribution,the expenditure shares as a function of technologies, prices, and trade costs can be derived:

$$\pi_{ni}^{j} = \frac{\lambda_i^j [c_i^j k_{ni}^j]^{-\theta^j}}{\sum_{h=1}^{N} \lambda_h^j [c_h^j k_{nh}^j]^{-\theta^j}} \qquad (6)$$

- **Proof of Equation**(**6**) :
  The expenditure in country n of sector$j$ goods from country i is denoted by X_{ni}^j;
  Total expenditure on sector j goods in country n is given by $X_n^j = P_n^j Q_n^j$.
- Denote the share of expenditure of country n on goods from i are given by $\pi_{ni}^j = X_{ni}^j / X_n^j$.

- Consider country n buys the a specific good from country i if $i = \arg\min \left\{ p^j_{n1}, \ldots, p^j_{nN} \right\}$.
- Then the probability that the country i provides a good at the lowest price in country n is simply:

$$\begin{aligned} \pi^j_{ni} &= Pr[p^j_{ni}(\omega^j) \leq \min_{h \neq i} p^j_{nh}(\omega^j)] \\ &= \int_0^\infty \prod_{h \neq i} \left(1 - Pr[p^j_{nh} \leq p]\right) dF_{p^j_{ni}}(p) \\ &= \int_0^\infty \prod_{h \neq i} [1 - F_{p^j_{nh}}(p)] dF_{p^j_{ni}}(p) \\ &= \int_0^\infty \prod_{h \neq i} [1 - (1 - e^{-T^j_{nh} p^{\theta^j}})] dF_{p^j_{ni}}(p) \end{aligned}$$

- $$= \int_0^\infty \prod_{h \neq i} e^{-T^j_{nh} p^{\theta^j}} dF_{p^j_{ni}}(p)$$
$$= \int_0^\infty e^{-\Phi^j_n p^{\theta^j}} T^j_{ni} \theta^j p^{\theta^j - 1} dp$$
$$= \frac{T^j_{ni}}{\Phi^j_n} \int_0^\infty \Phi^j_n \theta^j p^{\theta^j - 1} e^{-\Phi^j_n p^{\theta^j}} dp$$
$$= \frac{T^j_{ni}}{\Phi^j_n} \left(-e^{-\Phi^j_n p^{\theta^j}} |_0^\infty \right)$$
$$= \frac{T^j_{ni}}{\Phi^j_n} = \frac{\lambda^j_i [c^j_i k^j_{ni}]^{-\theta^j}}{\sum_{h=1}^N \lambda^j_h [c^j_h k^j_{nh}]^{-\theta^j}}.$$

Total Expenditure and Trade Balance

- Total expenditure on goods j is the sum of the expenditure on composite intermediate goods by firms and the expenditure by households, which is denoted by X_n^j.
- Households have income I_n, and they spend a share α_n^j of their income on goods from sector j. Firms also spend a share of their costs purchasing intermediate goods from all sectors. Then

$$X_n^j = \sum_{k=1}^{J} \gamma_n^{j,k} \sum_{i=1}^{N} X_i^k \frac{\pi_{in}^k}{1+\tau_{in}^k} + \alpha_n^j I_n \qquad (7)$$

where, $\gamma_n^{j,k}$ is the cost share in country n and sector k on goods from sector j, and $\sum_{i=1}^{N} X_i^k \frac{\pi_{in}^k}{1+\tau_{in}^k}$ is the world total expenditure (net of tariffs) on goods produced in sector k and country i.

Total Expenditure and Trade Balance

- $I_n = w_n L_n + R_n + D_n \qquad (8)$
  where, D_n is the national deficits, R_n is tariff revenues.
- $$R_n = \sum_{j=1}^{J} \sum_{i=1}^{N} \tau_{ni}^j M_{ni}^j$$
  where, $M_{ni}^j = X_n^j \frac{\pi_{ni}^j}{1+\tau_{ni}^j}$ are country $n's$ imports of sector j goods from country i.
- the national defices D_n are the sum of sectoral defices: $D_n = \sum_{k=1}^{J} D_n^k$, and
  $$D_n^j = \sum_{i=1}^{N} M_{ni}^j - \sum_{i=1}^{N} E_{ni}^j$$
  where, $E_{ni}^j = X_i^j \frac{\pi_{in}^j}{1+\tau_{in}^j}$ are country $n's$ exports of sector j goods to country i.

Total Expenditure and Trade Balance

- There is a fact that total expenditure, excluding tariff payments, in country n minus trade deficits equals the sum of each country's total expenditure, excluding tariff payments, on tradable goods from country n

$$\sum_{j=1}^{J}\sum_{i=1}^{N} X_n^j \frac{\pi_{ni}^j}{1+\tau_{ni}^j} - D_n = \sum_{j=1}^{J}\sum_{i=1}^{N} X_i^j \frac{\pi_{in}^j}{1+\tau_{in}^j} \qquad (9)$$

where, $\sum_{n=1}^{N} D_n = 0$.

- Equation (9) reflects the fact that country $n's$ imports, excluding tariff payments, minus trade deficits equals country $n's$ exports, excluding tariff payments, on tradable goods.
- Note that aggregate trade deficits in each country are exogenous in the model, however, sectoral trade deficits are endogenously determined.

Labour market clearing

- Here we use equation (7) , (8) and (9) to get the labour market clearing.
- Add equation (7) across sectors:

$$\sum_{j=1}^{J} X_n^j = \sum_{j=1}^{J} \left[\sum_{k=1}^{J} \gamma_n^{j,k} \sum_{i=1}^{N} X_i^k \frac{\pi_{in}^k}{1+\tau_{in}^k} \right] + \sum_{j=1}^{J} \alpha_n^j I_n \qquad (7-1)$$

let $\sum_{j=1}^{J} \left[\sum_{k=1}^{J} \gamma_n^{j,k} \sum_{i=1}^{N} X_i^k \frac{\pi_{in}^k}{1+\tau_{in}^k} \right] = C$ and given $\sum_{j=1}^{J} \alpha_n^j = 1$, we can re-write (7-1) as

$$\sum_{j=1}^{J} X_n^j = C + w_n L_n + R_n + D_n$$

- For the part of "C", expand it can get

$$\sum_{j=1}^{J}\gamma_n^{j,1}\left(X_1^1\frac{\pi_{1n}^1}{1+\tau_{1n}^1}+\dots+X_n^1\frac{\pi_{nn}^1}{1+\tau_{nn}^1}\right)+\dots$$

$$+\sum_{j=1}^{J}\gamma_n^{j,1}\left(X_1^n\frac{\pi_{1n}^n}{1+\tau_{1n}^n}+\dots+X_n^n\frac{\pi_{nn}^n}{1+\tau_{nn}^n}\right)$$

$$=\left(\gamma_n^{1,1}+\dots+\gamma_n^{J,1}\right)\left(X_1^1\frac{\pi_{1n}^1}{1+\tau_{1n}^1}+\dots+X_n^1\frac{\pi_{nn}^1}{1+\tau_{nn}^1}\right)+\dots$$

$$+\left(\gamma_n^{1,n}+\dots+\gamma_n^{J,n}\right)\left(X_1^n\frac{\pi_{1n}^n}{1+\tau_{1n}^n}+\dots+X_n^n\frac{\pi_{nn}^n}{1+\tau_{nn}^n}\right)$$

- $$= \sum_{j=1}^{J}(1-\gamma_n^j)\sum_{i=1}^{N}\left(X_i^j \frac{\pi_{in}^j}{1+\tau_{in}^j}\right)$$
  $$= \sum_{j=1}^{J}\sum_{i=1}^{N}\left(X_i^j \frac{\pi_{in}^j}{1+\tau_{in}^j}\right) - \sum_{j=1}^{J}\sum_{i=1}^{N}\gamma_n^j X_i^j \frac{\pi_{in}^j}{1+\tau_{in}^j}$$
- For the part of " $w_n L_n + R_n + D_n$ ", substitute equation (9) of D_n can obtain:

$$w_n L_n + \sum_{j=1}^{J}\sum_{i=1}^{N}\tau_{ni}^j X_n^j \frac{\pi_{ni}^j}{1+\tau_{ni}^j} + \sum_{j=1}^{J}\sum_{i=1}^{N} X_n^j \frac{\pi_{ni}^j}{1+\tau_{ni}^j} - \sum_{j=1}^{J}\sum_{i=1}^{N} X_i^j \frac{\pi_{in}^j}{1+\tau_{in}^j}$$

- combine all parts, then can cancel out $\sum_{j=1}^{J}\sum_{i=1}^{N}X_i^j\frac{\pi_{in}^j}{1+\tau_{in}^j}$, so

$$\sum_{j=1}^{J}X_n^j=-\sum_{j=1}^{J}\sum_{i=1}^{N}\gamma_n^jX_i^j\frac{\pi_{in}^j}{1+\tau_{in}^j}+w_nL_n+\sum_{j=1}^{J}\sum_{i=1}^{N}X_n^j\pi_{ni}^j$$

- because $\pi_{ni}^j=X_{ni}^j/X_n^j$, so
$\sum_{j=1}^{J}\sum_{i=1}^{N}X_n^j\pi_{ni}^j=\sum_{j=1}^{J}\sum_{i=1}^{N}X_{ni}^j=\sum_{j=1}^{J}X_n^j$
- Finally, we can get

$$w_nL_n=\sum_{j=1}^{J}\gamma_n^j\sum_{i=1}^{N}X_i^j\frac{\pi_{in}^j}{1+\tau_{in}^j}$$

which is the labour market clearing.

- **Definition1**. Given $L_n, D_n, \lambda_n^j, and\ d_{ni}^j$, an equilibrium under tariff structure τ is a wage vector $\mathbf{w} \in \mathbf{R}_{++}^N$ and price $\left\{P_n^j\right\}_{j=1,n=1}^{J,N}$ that satisfy equilibrium conditions(2),(4),(6),(7),and (9) for all j, n.
- Caliendo and Parro in this paper solove for changes in prices and wages after changing from policy τ to τ'.
- Define the equilibrium of the model under policy τ relative to a policy under tariff structure τ' .
- **Definition2**. Let $(\mathbf{w}, P)$ be an equilibrium under tariff structure τ and let $(\mathbf{w}', P')$ be an equilibrium under tariff structure τ'. Define $(\widehat{\mathbf{w}}, \widehat{P})$ as an equilibrium under τ' relative to τ.
- Let a variable with a hat $\widehat{x}$ represents the relative change of the variable, namely $\widehat{x} = x'/x$. Namely, $\widehat{x}$ is the change in the equilibrium values as a results of the changes in tariffs $\widehat{\tau}$.

- Using this notation and equations (2),(4),(6),(7),and (9), the equilibrium conditions in relative changes satisfy:
- Cost of the input bundles by equation (2):

$$\widehat{c}_n^j = \frac{c_n^{j\,\prime}}{c_n^j} = \widehat{w}_n^{\gamma_n^j} \prod_{k=1}^{J} \widehat{P}_n^{k\,\gamma_n^{k,j}} \qquad (10)$$

- Price index by equation (4) and (6):

$$\widehat{P}^j_n = \frac{P^{j\,\prime}_n}{P^j_n} = \left[\frac{\sum_{i=1}^N \lambda^j_i (c^{j\prime}_i k^{j\,\prime}_{ni})^{-\theta^j}}{\sum_{i=1}^N \lambda^j_i (c^j_i k^j_{ni})^{-\theta^j}}\right]^{-\frac{1}{\theta^j}}$$

$$= \left[\sum_{i=1}^N \frac{\lambda^j_i (\mathbf{c^j_i k^j_{ni}})^{-\theta^\mathbf{j}}}{\sum_{h=1}^N \lambda^j_h (c^j_h k^j_{nh})^{-\theta^j}} \frac{(c^{j\prime}_i k^{j\,\prime}_{ni})^{-\theta^j}}{(\mathbf{c^j_i k^j_{ni}})^{-\theta^\mathbf{j}}}\right]^{-\frac{1}{\theta^j}}$$

$$= \left[\sum_{i=1}^N \pi^j_{ni} (\widehat{k}^j_{ni} \widehat{c}^j_i)^{-\theta^j}\right]^{-\frac{1}{\theta^j}} \qquad (11).$$

Equilibrium in relative changes

- Bilateral trade shares:

$$\hat{\pi}^j_{ni} = \left[\frac{\hat{c}^j_i \hat{k}^j_{ni}}{\hat{P}^j_n} \right]^{-\theta^j} \qquad (12)$$

- To get (12), rearrange (4) to obtain
$P^{j\,-\theta^j}_n = A^{j\,-\theta^j} [\sum_{i=1}^N \lambda^j_i (c^j_i k^j_{ni})^{-\theta^j}] \qquad (\bullet)$
- Substitue $(\bullet)$ into equation (6), can get $\pi^j_{ni} = \frac{\lambda^j_i}{(A^j)^{\theta^j}} [\frac{c^j_i k^j_{ni}}{P^j_n}]^{-\theta^j}$.
- Because λ^j_i is the productivity, which will not change by the change of tariff, and A^j is constant, $\hat{\pi}^j_{ni} = \pi^{j\,\prime}_{ni} / \pi^j_{ni}$ is equation (12).

Equilibrium in relative changes

- Total expenditure in each country n and sector j by equation (7) and (8) :

$$X_n^{j\prime} = \sum_{k=1}^{J} \gamma_n^{j,k} \sum_{i=1}^{N} \frac{\pi_{in}^{k\prime}}{1+\tau_{in}^{k\prime}} X_i^{k\prime} + \alpha_n^j I_n' \qquad (13)$$

where,

$$I_n' = \widehat{w}_n w_n L_n + \sum_{j=1}^{J} \sum_{i=1}^{N} \tau_{ni}^{j\prime} \frac{\pi_{ni}^{j\prime}}{1+\tau_{ni}^{j\prime}} X_n^{j\prime} + D_n,$$

here, $w_n' = \widehat{w}_n w_n$,
and

$$R_n{}' = \sum_{j=1}^{J} \sum_{i=1}^{N} \tau_{ni}^{j}{}' M_{ni}^{j}{}' = \sum_{j=1}^{J} \sum_{i=1}^{N} \tau_{ni}^{j}{}' X_n^{j\prime} \frac{\pi_{ni}^{j}{}'}{1+\tau_{ni}^{j}{}'}.$$

- Trade balance by equation (9):

$$\sum_{j=1}^{J}\sum_{i=1}^{N}\frac{\pi_{ni}^{j'}}{1+\tau_{ni}^{j'}}X_n^{j'} - D_n = \sum_{j=1}^{J}\sum_{i=1}^{N}\frac{\pi_{in}^{j'}}{1+\tau_{in}^{j'}}X_i^{j'} \tag{14}$$

where, $\widehat{k}_{ni}^{j} = \frac{(1+\tau_{ni}^{j'})d_{ni}^{j}}{(1+\tau_{ni}^{j})d_{ni}^{j}} = (1+\tau_{ni}^{j'})/(1+\tau_{ni}^{j})$.

Relative change in real wages

- The logarithm change in real wages: (15)

$$\ln \frac{\widehat{w}_n}{\widehat{P}_n} = -\underbrace{\sum_{j=1}^{J} \frac{\alpha_n^j}{\theta^j} \ln \widehat{\pi}_{nn}^j}_{\text{Final goods}} - \underbrace{\sum_{j=1}^{J} \frac{\alpha_n^j}{\theta^j} \frac{1-\gamma_n^j}{\gamma_n^j} \ln \widehat{\pi}_{nn}^j}_{\text{Intermediate goods}} - \underbrace{\sum_{j=1}^{J} \frac{\alpha_n^j}{\gamma_n^j} \ln \prod_{k=1}^{J} \left(\frac{\widehat{P}_n^k}{\widehat{P}_n^j}\right)^{\gamma_n^{k,j}}}_{\text{Sectoral linkages}}$$

- Impact of free trade on real wages can be summarized by the impact it has on domestic shares, π_{nn}^k and sectoral price indices, P_n^k.
- Changes in real wages do not directly map into changes in welfare in this model because of trade deficits D_n and tariff revenues R_n (will talk about the welfare effects in the next section).

- **Proof of Equation(15)** :
- Using equations (10) and (12) to get

$$\widehat{\pi}^j_{nn} = [\frac{\widehat{c}^j_n \widehat{k}^j_{nn}}{\widehat{P}^j_n}]^{-\theta^j} = [\frac{\widehat{w}_n^{\gamma^j_n} \prod_{k=1}^J \widehat{P}^{k\,\gamma^{k,j}_n}_n}{\widehat{P}^j_n}]^{-\theta^j} \qquad (15-1)$$

note that because $d^j_{nn} = 1, and\ \tilde{\tau}^j_{nn} = 1 + \tau = 1, \widehat{k}^j_{nn} = 1.$

- Rearranging (15-1) can get

$$\frac{\widehat{w}_n^{\gamma^j_n}}{\widehat{P}^j_n} = \frac{[(\widehat{\pi}^j_{nn})^{-1/\theta^j}]}{\prod_{k=1}^J (\widehat{P}^k_n)^{\gamma^{k,j}_n}} \qquad (15-2)$$

Relative change in real wages

- From (15-2) get

$$\frac{\widehat{w}_n}{(\widehat{P}^j_n)^{\frac{1}{\gamma^j_n}}} = \frac{(\widehat{\pi}^j_{nn})^{-\frac{1}{\theta^j \gamma^j_n}}}{\prod_{k=1}^{J} (\widehat{P}^k_n)^{\frac{\gamma^{k,j}_n}{\gamma^j_n}}} \qquad (15-3)$$

- From (15-3), multiply $\frac{(\widehat{P}^j_n)^{\frac{1}{\gamma^j_n}}}{\widehat{P}^j_n}$ can get:

$$\frac{\widehat{w}_n}{\widehat{P}^j_n} = \frac{(\widehat{\pi}^j_{nn})^{-\frac{1}{\theta^j \gamma^j_n}}}{\prod_{k=1}^{J} (\widehat{P}^k_n)^{\frac{\gamma^{k,j}_n}{\gamma^j_n}}} (\widehat{P}^j_n)^{\frac{1-\gamma^j_n}{\gamma^j_n}} \qquad (15-4)$$

- From (15-4), can get expression for all sectors j:

$$\frac{\widehat{w}_n}{\widehat{P}_n^1} = \frac{(\widehat{\pi}_{nn}^1)^{-\frac{1}{\theta^1 \gamma_n^1}}}{\prod_{k=1}^J (\widehat{P}_n^k)^{\frac{\gamma_n^{k,1}}{\gamma_n^1}}} (\widehat{P}_n^1)^{\frac{1-\gamma_n^1}{\gamma_n^1}}$$

$$\vdots$$

$$\frac{\widehat{w}_n}{\widehat{P}_n^J} = \frac{(\widehat{\pi}_{nn}^J)^{-\frac{1}{\theta^J \gamma_n^J}}}{\prod_{k=1}^J (\widehat{P}_n^k)^{\frac{\gamma_n^{k,J}}{\gamma_n^j}}} (\widehat{P}_n^j)^{\frac{1-\gamma_n^J}{\gamma_n^J}}$$

Relative change in real wages

- weighted by α_n^j from both sides can obtain:

$$\frac{(\widehat{w}_n)^{\alpha_n^1}}{(\widehat{P}_n^1)^{\alpha_n^1}} = \frac{(\widehat{\pi}_{nn}^1)^{-\frac{\alpha_n^1}{\theta^1 \gamma_n^1}}}{\prod_{k=1}^J (\widehat{P}_n^k)^{\frac{\alpha_n^1 \gamma_n^{k,1}}{\gamma_n^1}}} (\widehat{P}_n^1)^{\frac{\alpha_n^1(1-\gamma_n^1)}{\gamma_n^1}}$$

$$\vdots$$

$$\frac{(\widehat{w}_n)^{\alpha_n^J}}{(\widehat{P}_n^J)^{\alpha_n^J}} = \frac{(\widehat{\pi}_{nn}^J)^{-\frac{\alpha_n^J}{\theta^J \gamma_n^J}}}{\prod_{k=1}^J (\widehat{P}_n^k)^{\frac{\alpha_n^J \gamma_n^{k,J}}{\gamma_n^J}}} (\widehat{P}_n^J)^{\frac{\alpha_n^J(1-\gamma_n^J)}{\gamma_n^J}}$$

- Taking the product for all sectors j weighted by α_n^j, can obtain:

$$\frac{(\widehat{w}_n)^{\sum_{j=1}^J \alpha_n^j}}{\prod_{j=1}^J (\widehat{P}_n^j)^{\alpha_n^j}} = \frac{\prod_{j=1}^J (\widehat{\pi}_{nn}^j)^{-\frac{\alpha_n^j}{\theta^j \gamma_n^j}}}{\prod_{j=1}^J \prod_{k=1}^J (\widehat{P}_n^k)^{\frac{\alpha_n^j \gamma_n^{k,j}}{\gamma_n^j}}} \prod_{j=1}^J (\widehat{P}_n^j)^{\frac{\alpha_n^j (1-\gamma_n^j)}{\gamma_n^j}}$$

- because $\sum_{j=1}^J \alpha_n^j = 1$, so

$$\frac{\widehat{w}_n}{\prod_{j=1}^J (\widehat{P}_n^j)^{\alpha_n^j}} = \frac{\prod_{j=1}^J (\widehat{\pi}_{nn}^j)^{-\frac{\alpha_n^j}{\theta^j \gamma_n^j}}}{\prod_{j=1}^J \prod_{k=1}^J (\widehat{P}_n^k)^{\frac{\alpha_n^j \gamma_n^{k,j}}{\gamma_n^j}}} \prod_{j=1}^J (\widehat{P}_n^j)^{\frac{\alpha_n^j (1-\gamma_n^j)}{\gamma_n^j}} \qquad (15-5)$$

Relative change in real wages

- For the part of $\prod_{j=1}^{J}(\widehat{P}_n^j)^{\alpha_n^j}$: based on euqation (5) $P_n = \prod_{j=1}^{J} \left(P_n^j/\alpha_n^j\right)^{\alpha_n^j}$, can get

$$\prod_{j=1}^{J}(P_n^j)^{\alpha_n^j} = \prod_{j=1}^{J}\left(\frac{P_n^j}{\alpha_n^j}\alpha_n^j\right)^{\alpha_n^j} = \prod_{j=1}^{J}\left(\frac{P_n^j}{\alpha_n^j}\right)^{\alpha_n^j}\left(\alpha_n^j\right)^{\alpha_n^j} = P_n\prod_{j=1}^{J}\left(\alpha_n^j\right)^{\alpha_n^j}$$

  according to the Definition 2, $\prod_{j=1}^{J}(\widehat{P}_n^j)^{\alpha_n^j} = \widehat{P}_n$
- So the LHS of (15-5) becomes $\frac{\widehat{w}_n}{\widehat{P}_n}$

- Taking logs from both sides, can obtain

$$\ln \frac{\widehat{w}_n}{\widehat{P}_n} = -\sum_{j=1}^{J} \frac{\alpha_n^j}{\theta^j \gamma_n^j} \ln \widehat{\pi}_{nn}^j - \sum_{j=1}^{J} \frac{\alpha_n^j}{\gamma_n^j} \ln \prod_{k=1}^{J} \left(\frac{\widehat{P}_n^k}{\widehat{P}_n^j}\right)^{\gamma_n^{k,j}}$$

$$= -\sum_{j=1}^{J} \frac{\alpha_n^j}{\theta^j} \ln \widehat{\pi}_{nn}^j - \sum_{j=1}^{J} \frac{\alpha_n^j}{\theta^j} \frac{1-\gamma_n^j}{\gamma_n^j} \ln \widehat{\pi}_{nn}^j$$

$$-\sum_{j=1}^{J} \frac{\alpha_n^j}{\gamma_n^j} \ln \prod_{k=1}^{J} \left(\frac{\widehat{P}_n^k}{\widehat{P}_n^j}\right)^{\gamma_n^{k,j}}$$

note that here $-\frac{\alpha_n^j}{\theta^j \gamma_n^j} = \frac{\alpha_n^j}{\theta^j}(-1 - \frac{1-\gamma_n^j}{\gamma_n^j})$, and $\sum_{k=1}^{J} \gamma_n^{k,j} = 1 - \gamma_n^j$. Now equation (15) is proved.

- Denote welfare of the representative consumer in country n by $W_n = I_n/P_n$.
- The change in welfare is given by: (16)

$$d \ln W_n = \frac{1}{I_n} \sum_{j=1}^{J} \sum_{i=1}^{N} \underbrace{(E_{ni}^j d \ln c_n^j - M_{ni}^j d \ln c_i^j)}_{\text{Terms of Trade}} + \frac{1}{I_n} \sum_{j=1}^{J} \sum_{i=1}^{N} \underbrace{\tau_{ni}^j M_{ni}^j (d \ln M_{ni}^j - d \ln c_i^j)}_{\text{Volume of Trade}}$$

where, $E_{ni}^j = X_i^j \frac{\pi_{in}^j}{1+\tau_{in}^j}$ are country $n's$ exports of sector j goods to country i; and $M_{ni}^j = X_n^j \frac{\pi_{ni}^j}{1+\tau_{ni}^j}$ are are country $n's$ imports of sector j goods from country i.

- **Proof of Equation(16)** :
- Welfare is given by $W_n = I_n / P_n$, taking logs from both sides, based on equation (8) can get:

$$\ln W_n = \ln I_n - \ln P_n = \ln \left(w_n L_n + R_n + D_n \right) - \ln P_n$$

where, $R_n = \sum_{j=1}^{J} \sum_{i=1}^{N} \tau_{ni}^{j} M_{ni}^{j}$ are the tariff revenues.

- Total differentiating:

$$\mathrm{d} \ln W_n = \frac{L_n}{I_n} \mathrm{d} w_n + \frac{1}{I_n} \mathrm{d} R_n - \mathrm{d} \ln P_n$$

$$= \frac{w_n L_n}{I_n} \mathrm{d} \ln w_n + \frac{R_n}{I_n} \mathrm{d} \ln R_n - \mathrm{d} \ln P_n \qquad (B1)$$

Note: assuming that exogenous trade deficits remain constant, $\mathrm{d} D_n = 0$; and in general $\mathrm{d} \ln x = \frac{1}{x} \mathrm{d} x$.

Welfare effects from tariff changes

- $\mathrm{d}R_n = \sum_{j=1}^{J} \sum_{i=1}^{N} \frac{\partial R_n}{\partial M_{ni}^j} \mathrm{d}M_{ni}^j + \sum_{j=1}^{J} \sum_{i=1}^{N} \frac{\partial R_n}{\partial \tau_{ni}^j} \mathrm{d}\tau_{ni}^j$
$$= \sum_{j=1}^{J} \sum_{i=1}^{N} \tau_{ni}^j \mathrm{d}M_{ni}^j + \sum_{j=1}^{J} X_n^j \sum_{i=1}^{N} \frac{\pi_{ni}^j}{1+\tau_{ni}^j} \mathrm{d}\tau_{ni}^j$$
- Then can get

$$\mathrm{d}R_n = \sum_{j=1}^{J} \sum_{i=1}^{N} \tau_{ni}^j M_{ni}^j \mathrm{d}\ln M_{ni}^j + \sum_{j=1}^{J} X_n^j \sum_{i=1}^{N} \pi_{ni}^j \mathrm{d}\ln \tilde{\tau}_{ni}^j \qquad (B2)$$

where, $1 + \tau_{ni}^j = \tilde{\tau}_{ni}^j$.

- Taking logs for the consumption price index equation (5) can obtain: $\ln P_n = \sum_{j=1}^{J} \alpha_n^j \ln P_n^j - \sum_{j=1}^{J} \alpha_n^j \ln \alpha_n^j$
- Totally differentiating equation (5), can get $\mathrm{d}lnP_n = \sum_{j=1}^{J} \alpha_n^j \mathrm{d}\ln P_n^j$, where $\sum_{j=1}^{J} \alpha_n^j \mathrm{d}\ln \alpha_n^j = 0$.
- Using the intermediate goods market clearing condition(7) to solve for α_n^j: $\alpha_n^j = \frac{X_n^j}{I_n} - \frac{1}{I_n} \sum_{k=1}^{J} \gamma_n^{j,k} \sum_{i=1}^{N} X_i^k \frac{\pi_{in}^k}{1+\tau_{in}^k}$
- Then get

$$\mathrm{d}lnP_n = \sum_{j=1}^{J} \alpha_n^j \mathrm{d}\ln P_n^j$$

$$= \sum_{j=1}^{J} \left[\frac{X_n^j}{I_n} - \frac{1}{I_n} \sum_{k=1}^{J} \gamma_n^{j,k} \sum_{i=1}^{N} X_i^k \frac{\pi_{in}^k}{1+\tau_{in}^k} \right] \mathrm{d}\ln P_n^j$$

- For $\mathrm{d}\ln P_n^j$, taking logs for equation (4) to get : $lnP_n^j = lnA^j - \frac{1}{\theta^j} ln \sum_{i=1}^N \lambda_i^j (c_i^j k_{ni}^j)^{-\theta^j}$ then totally differentiating it can get

$$dlnP_n^j = d\left[-\frac{1}{\theta^j} ln \sum_{i=1}^N \lambda_i^j (c_i^j k_{ni}^j)^{-\theta^j}\right]$$

$$= \sum_{i=1}^N \frac{\lambda_i^j (c_i^j k_{ni}^j)^{-\theta^j}}{\sum_{h=1}^N \lambda_h^j (c_h^j k_{nh}^j)^{-\theta^j}} dlnc_i^j + \sum_{i=1}^N \frac{\lambda_i^j (c_i^j k_{ni}^j)^{-\theta^j}}{\sum_{h=1}^N \lambda_h^j (c_h^j k_{nh}^j)^{-\theta^j}} dlnk_{ni}^j$$

- combined with equation (6), and holding iceberg trade costs fixed, namely $\mathrm{d}lnk_{ni}^j = \mathrm{d}ln\tilde{\tau}_{ni}^j$,

$$\mathrm{d}lnP_n^j = \sum_{i=1}^N \pi_{ni}^j [\mathrm{d}lnc_i^j + \mathrm{d}ln\tilde{\tau}_{ni}^j]$$

- Then get

$$
\begin{aligned}
\mathrm{d} lnP_n &= \sum_{j=1}^{J} \frac{X_n^j}{I_n} \mathrm{d} \ln P_n^j - \frac{1}{I_n} \sum_{j=1}^{J} \sum_{k=1}^{J} \gamma_n^{j,k} \sum_{i=1}^{N} X_i^k \frac{\pi_{in}^k}{1+\tau_{in}^k} \mathrm{d} \ln P_n^j \\
&= \sum_{j=1}^{J} \frac{X_n^j}{I_n} \sum_{i=1}^{N} \pi_{ni}^j \mathrm{d} \ln c_i^j + \sum_{j=1}^{J} \frac{X_n^j}{I_n} \sum_{i=1}^{N} \pi_{ni}^j \mathrm{d} \ln \tilde{\tau}_{ni}^j \\
&- \frac{1}{I_n} \sum_{j=1}^{J} \sum_{k=1}^{J} \gamma_n^{j,k} \sum_{i=1}^{N} X_i^k \frac{\pi_{in}^k}{1+\tau_{in}^k} \mathrm{d} \ln P_n^j \qquad (B3)
\end{aligned}
$$

Welfare effects from tariff changes

- Taking logs from both sides and totally differentiating equation (2) can obtain an expression for the change in wages:

$$\gamma_n^j \mathrm{d} ln w_n = \mathrm{d} ln c_n^j - \sum_{k=1}^{J} \gamma_n^{k,j} \mathrm{d} ln p_n^k \qquad (B4)$$

- Substituting (B2) and (B3) into (B1) obtain

$$\mathrm{d}\ln W_n = \frac{w_n L_n}{I_n} \mathrm{d}\ln w_n + \frac{1}{I_n} \sum_{j=1}^{J} \sum_{i=1}^{N} \tau_{ni}^j M_{ni}^j \mathrm{d}\ln M_{ni}^j$$

$$+ \frac{1}{I_n} \sum_{j=1}^{J} X_n^j \sum_{i=1}^{N} \pi_{ni}^j \mathrm{d}\ln \tilde{\tau}_{ni}^j - \sum_{j=1}^{J} \frac{X_n^j}{I_n} \sum_{i=1}^{N} \pi_{ni}^j \mathrm{d}\ln \tilde{\tau}_{ni}^j$$

$$- \sum_{j=1}^{J} \frac{X_n^j}{I_n} \sum_{i=1}^{N} \pi_{ni}^j \mathrm{d}\ln c_i^j + \frac{1}{I_n} \sum_{j=1}^{J} \sum_{k=1}^{J} \gamma_n^{j,k} \sum_{i=1}^{N} X_i^k \frac{\pi_{in}^k}{1+\tau_{in}^k} \mathrm{d}\ln p_n^j$$

- continued:

$$d\ln W_n = \frac{w_n L_n}{I_n} d\ln w_n + \frac{1}{I_n}\sum_{j=1}^{J}\sum_{i=1}^{N}\tau_{ni}^{j} M_{ni}^{j} d\ln M_{ni}^{j}$$

$$+\frac{1}{I_n}\sum_{j=1}^{J}\sum_{k=1}^{J}\gamma_n^{j,k}\sum_{i=1}^{N} X_i^k \frac{\pi_{in}^k}{1+\tau_{in}^k} d\ln p_n^j$$

$$-\sum_{j=1}^{J}\frac{X_n^j}{I_n}\sum_{i=1}^{N}\pi_{ni}^{j} d\ln c_i^j \qquad (B5)$$

Welfare effects from tariff changes

- Because $E_{ni}^k = X_i^k \frac{\pi_{in}^k}{1+\tau_{in}^k}$,

$$\frac{1}{I_n} \sum_{j=1}^{J} \sum_{k=1}^{J} \gamma_n^{j,k} \sum_{i=1}^{N} X_i^k \frac{\pi_{in}^k}{1+\tau_{in}^k} \mathrm{d} \ln P_n^j$$

$$= \frac{1}{I_n} \sum_{j=1}^{J} \sum_{k=1}^{J} \gamma_n^{j,k} \sum_{i=1}^{N} E_{ni}^k \mathrm{d} lnP_n^j$$

$$= \frac{1}{I_n} \sum_{j=1}^{J} \sum_{i=1}^{N} E_{ni}^j \sum_{k=1}^{J} \gamma_n^{k,j} \mathrm{d} lnP_n^k$$

The proof of the equation is on the next page.

- The proof of the equation:

$$\frac{1}{I_n}\sum_{j=1}^{J}\sum_{k=1}^{J}\gamma_n^{j,k}\sum_{i=1}^{N}E_{ni}^{k}\mathrm{d}lnP_n^j=\frac{1}{I_n}\sum_{j=1}^{J}\sum_{i=1}^{N}E_{ni}^{j}\sum_{k=1}^{J}\gamma_n^{k,j}\mathrm{d}lnP_n^k$$

- Rewrite $T_1=\sum_{j=1}^{J}\sum_{k=1}^{J}\gamma_n^{j,k}\sum_{i=1}^{N}E_{ni}^{k}\mathrm{d}lnP_n^j$

$$=\sum_{j=1}^{J}\mathrm{d}lnP_n^j\sum_{k=1}^{J}\gamma_n^{j,k}\sum_{i=1}^{N}E_{ni}^{k}$$

$$=\sum_{j=1}^{J}\mathrm{d}lnP_n^j\sum_{k=1}^{J}\gamma_n^{j,k}(E_{n1}^{k}+E_{n2}^{k}+\cdots+E_{nN}^{k})$$

- Let $\mathbf{A}_{1\times J} = \mathrm{d}lnP_n^j \begin{pmatrix} 1 & \dots & 1 \end{pmatrix}_{1\times J} = \begin{pmatrix} \mathrm{d}lnP_n^j & \dots & \mathrm{d}lnP_n^j \end{pmatrix}_{1\times J}$
- $\mathbf{G}_{J\times 1} = \begin{pmatrix} \gamma_n^{j,1}(E_{n1}^1 + E_{n2}^1 + \dots + E_{nN}^1) \\ \vdots \\ \gamma_n^{j,J}(E_{n1}^J + E_{n2}^J + \dots + E_{nN}^J) \end{pmatrix}_{J\times 1}$
- now, $T_1 = \sum_{j=1}^J \mathbf{A}_{1\times J}\mathbf{G}_{J\times 1}$

  $= \sum_{j=1}^J \mathrm{d}lnP_n^j \left[\gamma_n^{j,1}(E_{n1}^1 + \dots + E_{nN}^1) + \dots + \gamma_n^{j,J}(E_{n1}^J + \dots + E_{nN}^J)\right]$

  $= \mathrm{d}lnP_n^1 \left[\gamma_n^{1,1}(E_{n1}^1 + \dots + E_{nN}^1) + \dots + \gamma_n^{1,J}(E_{n1}^J + \dots + E_{nN}^J)\right] +$

  $\dots + \mathrm{d}lnP_n^J \left[\gamma_n^{J,1}(E_{n1}^1 + \dots + E_{nN}^1) + \dots + \gamma_n^{J,J}(E_{n1}^J + \dots + E_{nN}^J)\right]$

Welfare effects from tariff changes

- Let $T_2 = \sum_{j=1}^{J} \sum_{i=1}^{N} E_{ni}^{j} \sum_{k=1}^{J} \gamma_n^{k,j} \mathrm{d}lnP_n^k$
- $\mathbf{B}_{1\times J} = \begin{pmatrix} 1 & \dots & 1 \end{pmatrix}_{1\times J}$
- $\mathbf{V}_{J\times 1} = \sum_{i=1}^{N} E_{ni}^{j} \begin{pmatrix} \gamma_n^{1,j} \mathrm{d}lnP_n^1 \\ \vdots \\ \gamma_n^{J,j} \mathrm{d}lnP_n^J \end{pmatrix}_{J\times 1}$

$$= \begin{pmatrix} (E_{n1}^{j} + \dots + E_{nN}^{j}) \gamma_n^{1,j} \mathrm{d}lnP_n^1 \\ \vdots \\ (E_{n1}^{j} + \dots + E_{nN}^{j}) \gamma_n^{J,j} \mathrm{d}lnP_n^J \end{pmatrix}_{J\times 1}$$

- now, $T_2 = \sum_{j=1}^{J} \mathbf{B}_{1\times J} \mathbf{V}_{J\times 1}$

- $T_2 = \sum_{j=1}^{J} \mathbf{B}_{1\times J}\mathbf{V}_{1\times J}$
  $= \sum_{j=1}^{J} \left[(E_{n1}^{j} + \cdots + E_{nN}^{j})(\gamma_n^{1,j}\mathrm{d}lnP_n^1 + \cdots + \gamma_n^{J,j}\mathrm{d}lnP_n^J \right]$
  $= (E_{n1}^{1} + \cdots + E_{nN}^{1})(\gamma_n^{1,1}\mathrm{d}lnP_n^1 + \cdots + \gamma_n^{J,1}\mathrm{d}lnP_n^J)+$
  $\cdots + (E_{n1}^{J} + \cdots + E_{nN}^{J})(\gamma_n^{1,J}\mathrm{d}lnP_n^1 + \cdots + \gamma_n^{J,J}\mathrm{d}lnP_n^J)$
- So $T_1 = T_2$, the euqation is proved.

- The definition of $M_{ni}^{j} = X_{n}^{j} \frac{\pi_{ni}^{j}}{1+\tau_{ni}^{j}}$ can be rewriten as $(1+\tau_{ni}^{j}) M_{ni}^{j} = X_{n}^{j} \pi_{ni}^{j}$
- Based on (B5),

$$
\begin{aligned}
& -\sum_{j=1}^{J} \frac{X_{n}^{j}}{I_{n}} \sum_{i=1}^{N} \pi_{ni}^{j} \mathrm{d} \ln c_{i}^{j} \\
& = -\frac{1}{I_{n}} \sum_{j=1}^{J} \sum_{i=1}^{N} (1+\tau_{ni}^{j}) M_{ni}^{j} \mathrm{d} \ln c_{i}^{j} \\
& = -\frac{1}{I_{n}} \sum_{j=1}^{J} \sum_{i=1}^{N} M_{ni}^{j} \mathrm{d} \ln c_{i}^{j} - \frac{1}{I_{n}} \sum_{j=1}^{J} \sum_{i=1}^{N} \tau_{ni}^{j} M_{ni}^{j} \mathrm{d} \ln c_{i}^{j}
\end{aligned}
$$

- Now (B5) can be expanded to

$$
\begin{aligned}
d \ln W_n = \frac{w_n L_n}{I_n} d \ln w_n + \frac{1}{I_n} \sum_{j=1}^{J} \sum_{i=1}^{N} \tau_{ni}^{j} M_{ni}^{j} d \ln M_{ni}^{j} \\
+ \frac{1}{I_n} \sum_{j=1}^{J} \sum_{i=1}^{N} E_{ni}^{j} \sum_{k=1}^{J} \gamma_n^{k,j} d ln p_n^k \\
- \frac{1}{I_n} \sum_{j=1}^{J} \sum_{i=1}^{N} M_{ni}^{j} d \ln c_i^j - \frac{1}{I_n} \sum_{j=1}^{J} \sum_{i=1}^{N} \tau_{ni}^{j} M_{ni}^{j} d \ln c_i^j \qquad (B6)
\end{aligned}
$$

- Adding and substracting $\frac{1}{I_n}\sum_{j=1}^{J}\sum_{i=1}^{N} E_{ni}^{j} \mathrm{d}\ln c_n^j$ from (B6) to get:

$$\mathrm{d}\ln W_n = \frac{w_n L_n}{I_n}\mathrm{d}\ln w_n + \frac{1}{I_n}\sum_{j=1}^{J}\sum_{i=1}^{N}\tau_{ni}^{j} M_{ni}^{j}(\mathrm{d}\ln M_{ni}^{j} - \mathrm{d}\ln c_i^j)$$

$$-\frac{1}{I_n}\sum_{j=1}^{J}\sum_{i=1}^{N} E_{ni}^{j}(\mathrm{d}\ln c_n^j - \sum_{k=1}^{J}\gamma_n^{k,j}\mathrm{d}\ln p_n^k)$$

$$+\frac{1}{I_n}\sum_{j=1}^{J}\sum_{i=1}^{N}(E_{ni}^{j}\mathrm{d}\ln c_n^j - M_{ni}^{j}\mathrm{d}\ln c_i^j) \qquad (B7)$$

Welfare effects from tariff changes

- using (B4) and the labor market clearing condition

$$w_n L_n = \sum_{j=1}^{J} \gamma_n^j \sum_{i=1}^{N} X_i^j \frac{\pi_{in}^j}{1+\tau_{in}^j},$$

$$-\frac{1}{I_n} \sum_{j=1}^{J} \sum_{i=1}^{N} E_{ni}^j (\mathrm{d} \ln c_n^j - \sum_{k=1}^{J} \gamma_n^{k,j} \mathrm{d} \ln p_n^k)$$

$$= -\frac{1}{I_n} \sum_{j=1}^{J} \sum_{i=1}^{N} E_{ni}^j \gamma_n^j \mathrm{d} ln w_n$$

$$= -\frac{1}{I_n} \sum_{j=1}^{J} \sum_{i=1}^{N} \gamma_n^j X_i^j \frac{\pi_{in}^j}{1+\tau_{in}^j} \mathrm{d} ln w_n = -\frac{1}{I_n} (w_n L_n) \mathrm{d} ln w_n$$

- Finally, rearrange (B7) to get equation (16)

$$\mathrm{d} \ln W_n = \frac{1}{I_n} \sum_{j=1}^{J} \sum_{i=1}^{N} \underbrace{(E_{ni}^j \mathrm{d} \ln c_n^j - M_{ni}^j \mathrm{d} \ln c_i^j)}_{\text{Terms of Trade}} + \frac{1}{I_n} \sum_{j=1}^{J} \sum_{i=1}^{N} \underbrace{\tau_{ni}^j M_{ni}^j (\mathrm{d} \ln M_{ni}^j - \mathrm{d} \ln c_i^j)}_{\text{Volume of Trade}}$$

- From equation (16) we can define bilateral and sectoral measures of terms of trade and volume of trade that can be used to decompose the welfare effects across countries and sectors.
- The change in bilateral terms of trade between countries n and i is given by

$$\mathrm{d}\ln tot_{ni} = \sum_{j=1}^{J} (E_{ni}^{j} \mathrm{d}\ln c_{n}^{j} - M_{ni}^{j} \mathrm{d}\ln c_{i}^{j}) \qquad (17)$$

- The change in the bilateral volume of trade is given by

$$\mathrm{d}\ln vot_{ni} = \sum_{j=1}^{J} \tau_{ni}^{j} M_{ni}^{j} (\mathrm{d}\ln M_{ni}^{j} - \mathrm{d}\ln c_{i}^{j}) \qquad (18)$$

Welfare effects from tariff changes

- The change in sectoral terms of trade is measured by

$$d\ln tot_n^j = \sum_{i=1}^{N} (E_{ni}^j d\ln c_n^j - M_{ni}^j d\ln c_i^j) \qquad (19)$$

- The change in sectoral volume of trade is given by

$$d\ln vot_n^j = \sum_{i=1}^{N} \tau_{ni}^j M_{ni}^j (d\ln M_{ni}^j - d\ln c_i^j) \qquad (20)$$

- The change in welfare in country n can also be computed as

$$d\ln W_n = \frac{1}{I_n} \sum_{i=1}^{N} (d\ln tot_{ni} + d\ln vot_{ni})$$

$$= \frac{1}{I_n} \sum_{j=1}^{J} (d\ln tot_n^j + d\ln vot_n^j)$$

Data description

- The base year is 1993.
- In the data sample, the number of countries $N = 31$, 30 countries and a constructed rest of the world; the number of sectors $J = 40$ (20 tradable and 20 non-tradable).
- The data needed are bilateral trade flows (M_{ni}^j), value added (V_n^j), gross production (Y_n^j), and I–O tables.
  $*$ Bilateral trade flows (M_{ni}^j) are sourced from the United Nations Statistical Division (UNSD) Commodity Trade (COMTRADE) database.

Data description

- ∗ value added $V_n^j = w_n L_n$; and gross production (Y_n^j) is national accounts compatible production in current prices.

  $<OECDSTAN1998><OECDSTAN2020>$

  ∗ value added and gross production for the year 1993 come from three different sources:

  1. OECD STAN database for industrial analysis for OECD countries at the sectoral level;

  2. the Industrial Statistics Database INDSTAT2 for the remaining countries;

  3. the OECD Input-Output database for the rremaining countries and sectors are not available from the first two databases. The data obtained here was originally for year 1995; the authors converted value added and gross output to the year 1993.

- By combining information from the WIOD and the OECD Input-Output database to construct the share of intermediate inputs from each sector in sectoral gross output.
  $*$ WIOD contains I-O tables for 40 countries over period 1995-2011. The authors calculated the I-O coefficients from WIOD database.
  $*$ the information from OECD I-O tables was used to split these aggregate sectors into sectoral classification.
- The I-O table for the rest of the world was constructed using the median coefficients.

- With these data, the data counterparts of $\pi^j_{ni}, \gamma^j_n, \gamma^{j,k}_n$, *and* α^j_n can be calculated.
- Calibration of the observed parameters:
  $*\ \pi^j_{ni} = X^j_{ni} / \sum_{i=1}^N X^j_{ni}$, where, $X^j_{ni} = M^j_{ni}(1+\tau^j_{ni})$;
  $*\ \alpha^j_n = (Y^j_n + D^j_n - \sum_{k=1}^J \gamma^{j,k}_n Y^k_n)/I_n$, where $D^j_n = \sum_{i=1}^N M^j_{ni} - \sum_{i=1}^N M^j_{in}$
  $*\gamma^{j,k}_n$ is calculated from the I-O tables as the share of intermediate consumption of sector j in sector k over total intermediate consumption of sector k times $1-\gamma^j_n$; and $\gamma^j_n = V^j_n / Y^j_n$.
- The only parameters missing are the sectoral dispersion of productivity θ^j.

The trade elasticities θ^j

- The trade elasticities are the only parameters needed to estimate to identify the effects of tariff reductions.
- The trade elasticities are related to the dispersion of productivity parameter and it determines how trade flows react to changes in tariffs.
- If productivity is less dispersed, as indicated by a larger value of θ^j, then a change in tariffs will not change the share of traded goods in a substantial way. The reason is that goods are less substitutable.
- If the productivities are less concentrated —if there is high dispersion— small changes in tariffs can translate to large adjustments in the share of goods traded. The reason is that producers of the composite aggregate are more likely to change their suppliers, since goods are more substitutable.

New method estimating trade elasticities θ^j

- A new method is proposed to estimate the dispersion parameter: Consider three countries indexed by $n, i, and\ h.$ Take the cross-product of goods from sector j shipped in one direction between the three countries, from n to i, from i to h, and from h to$n$, and then the cross-product of the same goods shipped in the other direction, from n to h, from h to i, and from i to n.
- Based on $\pi^j_{ni} = X^j_{ni}/X^j_n$, we can get $X^j_{ni} = \pi^j_{ni}X^j_n$. In the same way, can get

$$X^j_{ih} = \pi^j_{ih}X^j_i \qquad X^j_{hn} = \pi^j_{hn}X^j_h$$

$$X^j_{nh} = \pi^j_{nh}X^j_n \qquad X^j_{hi} = \pi^j_{hi}X^j_h \qquad X^j_{in} = \pi^j_{in}X^j_i$$

- By cancling out all X_n^j, X_n^j, X_h^j can get:

$$\frac{X_{ni}^j X_{ih}^j X_{hn}^j}{X_{nh}^j X_{hi}^j X_{in}^j} = \frac{\pi_{ni}^j \pi_{ih}^j \pi_{hn}^j}{\pi_{nh}^j \pi_{hi}^j \pi_{in}^j}$$

- Using equation (6), $\pi_{ni}^j = \frac{\lambda_i^j [c_i^j k_{ni}^j]^{-\theta^j}}{\sum_{h=1}^N \lambda_h^j [c_h^j k_{nh}^j]^{-\theta^j}}$, we can calculate each expression and then take the ratio:

$$\frac{X_{ni}^j X_{ih}^j X_{hn}^j}{X_{nh}^j X_{hi}^j X_{in}^j} = \left(\frac{k_{ni}^j}{k_{in}^j} \frac{k_{ih}^j}{k_{hi}^j} \frac{k_{hn}^j}{k_{nh}^j} \right)^{-\theta^j} \qquad (21)$$

- The advantage of using equation (21) is that unobservable trade costs cancel out.

  Note: to see these effects more formally, from equation (6) note how changes in trade costs impact trade shares according to θ^j.

New method estimating trade elasticities θ^j

- From the definition of k^j_{ni} in equation (3), taking logs from both sides can get $\ln k^j_{ni} = \ln \tilde{\tau}^j_{ni} + \ln d^j_{ni}$
- Iceberg trade costs $\ln d^j_{ni}$ can be modelled quite generally as linear functions of cross-country characteristics, for example, $\ln d^j_{ni} = \nu^j_{ni} + \mu^j_n + \delta^j_i + \epsilon^j_{ni}$

  where, $\nu^j_{ni} = \nu^j_{in}$ captures symmetric bilateral trade costs like distance, language, commonborder, and belonging to an FTA or not;

  μ^j_n captures an importer sectoral fixed effect;

  δ^j_i is an exporter sectoral fixed effect that can also capture non-tariff barriers, and it is assumed to be common to all trading partners of country i;

  ϵ^j_{ni} is a random disturbance term.
- then can get

$$\ln k^j_{ni} = \ln \tilde{\tau}^j_{ni} + \nu^j_{ni} + \mu^j_n + \delta^j_i + \epsilon^j_{ni} \qquad (22)$$

- From equation (21), taking logs from both sides:

$$\ln\left(\frac{X^j_{ni}X^j_{ih}X^j_{hn}}{X^j_{nh}X^j_{hi}X^j_{in}}\right) = -\theta^j \ln\left(\frac{k^j_{ni}}{k^j_{in}}\frac{k^j_{ih}}{k^j_{hi}}\frac{k^j_{hn}}{k^j_{nh}}\right)$$

- For the right hand side, it can be expressed as

$$-\theta^j [\ln k^j_{ni} - \ln k^j_{in} + \ln k^j_{ih} - \ln k^j_{hi} + \ln k^j_{hn} - \ln k^j_{nh}]$$

by substituting equation (22), and rearranging to get:

$$-\theta^j \ln\left(\frac{\tilde{\tau}^j_{ni}}{\tilde{\tau}^j_{in}}\frac{\tilde{\tau}^j_{ih}}{\tilde{\tau}^j_{hi}}\frac{\tilde{\tau}^j_{hn}}{\tilde{\tau}^j_{nh}}\right) + [\epsilon^j_{in} - \epsilon^j_{ni} + \epsilon^j_{hi} - \epsilon^j_{ih} + \epsilon^j_{nh} - \epsilon^j_{hn}]$$

- Denote $\tilde{\epsilon}^j = \epsilon^j_{in} - \epsilon^j_{ni} + \epsilon^j_{hi} - \epsilon^j_{ih} + \epsilon^j_{nh} - \epsilon^j_{hn}$, we can get

$$\ln\left(\frac{X^j_{ni}X^j_{ih}X^j_{hn}}{X^j_{in}X^j_{hi}X^j_{nh}}\right) = -\theta^j \ln\left(\frac{\tilde{\tau}^j_{ni}}{\tilde{\tau}^j_{in}}\frac{\tilde{\tau}^j_{ih}}{\tilde{\tau}^j_{hi}}\frac{\tilde{\tau}^j_{hn}}{\tilde{\tau}^j_{nh}}\right) + \tilde{\epsilon}^j \tag{23}$$

- Note that the methodology is consistent with a wide class of gravity-trade models, and therefore, the estimated trade cost elasticity from using this method does not depend on the underlying microstructure assumed in the model.
- Caliendo and Parro estimate the dispersion-of-productivity parameter sector by sector using the proposed specification equation (23) for 1993, the year before NAFTA was active.
- Note that they estimate equation (23) by OLS, dropping the observations with zeros.

Solving the model for tariff changes

- Consider a change in policy from τ to the new policy τ', captured by $\widehat{k}^j_{ni}$ or a change in D_n to $D_n{}'$.
- *Step 1* : Guess a vector of wages $\widehat{\mathbf{w}} = (\widehat{w}_1, ..., \widehat{w}_N)$, *e.g.* $\widehat{\mathbf{w}} = 1$.
- *Step 2* : Use equilibrium conditions (10) and (11) to solve for prices in each sector and each country, $\widehat{p}^j_n(\widehat{\mathbf{w}})$ and input costs, $\widehat{c}^j_n(\widehat{\mathbf{w}})$ consistent with the vector of wages $\widehat{\mathbf{w}}$.
- *Step 3* : Use the information on π^j_{ni} and θ^j together with the solutions to $\widehat{p}^j_n(\widehat{\mathbf{w}})$ and $\widehat{c}^j_n(\widehat{\mathbf{w}})$ from step 2 and solve for $\pi^j_{ni}{}'(\widehat{\mathbf{w}})$ using (12).

- *Step 4* : Given $\pi_{ni}^{j\ \prime}(\widehat{\mathbf{w}})$ from step 3, the new tariff vector τ', and the data for $\gamma_n^j, \gamma_n^{j,k}$, *and* α_n^j, solve for total expenditure in each sector j and country n, $X_n^{j\prime}(\widehat{\mathbf{w}})$ consistent with the vector of wages $\widehat{\mathbf{w}}$ in the following way. Note that from equation (13), the total expenditure in the counterfactural scenario is given by: $(C1)$

$$X_n^{j\prime} = \sum_{k=1}^{J} \gamma_n^{j,k} \sum_{i=1}^{N} \frac{\pi_{in}^{k\ \prime}(\widehat{\mathbf{w}})}{1+\tau_{in}^{k\prime}} X_i^{k\prime} + \alpha_n^j(\widehat{w} w_n L_n + \sum_{j=1}^{J}\sum_{i=1}^{N} \tau_{ni}^{j\ \prime} M_{ni}^j(\widehat{\mathbf{w}}) + D_n{}')$$

- Equation (C1) is a system of $J \times N$ total expenditures.

- Rearrange (C1)

$$X_n^{j\prime} - \sum_{k=1}^{J} \gamma_n^{j,k} \sum_{i=1}^{N} \frac{\pi_{in}^{k\prime}(\widehat{\mathbf{w}})}{1+\tau_{in}^{k\prime}} X_i^{k\prime} - \alpha_n^j \sum_{j=1}^{J} \sum_{i=1}^{N} \tau_{ni}^{j\prime} M_{ni}^j(\widehat{\mathbf{w}})$$
$$= \alpha_n^j(\widehat{w} w_n L_n + D_n{}')$$

then can get:

$$\left[1 - \alpha_n^j \sum_{j=1}^{J} \sum_{i=1}^{N} \tau_{ni}^{j\prime} \frac{\pi_{ni}^{j\prime}(\widehat{\mathbf{w}})}{1+\tau_{ni}^{j\prime}}\right] X_n^{j\prime} - \left[\sum_{k=1}^{J} \gamma_n^{j,k} \sum_{i=1}^{N} \frac{\pi_{in}^{k\prime}(\widehat{\mathbf{w}})}{1+\tau_{in}^{k\prime}}\right] X_i^{k\prime}$$

$$= \alpha_n^j(\widehat{w} w_n L_n + D_n{}')$$

where, $M_{ni}^j(\widehat{\mathbf{w}}) = \frac{\pi_{ni}^{j\prime}(\widehat{\mathbf{w}})}{1+\tau_{ni}^{j\prime}} X_n^{j\prime}$.

Solving the model for tariff changes

- It is convenient to re-write the system of equations in matrix form:

$$\left[\mathbf{I} - \mathbf{F}(\widehat{\mathbf{w}})\right]\mathbf{X} - \tilde{H}(\widehat{\mathbf{w}})\mathbf{X} = \triangle(\widehat{\mathbf{w}})$$

$$\Longleftrightarrow \left[\left(\mathbf{I} - \mathbf{F}(\widehat{\mathbf{w}})\right) - \tilde{H}(\widehat{\mathbf{w}})\right]\mathbf{X} = \triangle(\widehat{\mathbf{w}})$$

where $\mathbf{X}$ is the vector of expenditures for each sector and country and $\triangle(\widehat{\mathbf{w}})$ is a vector containing the shares of each sector and country in final demand, value added and aggregate trade deficit by country:

$$\mathbf{X} = \begin{pmatrix} X_1^{1\,\prime} \\ \vdots \\ X_1^{J\,\prime} \\ \vdots \\ X_n^{1\,\prime} \\ \vdots \\ X_N^{J\,\prime} \end{pmatrix}_{JN\times 1} \; ; \; \triangle(\widehat{\mathbf{w}}) = \begin{pmatrix} \alpha_1^1(\widehat{w}_1 w_1 L_1 + D_1') \\ \vdots \\ \alpha_1^J(\widehat{w}_1 w_1 L_1 + D_1') \\ \vdots \\ \alpha_N^1(\widehat{w}_N w_N L_N + D_N') \\ \vdots \\ \alpha_N^J(\widehat{w}_N w_N L_N + D_N') \end{pmatrix}_{JN\times 1} .$$

- The matrix $\mathbf{I}$ is the identity matrix with dimensions $JN \times JN$.
- The square matrix $\mathbf{F}(\widehat{\mathbf{w}})$ is constructed using:

$$A_n = \begin{pmatrix} \alpha_n^1 \\ \vdots \\ \alpha_n^J \end{pmatrix}_{J\times 1}, \tilde{F}'_n(\widehat{\mathbf{w}}) = \left(\left(1 - F_n^{1\prime}(\widehat{\mathbf{w}})\right) \cdots \left(1 - F_n^{J\prime}(\widehat{\mathbf{w}})\right) \right)_{1\times J}$$

where, $F_n^{j\prime}(\widehat{\mathbf{w}}) = \sum_{i=1}^{N} \frac{\pi_{ni}^{j\ \prime}}{1+\tau_{ni}^{j\ \prime}}$.

- The matrix $\mathbf{F}(\widehat{\mathbf{w}})$ then is defined as:

$$\mathbf{F}(\widehat{\mathbf{w}}) = \begin{pmatrix} A_1 \otimes \tilde{F}'_1(\widehat{\mathbf{w}}) & 0_{J\times J} & \cdots & 0_{J\times J} & 0_{J\times J} \\ 0_{J\times J} & A_2 \otimes \tilde{F}'_2(\widehat{\mathbf{w}}) & \cdots & \vdots & \vdots \\ 0_{J\times J} & 0_{J\times J} & \ddots & 0_{J\times J} & 0_{J\times J} \\ \vdots & \vdots & \cdots & A_{N-1} \otimes \tilde{F}'_{N-1}(\widehat{\mathbf{w}}) & 0_{J\times J} \\ 0_{J\times J} & 0_{J\times J} & \cdots & 0_{J\times J} & A_N \otimes \tilde{F}'_N(\widehat{\mathbf{w}}) \end{pmatrix}_{JN\times JN}.$$

- The square matrix $\tilde{H}(\widehat{\mathbf{w}})$ is given by:

$$\tilde{H}(\widehat{\mathbf{w}}) = \begin{pmatrix} \gamma_1^{1,1}\tilde{\pi}_{1,1}^{1'}(\widehat{\mathbf{w}}) & \dots & \gamma_1^{1,J}\tilde{\pi}_{1,1}^{J'}(\widehat{\mathbf{w}}) & \cdots & \gamma_1^{1,1}\tilde{\pi}_{N,1}^{1'}(\widehat{\mathbf{w}}) & \cdots & \gamma_1^{1,J}\tilde{\pi}_{N,1}^{J'}(\widehat{\mathbf{w}}) \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ \gamma_1^{J,1}\tilde{\pi}_{1,1}^{1'}(\widehat{\mathbf{w}}) & \dots & \gamma_1^{J,J}\tilde{\pi}_{1,1}^{J'}(\widehat{\mathbf{w}}) & \cdots & \gamma_1^{J,1}\tilde{\pi}_{N,1}^{1'}(\widehat{\mathbf{w}}) & \cdots & \gamma_1^{J,J}\tilde{\pi}_{N,1}^{J'}(\widehat{\mathbf{w}}) \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ \gamma_N^{1,1}\tilde{\pi}_{1,N}^{1'}(\widehat{\mathbf{w}}) & \dots & \gamma_N^{1,J}\tilde{\pi}_{1,N}^{J'}(\widehat{\mathbf{w}}) & \cdots & \gamma_N^{1,1}\tilde{\pi}_{N,N}^{1'}(\widehat{\mathbf{w}}) & \cdots & \gamma_N^{1,J}\tilde{\pi}_{N,N}^{J'}(\widehat{\mathbf{w}}) \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ \gamma_1^{J,1}\tilde{\pi}_{1,N}^{1'}(\widehat{\mathbf{w}}) & \dots & \gamma_N^{J,J}\tilde{\pi}_{1,N}^{J'}(\widehat{\mathbf{w}}) & \cdots & \gamma_N^{J,1}\tilde{\pi}_{N,N}^{1'}(\widehat{\mathbf{w}}) & \cdots & \gamma_N^{J,J}\tilde{\pi}_{N,N}^{J'}(\widehat{\mathbf{w}}) \end{pmatrix}_{JN\times JN},$$

where, $\tilde{\pi}_{in}^{k'}(\widehat{\mathbf{w}}) = \pi_{in}^{k'}(\widehat{\mathbf{w}})/(1+\tau_{in}^{k'})$.

Solving the model for tariff changes

- Let $\Omega(\widehat{\mathbf{w}}) = \mathbf{I} - \mathbf{F}(\widehat{\mathbf{w}}) - \tilde{H}(\widehat{\mathbf{w}})$.
- Then the vector $\mathbf{X}(\widehat{\mathbf{w}})$ can be solved by inverting the matrix $\Omega(\widehat{\mathbf{w}})$:

$$\mathbf{X}(\widehat{\mathbf{w}}) = \Omega^{-1}(\widehat{\mathbf{w}})\Delta(\widehat{\mathbf{w}}).$$

  where, denote by $X_n^{j\prime}(\widehat{\mathbf{w}})$ the entry j of the vector $\mathbf{X}(\widehat{\mathbf{w}})$ (the expenditure in sector j and country n.)
- This expression is crucial to solve for the general equilibrium, since it allows us to express all the equilibrium conditions as a function of one vector of unknowns: the vector of factor prices, $\widehat{\mathbf{w}}$.

- *Step 5* : Substitute $\pi^{j\prime}_{in}(\widehat{\mathbf{w}}), \mathbf{X}(\widehat{\mathbf{w}}), \tau', \textit{and } D'_n$ into equation (14) and obtain the equilibrium condition:

$$\sum_{j=1}^{J}\sum_{i=1}^{N}\frac{\pi^{j\prime}_{ni}(\widehat{\mathbf{w}})}{1+\tau^{j\prime}_{ni}}X^{j\prime}_{n}(\widehat{\mathbf{w}}) - D'_n = \sum_{j=1}^{J}\sum_{i=1}^{N}\frac{\pi^{j\prime}_{in}(\widehat{\mathbf{w}})}{1+\tau^{j\prime}_{in}}X^{j\prime}_{i}(\widehat{\mathbf{w}}). \qquad (C2)$$

  Note that the system of equilibrium conditions (equation (10) through (13)) have been reduced to a system of N equations (one trade balance per country) and N unknowns (one wage per country).

- *Step 6* : Verify if equation (C2) holds. If not, adjust the guess of $\widehat{\mathbf{w}}$ and proceed to step 1 again until equilibrium condition (C2) is obtained.

Solving the model for tariff changes

```matlab
function [wf0 pf0 PQ Fp Dinp ZW Snp c DP PF] = equilibrium_LC(tau_hat,taup,alphas,T,B,G,Din,J,N,maxit,tol,VAn,Sn,vfactor)

% initialize vectors of ex-post wage and price factors
 wf0      =  ones(N,1);  pf0      =  ones(J,N);

 wfmax = 1; e = 1;
while (e <= maxit) && (wfmax > tol);

[pf0 c] = PH(wf0,tau_hat,T,B,G,Din,J,N,maxit,tol);

% Calculating trade shares
Dinp = Dinprime(Din,tau_hat,c,T,J,N);
Dinp_om = Dinp./taup;

for j   = 1:1:J
irow    = 1+N*(j-1):1:N*j;
Fp(j,:) = sum((Dinp(irow,:)./taup(irow,:))');
end

% % Expenditure MATRIX
PQ = expenditure(alphas,B,G,Dinp,taup,Fp,VAn,wf0,Sn,J,N);

% Iterating using LMC
wf1 =LMC(PQ, Dinp_om, J,N,B,VAn);

%Excess function
ZW = (wf1-wf0);

PQ_vec   = reshape(PQ',1,J*N)'; % expenditures Xji in long vector: PQ_vec=(X11 X12 X13...)'

for n = 1:1:N
    DP(:,n)  = Dinp_om(:,n).*PQ_vec;
end
LHS = sum(DP)'; %exports

% calculating RHS (Imports) trade balance
PF = PQ.*Fp;
RHS = sum(PF)'; %imports

% excess function (trade balance)
Snp = (RHS - LHS + Sn);
ZW2 = -(RHS - LHS + Sn)./(VAn);
%Iteration factor prices
wf1 =wf0.*(1-vfactor*ZW2./wf0);

wfmax=sum(abs(wf1-wf0));
wfmax=sum(abs(Snp));

wfmax0 = wfmax;
wf0=(wf1);


e=e+1;
end
```