

Absorption and Scattering of Light by Small Particles

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To our families

Preface

When asked during the writing of this book what topic could divert us for so long from the pleasures of a normal life we would answer: “It is about how small particles absorb and scatter light.” “My goodness,” would be the response, “who could possibly be interested in that?” As it happens, scientists and engineers from a surprising variety of disciplines—solid-state physics, electrical engineering, meteorology, chemistry, biophysics, astronomy—make forays into this field, some never to escape. To completely satisfy such diverse groups, each with its peculiar conventions, notation, terminology, and canons, is an impossible task: physicists prefer the language of elementary excitations—phonons, plasmons, and all that; electrical engineers are more comfortable with the language of antennas and waveguides; chemists and biophysicists might not like either. We have therefore striven for the middle ground with the hope of, if not pleasing everyone, at least not antagonizing anyone. Ultimately, however, our point of view is that of physicists. Quantum-mechanical concepts are introduced where they serve to elucidate physical phenomena, but otherwise our approach is primarily classical.

Like so many other books, this one began its existence as lecture notes. Separately and jointly we have given lectures to graduate students and researchers with the kinds of diverse backgrounds and interests we expect our readers will have. Although more of an advanced monograph than a textbook, this book has a pedagogical flavor because of its origins. Indeed, many of the topics covered are in direct response to questions asked either in classrooms or by our colleagues.

There is one important idea, the *raison d'être* of this book, that we should like to implant firmly in the minds of our readers: scattering theory divorced from the optical properties of bulk matter is incomplete. Solving boundary-value problems in electromagnetic theory may be great fun and often requires considerable skill; but the full physical ramifications of mathematical solutions are hidden to those with little knowledge of how refractive indices of various solids and liquids depend on frequency, the values they take, and the constraints imposed on them. Accordingly, this book is divided into three parts.

Part 1, Chapters 1 through 8, is primarily scattering theory. After an introduction there is a chapter on those topics from electromagnetic theory essential to an understanding of the succeeding six chapters on exact and

approximate solutions to various scattering problems. Because uninterrupted strings of mathematical formulas tend to pall, computational and experimental results are interspersed throughout these chapters.

Bulk matter, rather than particles, is the subject of Part 2. In Chapter 9 we discuss classical theories of optical properties based on idealized models. Such models rarely conform strictly to reality, however, so Chapter 10 presents measurements for three representative materials over a wide range of frequencies, from radio to ultraviolet: aluminum, a metal; magnesium oxide, an insulator; and water, a liquid.

Part 3 is a marriage of Parts 1 and 2, the offspring of which are chapters on extinction (Chapter 11), surface modes (Chapter 12), and angular scattering (Chapter 13). Applications are not totally absent from the first thirteen chapters, but there is a greater concentration of them in Chapter 14.

We did not attempt an exhaustive list of references, even assuming that were possible. Instead, we concentrated on the years since publication of Kerker's book (1969), which cites nearly a thousand references. Even with this restriction we were selective, guided by our tastes rather than some ideal notion of completeness.

We avoided irritating statements such as "it can be shown"; while implying calm, they usually signal rough sailing ahead. Of course, we do not give all the details of lengthy derivations, but we do provide enough guideposts so that a reader can, with a bit of effort, duplicate our results. We always chose the simplest derivations, preferring physical plausibility over mathematical rigor. Those who demand the latter are reminded that one man's rigor is another man's mortis.

This book was not written with scissors: all derivations are our own, as are most of the figures, many of them generated with the computer programs in the appendixes. Even much of the experimental data was taken with an eye toward examples for the book. Any errors, therefore, are solely ours.

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January 1983*

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D.R.H.

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