Honors Math

About

This is a summery of **Honors Calculus** (117 + 118) (or as some people call it: "**Abstract Calculus**")
This course will focus on set theory and number theory (and logic)

* note this won't display properly on GitHub, I use a program called [Typora](#) with inline math enabled

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Notes

0. Shorthand

Ex. $\rightarrow$ example

Thm $\rightarrow$ Theorem (will be in block quotes)

s.t. $\rightarrow$ such that

Formal Logic

$\in \rightarrow$ element of

$\subset \ or \ \subseteq \ \rightarrow$ subset of

$\cup \rightarrow$ union

$\cap \rightarrow$ intercept

$\wedge \rightarrow$ and

$\vee \rightarrow$ or

$\rightarrow \ \rightarrow$ then

$\Longleftrightarrow \rightarrow$ if and only if (this is the same as '=')

$=: \rightarrow$ is defined to be

$\forall \rightarrow$ for all

$\exists \rightarrow$ there exists

$\exists! \rightarrow$ there exists unique

$Q.E.D. \rightarrow$ and it is proved

$\therefore \ \rightarrow$ therefore

$\because \ \rightarrow$ because

Fields

$\mathbb{N} \rightarrow$ natural numbers (i.e. 1,2,3,4...) [* note 0 might be included in natural numbers depending on what text book is in use]

$W \rightarrow$ whole numbers ($\{0\} \cup \mathbb{N}$)

$\mathbb{Z} \rightarrow$ integers (i.e. ..-2,-1,0,1,2...)

$\mathbb{Q} \rightarrow$ rational numbers (anything that can be expressed as $\frac{p}{q}, where \ p \in \mathbb{Z} \ and \ q \in \mathbb{N}$)

$\mathbb{R} \rightarrow$ will be discussed later

1. Sets

a set is defined as a number (can be 0 but then that would be the trivial or stupid set) of elements belonging to the same group

> Ex.
>
> - {1,2,3} = {2,1,3}
> - {a,b}
> - {even, odd}
> - $\mathbb{N}$ = {1,2,3,4,5...}
> - {} (trivial set)

If an element 'x' is part of a set 'A', it is described as $x \in A$

If $A \cup B = A,\ B \subset A$. If for all elements of B are in A, then B is considered a subset of A

If elements of A are present in B, then those elements 'C' are said to be $A \cap B$ or $A \cap B = C$
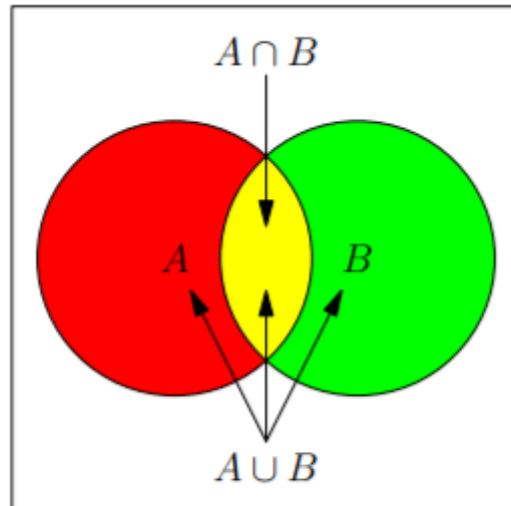

Figure 1.1: Venn Diagram

From Bowmen notes

2. Logic

Truth Values

There are 2 truth values, true (T or topology) and false (F or contradiction)

In math, there are established rules (which were set arbitrarily but is important for consistence around the world) for operations on truth values

Most of the time, a truth table will be used to illustrate how operators effect the truth value of a statement

Some common ones are:

$A \wedge B$; (A and B)

		A	A
		T	F
B	T	T	F
B	F	F	F

$A \vee B$; (A or B)

		A	A
		T	F
B	T	T	T
B	F	T	F

$A \rightarrow B$; (If A then B)

		A	A
		T	F
B	T	T	T
B	F	F	T

$A \iff B$; (A if and only if B or A = B)

* note this is a different way for displaying the truth table (important for chain multiple operators with more than 2 variables)

A	B	$A \iff B$	$A \to B \ and \ B \to A$
T	T	T	T
T	F	F	F
F	T	F	F
F	F	T	T

To chain multiple operators together, you can use an extended truth table that considers all the variables (like the one above, but you must consider all variables and all the cases possible use a)

As seen from the table above, an iff is logically equivalent to a if b and b if a, therefore for proving an iff, the easiest way is to prove it using the A premise to get to B and then using the B premise to derive A

Contraction

The easiest way to prove something is to use *proof by contradiction*. Assume the opposite of the theorem or a fact that is true and find a contraction in reasoning

> Ex.
>
> $\sqrt{2} \ is \ not \ rational$
>
> Start of proof by contraction:
>
>   Suppose for contradiction, $\sqrt{2} \ is \ rational$, i.e. it can be expressed as $\frac{p}{q}, where \ p \in \mathbb{Z} \ and \ q \in \mathbb{N}.$
>
> $$\sqrt{2} = \frac{p}{q}$$
> $$\to \sqrt{2}^2 = \frac{p^2}{q^2}$$
> $$2q^2 = p^2$$
>
> * note even number are expressed as 2n and odd number can be expressed as 2n+1 for $n \in \mathbb{Z}$
>
> Therefore p must be even because only an even squared is even $((2n)^2 = 4n^2 \ and \ (2n+1)^2 = 4n^2 + 2n + 1 \therefore odd)$

3. What is a number? (Algebraic Properties)

For real numbers, the following must be true

* assume $a, b, c \in \mathbb{R}$

1. Must be associative

$a + (b + c) = (a + b) + c$

2. Must have an additive identity

$a + 0 = 0 + a = a$

3. Must have an *additive inverse* -a s.t.

$a + -a = -a + a = 0$

4. Follows additive commutativity

$a + b = b + a$

5. Are associate

$a(bc) = (ab)c$

6. There exists are multiplicative identity where it is not 0

$a \times 1 = a$

7. It can distribute

$a \times (b + c) = a \times b + a \times c$

8. Has an inverse that is not 0, i.e. $a^{-1} \neq 0$

$a \times a^{-1} = 1$

9. Follows multiplicative commutativity

$a \times b = b \times a$

10. Trichotomy Law

must be one and only one of the following relations:

$a < b, \ a = b, \ a > b$

11. Closed under addition

$a > 0 \ and \ b > 0 \rightarrow a + b > 0$

12. Closed under multiplication

$a > 0 \ and \ b > 0 \rightarrow a \times b > 0$

More rules to come

Also note the first lemma (lemma is like a theorem but arbitrary defined to be a small theorem)

* lemmas or thm can be assumed

Lemma 1.1 (Midpoint Lemma):

$$a < b \quad \Rightarrow \quad a < \frac{a+b}{2} < b.$$

Proof:

$$a < b \quad \Rightarrow \quad a + a \; < \; a + b < b + b$$

$$\Rightarrow \quad a = \frac{a+a}{2} \; < \; \frac{a+b}{2} < \frac{b+b}{2} = b.$$

4. Induction

Steps

1. Prove for any element in set (usually 1 or 0 if they are in the set)
2. Suppose true for n
3. Prove for n+1

==Notice==: This makes it so it holds true for the first tested element and for every subsequent element (like a domino effect)

> Ex.
>
> Gauss' claim:
>
> $$1 + 2 + \ldots + n \equiv \sum_{i=1}^{n} i = \frac{n(n+1)}{2}$$
>
> $$Let\ S\ be\ set\ n$$
> $$Step\ 1: \ Check\ 1 \in S$$
> $$1 = \frac{1(1+1)}{2} = 1$$
> $$Step\ 2: \ Suppose\ k \in S$$
> $$\therefore \sum_{i=1}^{n} i = \frac{k(k+1)}{2}$$
> $$Then\ prove\ k + 1$$
>
> $$\sum_{i=1}^{k+1} i = \frac{k(k+1)}{2} = 1 + 2 + 3 + \ldots + k + (k+1)$$
>
> $$= \frac{k(k+1)}{2} + (k+1)$$
>
> $$= (k+1)\left(\frac{k}{2} + 1\right)$$
>
> $$= \frac{(k+1)((k+1)+1)}{2}$$
>
> Hence $k + 1 \in S$ or $k \in S \to k + 1 \in S$
>
> * see Bowmen notes (starting from page 19) for more examples

5. Absolute Values

Resources

- school notes
- [Bowmen notes](#)

Note about this course and my experience

My school had an extraordinary honors calculus year. The main prof left by second semester and a German prof can in to sub, but, because of his Germanicness (he literally goes off of a German textbook), it was difficult to follow his sometimes rigorous and sometimes not rigorous (he called them trivial) proofs.

Also, at my university, compute 272 is trivially similar to 117.

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Contributions

Contribute if and only if you know what you are doing (and optionally, it would help if you where Germanic or fluent in Germanics)

* If someone wants to convert this to Latex, go ahead and pull request (must have a index or contents page with links). I tried with pandoc, but it was not as complete as I thought so if anyone wants to do clean up, go ahead, also I can just export as pdf